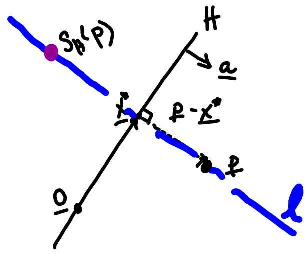
MAT2500 08.09.2020

OPPG. 1 - 6



b) $\|p-x^*\| = \frac{|p\cdot a|}{\|a\|}$ (sette inn og regne)

H har normalvektor a

$p = \frac{\langle p, x^*\rangle}{\|x^*\|^2}\cdot x^* + \frac{\langle p, a\rangle}{\|a\|^2}\cdot a$ (s. 9 for ortonormale)

$= \underbrace{\frac{\langle p, x^*\rangle}{\|x^*\|}\cdot\frac{x^*}{\|x^*\|}}_{\text{langs } H} + \underbrace{\frac{\langle p, a\rangle}{\|a\|}\cdot\frac{a}{\|a\|}}_{\text{langs } \perp H}$

$\frac{\langle p, a\rangle}{\|a\|}$ gir oss avstanden fra p til H.

c) $\ell \perp H$

$\ell = \{t\cdot a + p : t \in \mathbb{R}\}$

retning a — gjennom p

$S_H(p)$ på ℓ, motsatt av p, $d(S_H(p), x^*) = d(p, x^*)$

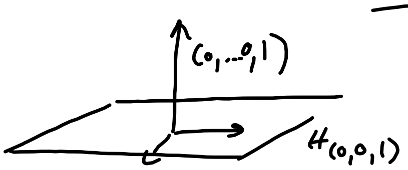
Skal vise: $S_H^2 = id$ $\quad S_H(S_H(p)) \in \ell$, $d(S_H(S_H(p)), x^*) = d(S_H(p), x^*)$

men da må $S_H(S_H(p)) = p$ (S_H er ikke id selv.)

d) Formel for $S_H(p)$

$S_H(p) = p + t\cdot\frac{a}{\|a\|}$ for en $t\in\mathbb{R}$

$= p - \frac{2|p\cdot a|}{\|a\|}\cdot\frac{a}{\|a\|}$



Har $a = (0,\dots,0,1)$

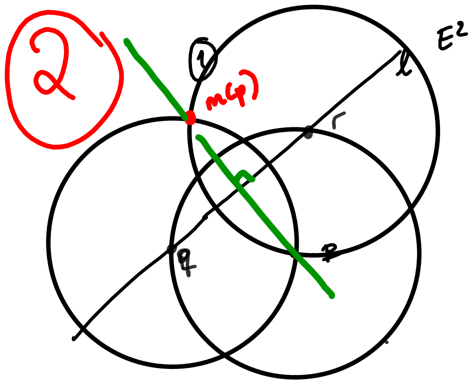
$S_H(p) = p - 2p_n\cdot(0,\dots,0,1)$

$= (p_1,\dots,p_n) - (0,\dots,0,-2p_n)$

$= (p_1,\dots,p_{n-1},-p_n)$

$= \begin{pmatrix} 1 & & & 0 \\ & 1 & & \\ & & \ddots 1 & \\ 0 & & & -1 \end{pmatrix}\cdot\begin{pmatrix} p_1 \\ \vdots \\ p_n\end{pmatrix}$

S om x-aksen $\quad S = \begin{pmatrix} 1 & 0 \\ 0 & -1\end{pmatrix}$

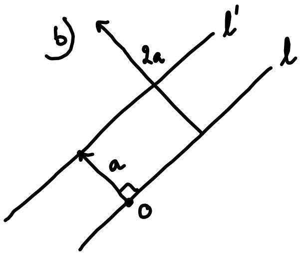


Alle pkt på l er fikspkt for m
Da er $m = s_l$

2) a) $t_a \circ t_b(x) = t_a(x+b)$
$= x + b + a$
$= t_{a+b}(x)$

$\underline{t_a \circ t_b = t_{a+b}}$

b)



Skal vise
$\underline{t_a \circ s_l \circ t_{-a} = s_{l'}}$

Har at $s_l(a) = -a$

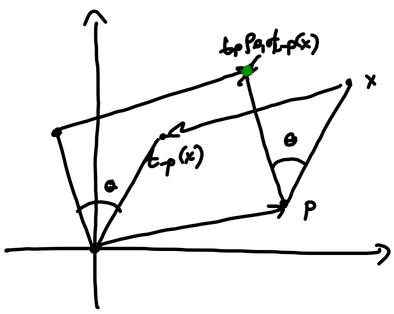
$t_a \circ s_l \circ t_{-a}(x) = t_a \circ s_l(x-a)$
$= t_a(s_l(x) - s_l(a))$
$= s_l(x) - (-a) + a$
$= s_l(x) + 2a$
$= t_{2a}\, s_l(x)$
$= s_{l'}(x)$

$t_a \circ s_l \circ t_a = s_l$
$t_a \circ s_l \circ t_a(x) = t_a \circ s_l(x+a)$
$= t_a(s_l(x) - a)$
$= s_l(x) - a + a$
$= s_l(x)$.

t_{-a} $\underline{s_l \circ t_a = t_{-a} \circ s_l}$

Setn 2.4.
$l \parallel l'$ med $\perp$ avst $\frac{1}{2} w_a$

c) $t_p \circ \rho_{\theta,0} \circ t_{-p} = \rho_{\theta,p}$



$t_p \circ \rho_{\theta,0} \circ t_{-p}(x) = t_p \circ \rho_{\theta,0}(x-p)$
$= t_p(\rho_{\theta,0}(x) - \rho_{\theta,0}(p))$
$= \rho_{\theta,0}(x) - \rho_{\theta,0}(p) + p$
$= t_{p-\rho_{\theta,0}(p)}\, \rho_{\theta,0}(x)$
er or. bev.
$t_{p-\rho_{\theta,0}(p)}\rho_{\theta,0}(p) = \rho_{\theta,0}(p) - \rho_{\theta,0}(p) + p = p$
så p er et fikspunkt
$= \rho_{\theta,p}(x)$

3) $\triangle ABC$ og $\triangle A'B'C'$ kongruente i $\mathbb{R}^2$

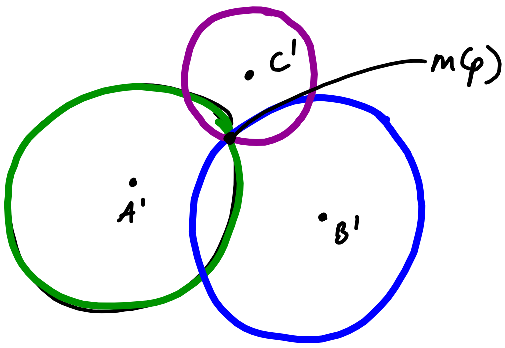
a) Skal vise m isometri s.a. $m(A)=A'$, $m(B)=B'$, $m(C)=C'$

$$m: \mathbb{R}^2 \to \mathbb{R}^2$$
$$A \mapsto A'$$
$$B \mapsto B'$$
$$C \mapsto C'$$

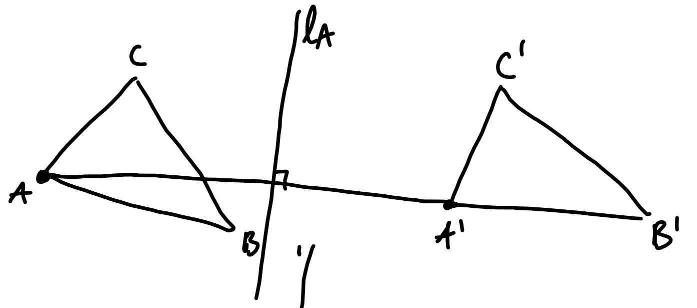
- Siden $\triangle$ er kongruente
$$d(A,B) = d(A',B') = d(m(A),m(B))$$
osv.
så m bevarer avstand.

- Per konstruksjon surjektiv, så m er en isometri av enhet form.

b) Skal vise m entydig bestemt

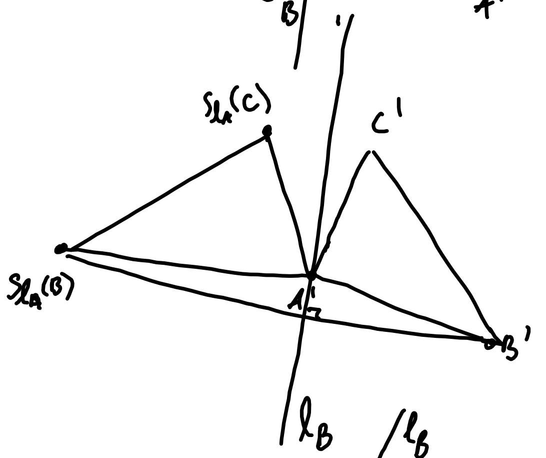


(4) Enhver ikke-triviell isometri m kan skrives som sammensetn. av 1, 2 eller 3 speilinger



$S_{l_A}(A) = A'$

Hvis $S_{l_A}(B) = B'$ og $S_{l_A}(C) = C'$ er jeg ferdig.

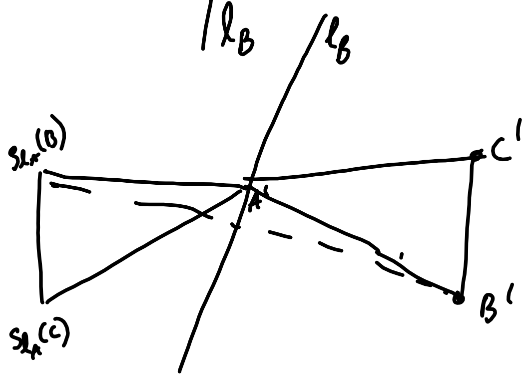


Ta midtnormal mellom B' og $S_{l_A}(B)$,

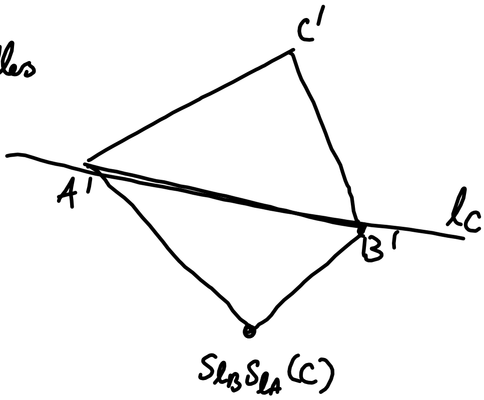
$A' \in l_B$

$S_{l_B} S_{l_A}(B) = B'$.

Hvis $S_{l_B} \circ S_{l_A}(C) = C'$, så er jeg ferdig.



Ellers



Velg $l_C = A'B'$

$S_{l_C} \circ S_{l_B} \circ S_{l_A}(C) = C'$