

SETS

## Building sets

Set builder notation:

$$\begin{aligned}(0, 1] &= \{x \in \mathbb{R} \mid 0 < x \text{ and } x \leq 1\} \\ &= \{x \in \mathbb{R} : 0 < x \text{ and } x \leq 1\}\end{aligned}$$

$$\begin{aligned}E &= \{\text{all even numbers}\} \\ &= \{2n : n \in \mathbb{Z}\}\end{aligned}$$

Some common sets (NO: menge)

$$\mathbb{N} = \{1, 2, 3, \dots\}$$

natural numbers

$$\mathbb{N}_0 = \{0, 1, 2, 3, \dots\}$$

nonnegative integers

$$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$$

integers (NO: heftall)

$$\mathbb{Q} = \{p/q : p \in \mathbb{Z}, q \in \mathbb{N}\}$$

rational numbers

$$\mathbb{R} = \{\text{all real numbers}\}$$

$$\emptyset = \{\}$$

the empty set

$$\mathbb{R}^n = \{(x_1, \dots, x_n) : x_1, \dots, x_n \in \mathbb{R}\}$$

Euclidean space

## Comparing sets

We can compare two sets  $A, B$ :

$A \subset B$

$A$  is a subset (NO: delmenge) of  $B$

Means that  $x \in A \Rightarrow x \in B$  (Also written  $A \subseteq B$ )

$A = B$

$A$  equals  $B$

Means that both  $A \subset B$  and  $B \subset A$

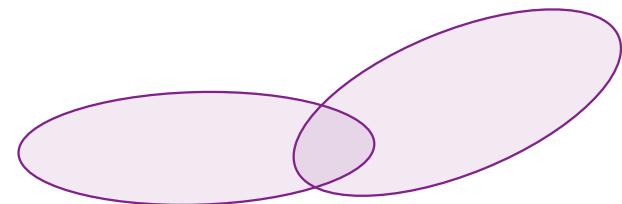
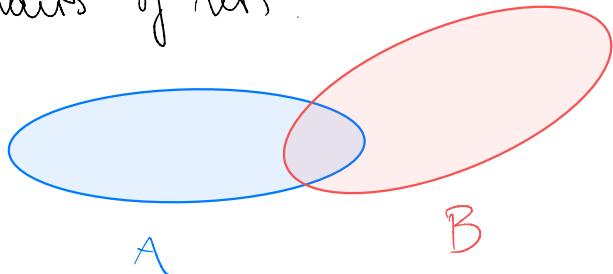
$A \subsetneq B$

$A$  is a proper subset (NO: ekte delmenge) of  $B$

Means that  $A \subset B$ , and that  $\exists x \in B$  such that  $x \notin A$

## Binary operations

We can perform operations on pairs of sets.



$$\begin{aligned} A \cup B &= \text{union of } A \text{ and } B \\ &= \{x : x \in A \text{ or } x \in B\} \end{aligned}$$



$$\begin{aligned} A \cap B &= \text{intersection (NO: null) of } A, B \\ &= \{x : x \in A \text{ and } x \in B\} \end{aligned}$$



$$\begin{aligned} A \setminus B &= \text{difference of } A \text{ and } B \\ &= \{x \in A : x \notin B\} \end{aligned}$$



A



B

A and B are disjoint

(NO: disjunkte) if  $A \cap B = \emptyset$

$A^c$

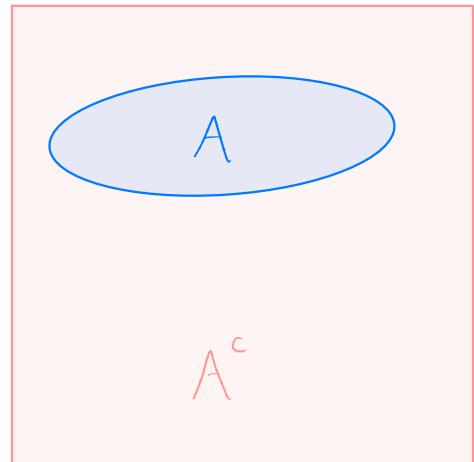
is the complement (NO: Komplement)  
of A - everything not in A:

$$A^c = \{x : x \notin A\}.$$

$A^c$  is sometimes written as  $\bar{A}$ .

Note: It is clear from the context what  
"everything" is. If  $A \subset \mathbb{R}$  then

also  $A^c \subset \mathbb{R}$ .



$A \times B$

is the Cartesian product (NO: kartesisch produkt) of A, B:

$$A \times B = \{(x, y) : x \in A, y \in B\}.$$

$A^2$

$$= A \times A$$

Thus,  $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$

$$\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$$

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QUESTIONS ?

COMMENTS ?