

MAT 2400: Mandatory assignment 2, S-15'

Deadline: You must turn in your paper before 2.30 p.m., Thursday, April 30., 2015, in the designated area on the 7th floor of NHA. Remember to use the official front page available on the 7th floor and at

<http://www.mm.uio.no/math/studier/admin/obligatorisk-innlevering/obligforside.pdf>

If you due to illness or other circumstances want to extend the deadline, you must apply for an extension to studieinfo@math.uio.no Remember that illness has to be documented by a medical doctor! See

<http://www.mm.uio.no/math/studier/admin/obligatorisk-innlevering/index.html>

for more information about the rules for mandatory assignments.

Instructions: The assignment is compulsory, and students who do not get their paper accepted, will not get access to the final exam. To get the assignment accepted, you need a score of at least 60%. In the evaluation, credit will be given for a clear and well-organized presentation. All questions (points 1a), 1b) etc.) have equal weight. Students who do not get their original paper accepted, but who have made serious and documented attempts to solve the problems, will get one chance of turning in an improved version.

In solving the problems you may collaborate with others and use tools of all kinds. However, the paper you turn in should be written by you (by hand or computer) and should reflect your understanding of the material. If we are not convinced that you understand your own paper, we may ask you to give an oral presentation.

Problem 1. Let (X, d) be a bounded metric space, and let $P(X)$ denote the collection of non-empty closed subsets of X . For A and B in $P(X)$, let

$$h(A, B) = \sup_{x \in X} |\text{dist}(x, A) - \text{dist}(x, B)|,$$

where $\text{dist}(x, C)$ is given by

$$\text{dist}(x, C) = \inf_{c \in C} d(x, c).$$

The function h is called *the Hausdorff metric*.

a) Show that if $h(A, B) = 0$ then $A = B$. Here A and B are two non-empty closed subsets of X .

b) Show that h is a metric on $P(X)$.

c) For A and B in $P(X)$, let \hat{h} be defined as

$$\hat{h}(A, B) = \max \left\{ \sup_{a \in A} \text{dist}(a, B), \sup_{b \in B} \text{dist}(b, A) \right\}.$$

Show that

$$\hat{h}(A, B) = h(A, B) \quad \text{for all } A, B \text{ in } P(X).$$

(**Hint:** Show the two inequalities $h(A, B) \geq \hat{h}(A, B)$ and $\hat{h}(A, B) \geq h(A, B)$.)

Problem 2.

a) Let $0 < r < 1$ and consider the series

$$\sum_{n=-\infty}^{\infty} r^{|n|} e^{inx}.$$

Show that this series converges uniformly for all $x \in \mathbb{R}$, and that its sum equals

$$P_r(x) = \frac{1 - r^2}{1 - 2r \cos(x) + r^2}.$$

b) Show that

$$(1) \quad P_r(x) \geq 0 \quad \text{for all } x.$$

c) Show that for every $\delta > 0$, $\delta < \pi$, $P_r(x) \rightarrow 0$ uniformly on the intervals $[-\pi, -\delta] \cup [\delta, \pi]$, as $r \uparrow 1$.

d) Show that

$$(2) \quad \int_{-\pi}^{\pi} P_r(x) dx = 2\pi.$$

e) Let f be a continuous 2π periodic function. Show that

$$\lim_{r \uparrow 1} \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x-y) P_r(y) dy = f(x).$$

f) Assume that f has Fourier series

$$\sum_{n=-\infty}^{\infty} c_n e^{inx},$$

show that

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} f(x-y) P_r(y) dy = \sum_{n=-\infty}^{\infty} c_n r^{|n|} e^{inx},$$

and that the series converges absolutely and uniformly. (**Hint:** Show that the function on the left is differentiable in x)

g) Show that

$$\lim_{r \uparrow 1} \sum_{n=-\infty}^{\infty} c_n r^{|n|} e^{inx} = f(x),$$

uniformly in x .