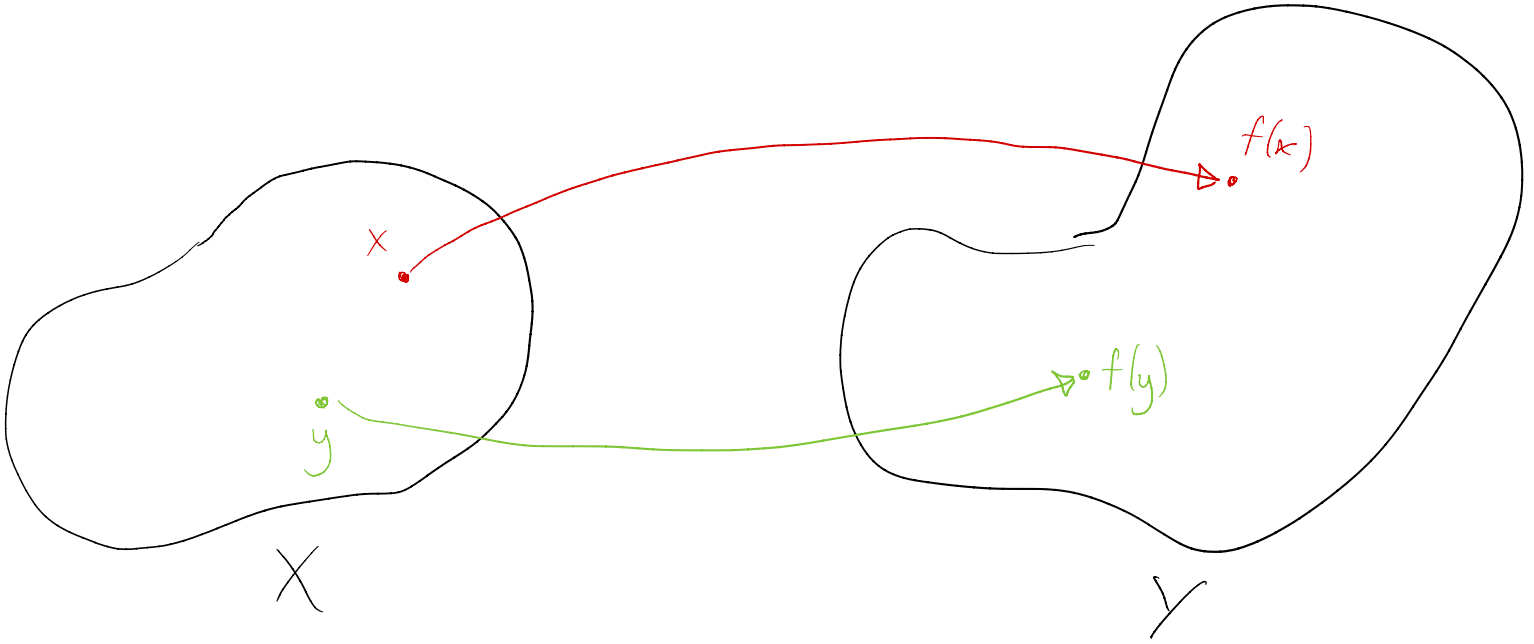
FUNCTIONS

A function maps objects from one set (its domain) to another set (its codomain or image)



A function can be defined in different ways:

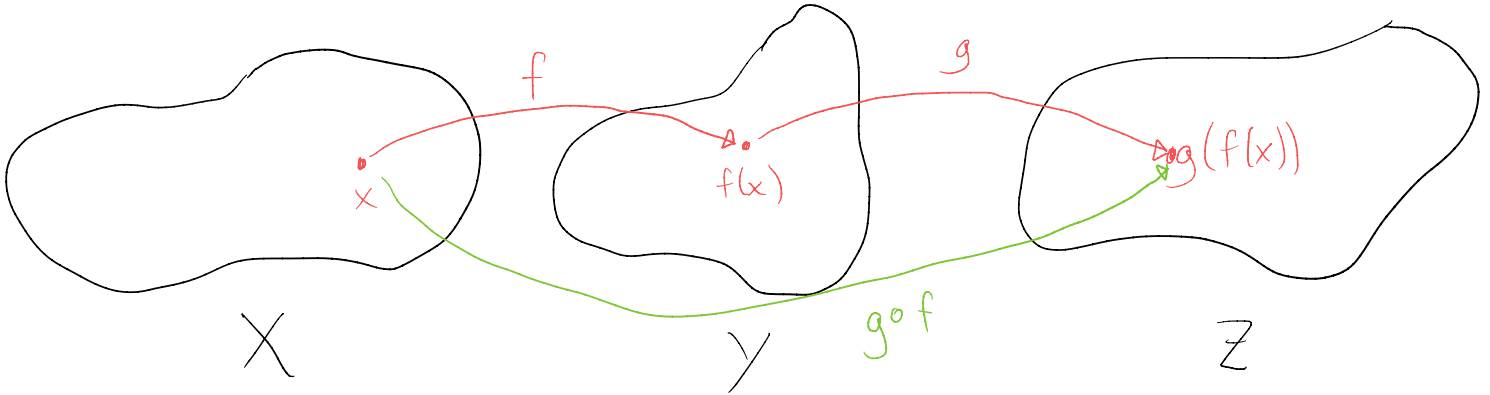
f is the function which squares real numbers

$f(x) = x^2 \quad \text{for} \quad x \in \mathbb{R}$

$f = \left(x \longmapsto x^2, \quad x \in \mathbb{R}\right)$

$\longmapsto$ is pronounced "maps to" (NO: avbildes til)

Composition of functions



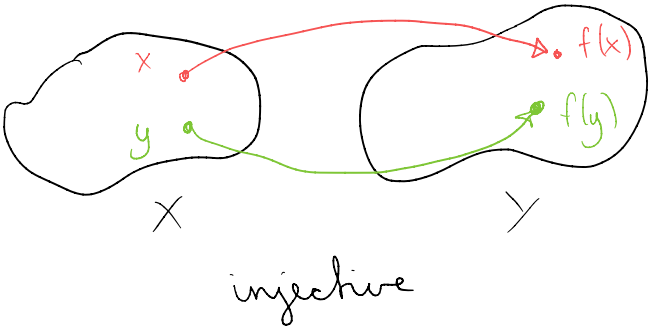
Functions $f: X \to Y$ and $g: Y \to Z$ can be composed:

$$g \circ f = \left(x \longmapsto g(f(x)), \quad x \in X \right)$$

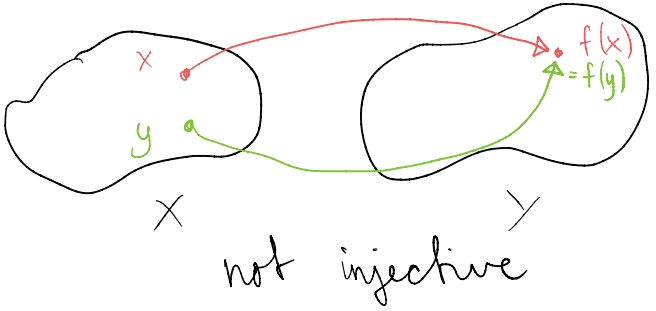
(NO: sammensetting / komposisjon)

Injective, surjective, bijective

$f: X \to Y$ is injective if $f(x) \neq f(y)$ whenever $x \neq y$



injective



not injective

Injective:

$f(x) = 2x$ for $x \in \mathbb{R}$

$f(x) = x^2$ for $x \geq 0$

$f(x) = \sin(x)$ for $x \in [0, \pi/2]$

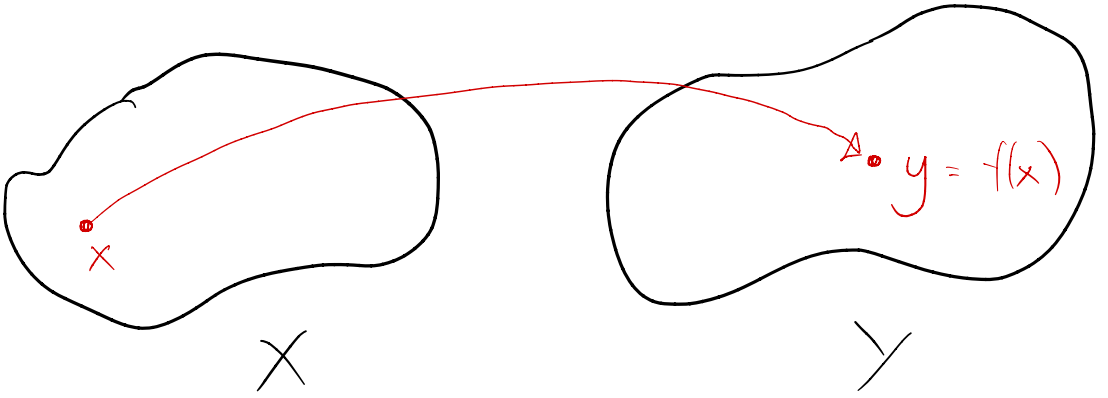
Not injective:

$f(x) = x^2$ for $x \in \mathbb{R}$

$f(x) = \sin(x)$ for $x \in [0, \pi]$

Injective, surjective, bijective

$f: X \to Y$ is surjective if for every $y \in Y$, there is some $x \in X$ such that $f(x) = y$.



Injective, surjective, bijective

$f: X \to Y$ is surjective if for every $y \in Y$, there is some $x \in X$ such that $f(x) = y$.

Surjective

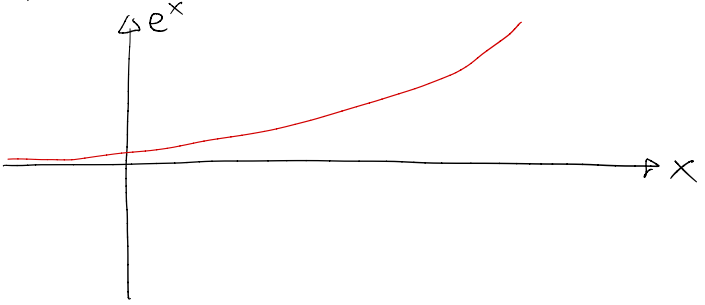
$f: \mathbb{R} \to \mathbb{R}, \quad f(x) = 2x$

$f: \mathbb{R} \to (0, \infty), \quad f(x) = e^x$

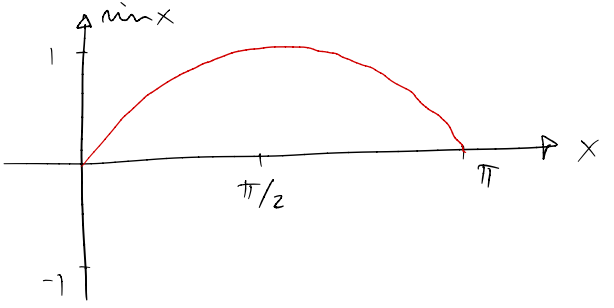
$f: \mathbb{R} \to [-1, 1], \quad f(x) = \cos(x)$

Not surjective:

$f: \mathbb{R} \to \mathbb{R}, \quad f(x) = e^x$



$f: [0, \pi] \to [-1, 1], \quad f(x) = \sin(x)$



Injective, surjective, bijective

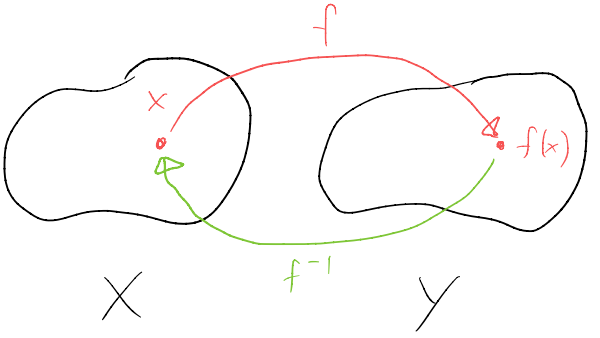
$f: X \to Y$ is bijective if it is both injective and surjective

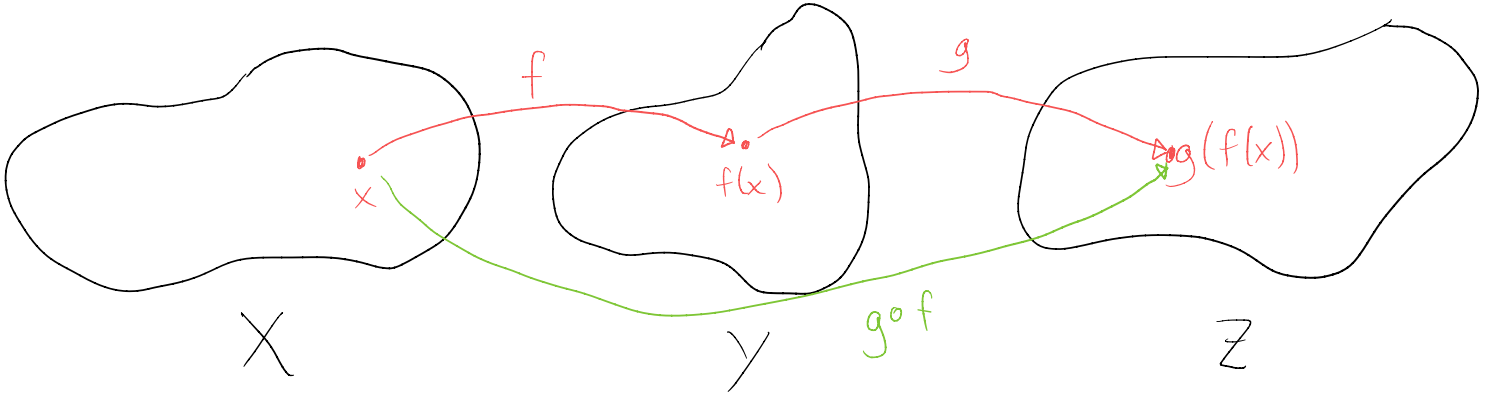
Every bijective function $f: X \to Y$ has an inverse function $f^{-1}: Y \to X$:

$f(f^{-1}(y)) = y$, $f^{-1}(f(x)) = x$

equivalently:

$f: X \to Y$ is bijective if for every $y \in Y$ there is one, and only one, $x \in X$ such that $f(x) = y$.



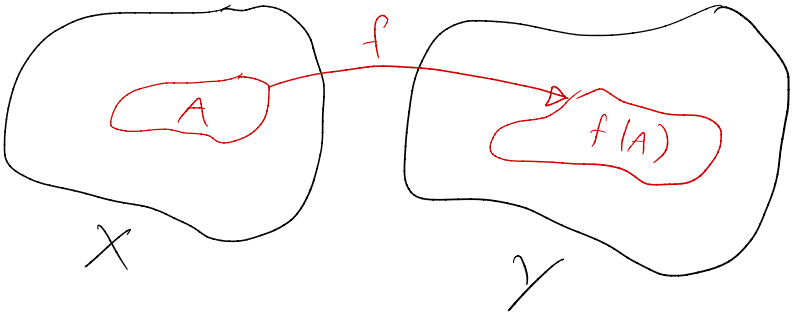


If $f: X \to Y$ and $g: Y \to Z$ are bijective then so is $g \circ f$, and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

Images and preimages

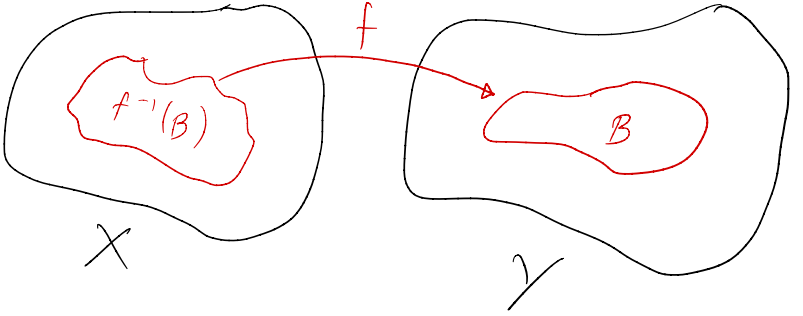
Given $f: X \to Y$ and $A \subset X$, the image of A (under f) is

$$f(A) = \{ f(x) : x \in A \}$$



The preimage of $B \subset Y$ (under f) is

$$f^{-1}(B) = \{ x \in X : f(x) \in B \}$$



Proposition:

$$f^{-1}\left(\bigcup_{k\in I} B_k\right) = \bigcup_{k\in I} f^{-1}(B_k)$$

$$f^{-1}\left(\bigcap_{k\in I} B_k\right) = \bigcap_{k\in I} f^{-1}(B_k)$$

$$f^{-1}(B^c) = \left(f^{-1}(B)\right)^c$$

$$f\left(\bigcup_{k\in I} A_k\right) = \bigcup_{k\in I} f(A_k)$$

$$f\left(\bigcap_{k\in I} A_k\right) \subseteq \bigcap_{k\in I} f(A_k), \text{ with equality if } f \text{ is injective.}$$

QUESTIONS ?

COMMENTS ?