

15th October, 2026

MAT2410 — Introduction to complex analysis

Mandatory assignment 2 of 2

Submission deadline

Thursday 29 October 2026, 14:30 in Canvas (canvas.uio.no).

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

You may answer in Norwegian or English. Every submitted solution will be awarded a passing grade. Collaboration is permitted and encouraged; LLM assistance is permitted but strongly discouraged.

Problem 1. Let $f(z) = \frac{e^z}{(1-z)^2}$, and let $\sum_{k=0}^{\infty} a_k z^k$ be the power series expansion of f around 0.

- (a) Determine a_0 , a_1 and a_2 .
- (b) Determine the radius of convergence of the power series.
- (c) Compute

$$\int_{|z|=2} f(z) dz,$$

where the path around the circle $|z| = 2$ is oriented counterclockwise.

Problem 2. Let $D \subseteq \mathbb{C}$ be a bounded domain with piecewise smooth boundary. Let f be analytic on D , extending smoothly to $\overline{D}$.

- (a) Show that if f is constant on ∂D , then f is constant on $\overline{D}$.
- (b) Show that if $f(z) \neq 0$ for all $z \in \partial D$ and

$$\int_{\partial D} \frac{1}{f(z)} dz \neq 0,$$

then f has a zero in D .

Problem 3. Let $\alpha \in \mathbb{C}$. For each integer $k \geq 1$, let

$$\binom{\alpha}{k} = \frac{\alpha(\alpha-1)\dots(\alpha-k+1)}{k!},$$

and let

$$\binom{\alpha}{0} = 1.$$

Consider the power series

$$f(z) = \sum_{k=0}^{\infty} \binom{\alpha}{k} z^k.$$

- (a) Determine the radius of convergence of the power series; the answer will depend on whether $\alpha \in \mathbb{Z}_{\geq 0}$ or not.

- (b) Recall that for $w \in \mathbb{C} \setminus \mathbb{R}_{\leq 0}$ we define the principal branch P_α of the α -th power function w^α by

$$P_\alpha(w) = e^{\alpha \operatorname{Log}(w)},$$

where $\operatorname{Log}(w)$ denotes the principal branch of the logarithm. Show that

$$P'_\alpha(w) = \alpha P_{\alpha-1}(w)$$

for all $w \in \mathbb{C} \setminus \mathbb{R}_{\leq 0}$.

- (c) Determine $P_\alpha^{(k)}(1)$ for all $k \geq 0$ and use this to show that for $|z| < 1$, we have

$$f(z) = P_\alpha(1+z).$$

Problem 4. Let $f(z) = \frac{e^z}{z}$, let $r > 0$, and let γ_r be the path

$$\gamma_r(t) = -r \sin t + ir \cos t, \quad 0 \leq t \leq \pi.$$

Show that

$$\left| \int_{\gamma_r} f(z) dz \right| \leq \pi.$$

Problem 5. For each of the hypothetical function descriptions below, either give an example of a function satisfying the description, or prove that no such function can exist.

- (a) A function which is defined and analytic on $\mathbb{C} \setminus \mathbb{Z}$, but does not admit an analytic extension to any point of $\mathbb{Z}$.
- (b) An entire function such that $|f^{(k)}(0)| \geq k!$ for all $k \geq 0$.
- (c) An entire function such that $f\left(\frac{1}{n}\right) = \frac{1}{2-1/n}$ for all integers $n \geq 1$.