

UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Exam in: MAT2410 — Introduction to Complex Analysis

Day of examination: Friday December 11th 2015

Examination hours: 14:30–18:30

This problem set consists of 2 pages.

Appendices: None

Permitted aids: None

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1

Consider the rational function

$$f(z) = \frac{z^2 + z - 2}{z^2(z+2)^2}.$$

a

Find all the poles of f and determine their orders.

b

Find the residue of f at each pole.

c

Let γ be the circle of radius 1 centered at 0 with positive orientation. Compute the integral

$$\int_{\gamma} f(z) dz.$$

Problem 2

Let R be a real number with $R > 1$. Let

$$f(z) = \frac{1}{z^2}.$$

a

Consider the line parameterized by $\alpha(t) = t + i$. Write $f(\alpha(t))$ in the form $u(t) + iv(t)$ where u and v are real valued functions.

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b

Let Ω_R be the closed region defined by $\operatorname{Re} z \geq 0$, $\operatorname{Im} z \geq 1$ and $|z| \leq R$. Let γ_R be the boundary with positive orientation. Sketch Ω_R and find $\int_{\gamma_R} f(z) dz$.

c

Let C_R be the circle arc part of γ_R . Show that $\int_{C_R} f(z) dz \rightarrow 0$ when $R \rightarrow \infty$.

d

Compute the real integral

$$\int_0^\infty \frac{x^2 - 1}{(x^2 + 1)^2} dx.$$

Problem 3

Show that the polynomial $z^7 - 4z^3 + z - 1$ has 3 zeroes counted with multiplicity inside the disc $|z| < 1$.

Problem 4

Let $\mathbb{P}$ be the Riemann sphere or extended complex plane $\mathbb{C} \cup \{\infty\}$. Consider the Möbius transformation $f : \mathbb{P} \rightarrow \mathbb{P}$ given by

$$f(z) = \frac{z - 1}{z + 1}.$$

a

Find $f(\infty)$ and all the fixed points of f .

b

Find the Möbius transformation g with the property $g(-i) = -i$, $g(i) = i$ and $g(1) = \infty$.

END