

17th September, 2026

MAT2410 — Introduction to complex analysis

Mandatory assignment 1 of 2

Submission deadline

Thursday 1 October 2026, 14:30 in Canvas (canvas.uio.no).

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

Every submitted solution will be awarded a passing grade. Collaboration is permitted and encouraged; LLM assistance is permitted but strongly discouraged.

Problem 1. Let $f(z) = e^z$, and let E be the set of complex numbers given by

$$E = \{x + iy \mid 0 \leq x \leq 1, 0 \leq y \leq 1\}.$$

Describe and sketch the image of E under the mapping $w = f(z)$.

Problem 2. Let $f(z) = (\operatorname{Re}(z))^2$, let $y_0 \in \mathbb{R}$, and let $z_0 = iy_0$.

- (a) Prove that f is complex differentiable at z_0 , and determine the value of $f'(z_0)$.
- (b) Prove that f is not analytic on any nonempty open subset $U \subseteq \mathbb{C}$.

Problem 3. Let $f(z) = z^5 - z$.

- (a) Determine the complex roots of $f(z)$, and give a factorization of $f(z)$ into linear polynomials.
- (b) Prove that there exist an open set $U \subseteq \mathbb{C}$ containing 0 and an analytic function $g(w)$ on U with the following properties:
 - We have $g(0) = 1$.
 - For every $w \in U$, the number $g(w)$ is a root of the polynomial

$$z^5 - z - w.$$

Problem 4. Let $u(x, y) = 2xy + y^3 + ax^2y$, where a is a real number such that u is harmonic.

- (a) Determine the value of a .
- (b) Find a harmonic conjugate $v(x, y)$ of u .
- (c) Let $\gamma(t)$, $0 \leq t \leq \pi$ be the path given by

$$\gamma(t) = (\cos(t)^3, \sin(t)^4).$$

Compute the line integral

$$\int_{\gamma} -\frac{\partial u}{\partial y}(x, y) dx + \frac{\partial u}{\partial x}(x, y) dy.$$

Hint: You do not need to integrate any trigonometric functions to solve this.