COMPUTING WITH SETS

Unions and intersections

$\boxed{A \cup B}$ = union of A and B
$= \{x : x \in A \text{ or } x \in B\}$

$\boxed{A \cap B}$ = intersection (NO: snitt) of A, B
$= \{x : x \in A \text{ and } x \in B\}$

Given n sets $A_1, A_2, \dots, A_n$, we have:

$$A_1 \cup A_2 \cup \cdots \cup A_n = \bigcup_{k=1}^{n} A_k = \{x : x \text{ lies in at least one of the sets } A_1, \dots, A_n\}$$

$$A_1 \cap A_2 \cap \cdots \cap A_n = \bigcap_{k=1}^{n} A_k = \{x : x \text{ lies in all of the sets } A_1, \dots, A_n\}$$

Unions and intersections are associative:

$$\left(\bigcup_{k=1}^{n} A_k\right) \cup B = \bigcup_{k=1}^{n} A_k \cup B \qquad \left(\bigcup_{k=1}^{n} A_k\right) \cap B = \bigcup_{k=1}^{n} A_k \cap B$$

$$\left(\bigcap_{k=1}^{n} A_k\right) \cup B = \bigcap_{k=1}^{n} A_k \cup B \qquad \left(\bigcap_{k=1}^{n} A_k\right) \cap B = \bigcap_{k=1}^{n} A_k \cap B$$

<u>Proof</u>: Exercise!

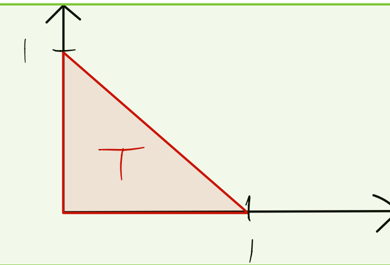
We can also take unions/intersections of larger collections of sets:

Definition: A <u>family of sets</u> is a collection of sets A_k indexed over a set $k \in I$

$$\bigcup_{k \in I} A_k = \{x : x \text{ lies in at least one of the sets } A_k \text{ (for } k \in I)\}$$

$$\bigcap_{k \in I} A_k = \{x : x \text{ lies in all of the sets } A_k \text{ (for } k \in I)\}$$

Example: Write T as a union of line segments



$$T = \{(x,y) : 0 \le y \le 1-x, \quad 0 \le x \le 1\}$$

$$= \bigcup_{x \in [0,1]} \{(x,y) : 0 \le y \le 1-x\}$$

$$= \bigcup_{x \in [0,1]} \{x\} \times [0, 1-x]$$

The associative laws still hold:

$$\left(\bigcup_{k\in I} A_k\right) \cup B = \bigcup_{k\in I} A_k \cup B$$

$$\left(\bigcup_{k\in I} A_k\right) \cap B = \bigcup_{k\in I} A_k \cap B$$

$$\left(\bigcap_{k\in I} A_k\right) \cup B = \bigcap_{k\in I} A_k \cup B$$

$$\left(\bigcap_{k\in I} A_k\right) \cap B = \bigcap_{k\in I} A_k \cap B$$

De Morgan's laws

What is the complement of a union/intersection of sets?

$$(A \cup B)^c = A^c \cap B^c \qquad (A \cap B)^c = A^c \cup B^c$$

For families of sets:

$$\left(\bigcup_{k \in I} A_k\right)^c = \bigcap_{k \in I} A_k^c \qquad \left(\bigcap_{k \in I} A_k\right)^c = \bigcup_{k \in I} A_k^c$$

QUESTIONS ?

COMMENTS ?