

October 11<sup>th</sup>, 2018

# MAT2410

## Mandatory assignment 2 of 2

### Submission deadline

Thursday October 24<sup>th</sup> 2019, 14:30 in Canvas

### Instructions

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with  $\text{\LaTeX}$ ). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. Students who fail the assignment, but have made a genuine effort at solving the exercises, are given a second attempt at revising their answers. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

In exercises where you are asked to write a computer program, you need to hand in the code along with the rest of the assignment. It is important that the submitted program contains a trial run, so that it is easy to see the result of the code.

### Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: [studieinfo@math.uio.no](mailto:studieinfo@math.uio.no)) well before the deadline.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

### Complete guidelines about delivery of mandatory assignments:

[uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html](http://uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html)

GOOD LUCK!

**IMPORTANT REMARKS:** Requirement to pass: 50%. You need to write complete arguments for the answers. Answers such as "The integral is -1", **WITHOUT** sufficient arguments to support, will **NOT** be counted.

**Problem 1.** (25%)

The velocity field of a fluid in  $\mathbb{C} \setminus \{0\}$  is given by

$$\mathbf{V}(x, y) = \left( \frac{-2xy}{(x^2 + y^2)^2}, \frac{x^2 - y^2}{(x^2 + y^2)^2} \right)$$

- Show that the fluid is incompressible and irrotational.
- Find a complex velocity potential. What is the flux of  $\mathbf{V}$  across the straight line from  $2i$  to  $1$  in the direction from right to left ?
- Draw the flowlines through  $2$  and  $-2$ , indicating the direction of the flow.

**Problem 2.** (15%) Compute  $(\frac{3}{4} - \frac{1}{4}i)^2$ . Let  $\gamma$  be the straight line from  $0$  to  $(\frac{3}{4} + \frac{1}{4}i)$ . Find

$$\int_{\gamma} \frac{dz}{\sqrt{1 - z^2}}$$

where we use the principal branch of the square root. Express the answer in the form  $a + bi$ .

**Problem 3.** (15%) a) Compute

$$\int_{|z|=1} \frac{dz}{z^2 + 2z}$$

b) Compute

$$\int_{|z|=2} \frac{\sin(z) dz}{z^3 - z^2}$$

**Problem 4.** (20%)

a) What type of singularity does

$$f(z) = \frac{1 - \cos(z)}{\sin(z^2)}$$

have at  $z = 0$ ?

- Compute the first three terms in the power series of  $f$  at  $0$ .
- What is the radius of convergence of this power series ?

**Problem 5.** (25%) Let  $f(z) = \frac{1}{z^3 - z^2}$ .

- Classify the singularities of  $f$ . What type of point is  $\infty$  for  $f$ ?
- Compute the Laurent series centered at the origin for the domains  $D_1 = \{0 < |z| < 1\}$  and  $D_2 = \{1 < |z|\}$ .
- Compute the Laurent series centered at  $z = -1$  in the annulus  $1 < |z + 1| < 2$ .