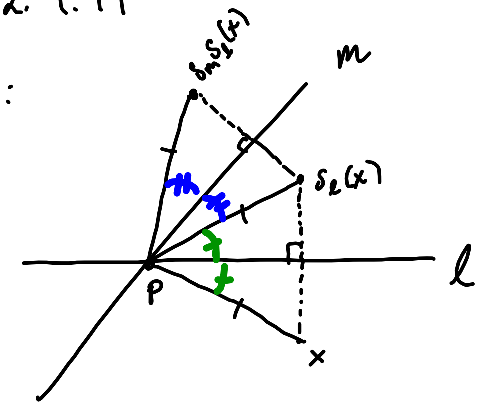
MAT 2500 - 11.09.2020

5) 2.4.11

I :



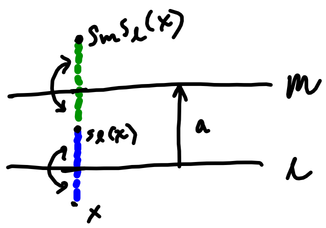
$m \not\parallel \ell$

$p \in m \cap \ell \qquad \angle(m,\ell) = \alpha$

$s_m \circ s_\ell = \rho_{2\alpha, p}$

- Sammensetn av to speilinger er or. bev siden en speiling er or. rev.
- Så enten translasjon eller rotasjon.
- s_ℓ har ℓ som fikspunkter. s_m har m som fikspunkter. } $s_m \circ s_\ell$ har $m \cap \ell = p$ som fikspunkt!
- Så $s_\ell \circ s_m$ er en rotasjon.

II



$s_m \circ s_\ell = t_{2a}$

$$s_m = t_a s_\ell t_{-a} \qquad (2\text{-}2)$$

$$\begin{aligned}
s_m \circ s_\ell(x) &= t_a s_\ell t_{-a} \circ s_\ell(x) \\
&= t_a s_\ell(s_\ell(x) - a) \\
&= t_a(s_\ell \circ s_\ell(x) - s_\ell(a)) \\
&= t_a(x + a) \\
&= x + a + a \\
&= x + 2a \\
&= \underline{\underline{t_{2a}(x)}}
\end{aligned}$$

(6) 2.4.6 $t_a \rho_\theta$ eller $t_a \rho_\theta s$

(A) I $\rho_\theta t_a(x) = \rho_\theta(x+a)$

$= \rho_\theta(x) + \rho_\theta(a)$

$= t_{\rho_\theta(a)} \rho_\theta(x)$

II $s t_a(x) = s(x+a)$

$= s(x) + s(a)$

$= t_{s(a)} \circ s(x)$

$= t_{s(a)} \rho_0 s(x)$

§7 i)
ii)
iii)

III $s\rho_\theta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

$= \begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{pmatrix}$

Vet at $t_a \rho_\varphi s$ $= t_a \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\rho_\varphi s$

$= t_{a=0} \begin{pmatrix} \cos\varphi & \sin\varphi \\ \sin\varphi & -\cos\varphi \end{pmatrix}$

$\left.\begin{matrix} \cos\theta = \cos\varphi \\ -\sin\theta = \sin\varphi \end{matrix}\right\} \Rightarrow \varphi = -\theta.$

$s\rho_\theta = \rho_{-\theta} s$

(B) 2.4.7. m or.rev $\Rightarrow m^2$ translasjon

v/s. 2.3 i heftet $m = t_a \rho_\theta s$

$$m^2 = t_a \rho_\theta s t_a \rho_\theta s$$

$$= t_a \rho_\theta t_{s(a)} s s \rho_{-\theta}$$

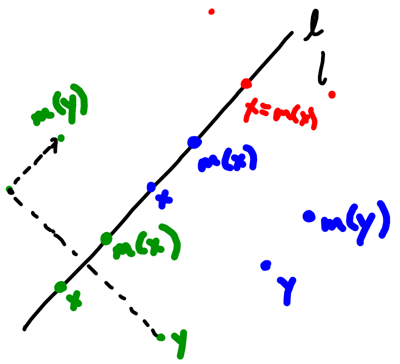
$$= t_a \rho_\theta t_{s(a)} \rho_{-\theta}$$

$$= t_a t_{\rho_\theta(s(a))} \rho_\theta \rho_{-\theta}$$

$$= t_{a+\rho_\theta(s(a))}$$, som er en translasjon

⑦ 2.4.9. m, $m(\ell) = \ell$

$m|_\ell = t_a$ $m(x) = x + a$ for alle $x \in \ell$.

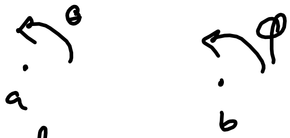


- m speiling $m|_\ell = t_0$
- m glidespeiling $m|_\ell = t_a$ $a \parallel \ell$
- m translasjon $m|_\ell = t_a$ $a \parallel \ell$

- m rotasjon? $m(\ell) = \ell$ for eneste mulighet, men da er $m|_\ell$ IKKE en translasjon.

(8) 2.4.10 To rotasjoner om to forskjellige punkter

$\rho_{\theta,a}$ $\rho_{\varphi,b}$



$\rho_{\theta,a} \circ \rho_{\varphi,b}$ må være orienteringsbevarende.
så enten translasjon eller rotasjon.

2-2c) $\rho_{\theta,p} = t_p \rho_{\theta,0} t_{-p}$

$$\rho_{\theta,a} \circ \rho_{\varphi,b} = t_a \rho_{\theta,0} t_{-a} t_b \rho_{\varphi,0} t_{-b}$$

$$= t_a \rho_\theta t_{-a} t_{b+\rho_\varphi(-b)} \rho_\varphi$$

$$= t_{a+\rho_\theta(-a)+\rho_\theta(b)+\rho_\theta\rho_\varphi(-b)} \rho_\theta \rho_\varphi$$

$$= t_{a+\rho_\theta(-a)+\rho_\theta(b)+\rho_{\theta+\varphi}(-b)} \rho_{\theta+\varphi}$$

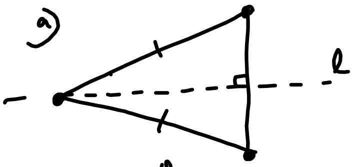
Så stort sett rotasjon, men
hvis $\theta + \varphi = 0 + 2k\pi$, $k \in \mathbb{Z}$.

3

(4) 2.4.13

Symmetrigruppe

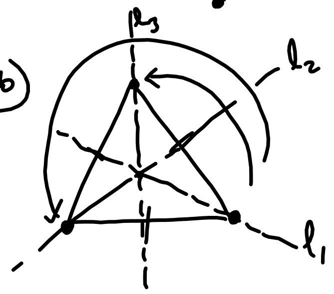
a)



id

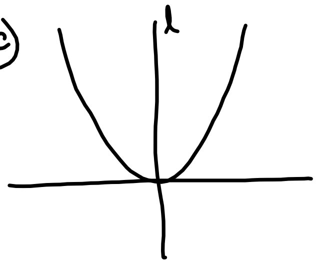
s_ℓ for ℓ midtnormal til gitt side av trekanten

b)



id	s_1	D_3
$\rho_{\frac{2\pi}{3}}$	s_2	
$\rho_{\frac{4\pi}{3}}$	s_3	

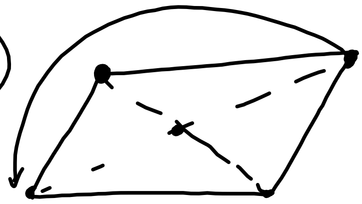
c)



id

s_ℓ der ℓ er $x=0$

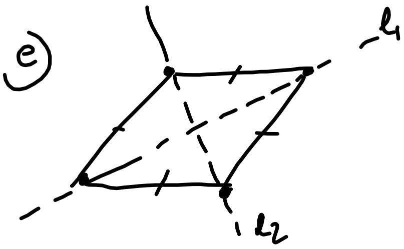
d)



id

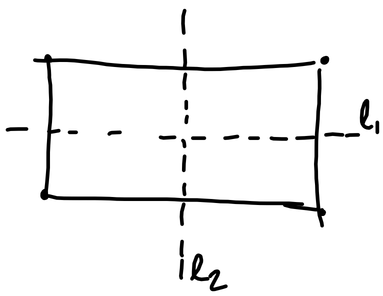
ρ_π

e)



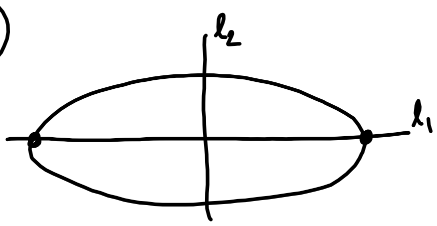
id	s_1
ρ_π	s_2

f)



id	s_1
ρ_π	s_2

g)



id	s_1
ρ_π	s_2