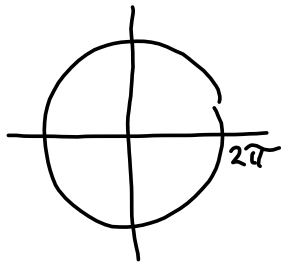
MAT 2500 15.09.2020

3 1 5 hjørner jevnt fordelt på en sirkel

$(\rho_{\frac{2\pi}{5}})^k$, $k = 0, \ldots, 4$



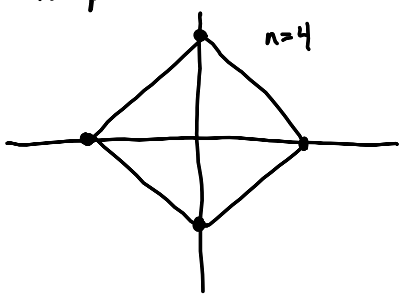
Ingen speilinger

$S = \{ \rho_{\frac{2\pi k}{5}} \; ; \; k = 0, \ldots, 4 \}$

(2) Symmetrigruppen for regulær n-kant

$D_n = \{ \rho_{\frac{2\pi k}{n}}, \rho_{\frac{2\pi k}{n}} \cdot s \;:\; k = 0, \ldots, n-1 \}$ (2n elementer)

n partall



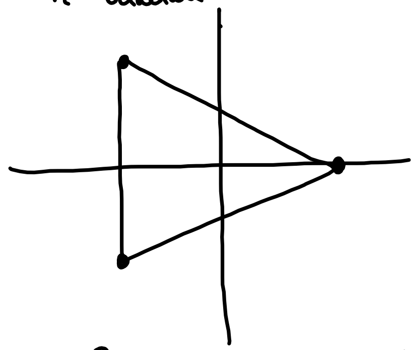
$\rho_{\frac{2\pi}{n}}$ genererer rotasjonene

$\rho_{\frac{2\pi k}{n}}, k = 0, \ldots, n-1$

s er en symmetri for det er like mange hjørner jevnt fordelt over og under x-aksen

$\rho_{\frac{2\pi k}{n}} s$ er også symmetrier $k = 0, \ldots, n-1$.

n oddetall



$\rho_{\frac{2\pi k}{n}}, k = 0, \ldots, n-1$

s er en symmetri som tar hjørner i øvre halvplan på hjørner i nedre halvplan

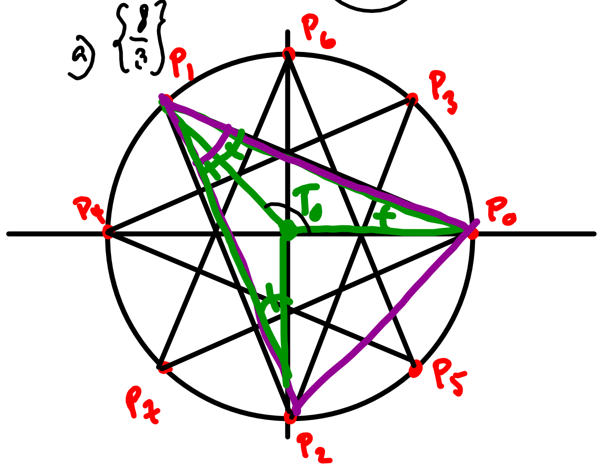
$\rho_{\frac{2\pi k}{n}} \cdot s, k = 0, \ldots, n-1$

3) STJERNEPOLYGON

$n = \frac{p}{d}$ der $\gcd(p,d)=1$

$\rho_{\frac{2\pi}{n}} = \rho_{\frac{2\pi d}{p}}$

a) $\left\{\frac{8}{3}\right\}$



$P_0 = (1,0)$

$P_{i+1} = \rho(P_i)$

$P_1 = \rho_{\frac{2\pi d}{p}}(P_0)$

$F = \left\{\frac{p}{d}\right\}$ vil ha p hjørner $\{P_0, \ldots, P_{p-1}\}$

og p kanter

vil skjære hvis $d > 1$.

$\rho_{\frac{2\pi \cdot 3}{8}} = \rho_{\frac{3\pi}{4}}$

b) Skal vise at $\left\{\frac{p}{d}\right\}$ har D_p som symmetrigruppe

- De p hjørnene ligger jevnt fordelt med vinkel $\frac{2\pi}{p}$ (vis!)
- $\rho_{\frac{2\pi k}{p}}$ for $k=0,\ldots,p-1$ er i symmetrigruppen
- Speilinger som for regulære p-kanter
- Symmetrigruppen er D_p.

c) T_0 : $\Delta P_0 O P_1$

$\angle P_0 O P_1 = \frac{2\pi \cdot d}{p}$ (rotasjonsvinkel)

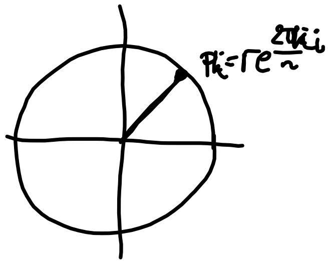
$\Delta P_0 P_1 P_2$ $\quad \angle P_1 = \pi - \angle P_0 O P_1$

$= \pi - 2\pi \cdot \frac{d}{p}$

$= \pi\left(1 - \frac{2d}{p}\right)$

5 2.4.14

$\{p_1, \dots, p_n\}$ reg. n-kant senter i O.
Skal vise at def. av tyngdepkt stemmer!



$$T = \frac{1}{n}\sum_{k=1}^{n} p_k \qquad p_k = r\underbrace{e^{\frac{2\pi k}{n}i}}$$

$$z = e^{\frac{2\pi}{n}i}$$

$$i^2 = -1$$

$$= \frac{1}{n}\sum_{k=1}^{n} r e^{\frac{2\pi k}{n}i}$$

$$= \frac{1}{n}\sum_{k=1}^{n} r z^k$$

$$\boxed{\text{Geometrisk rekke: } \sum_{k=0}^{n} z^k = \frac{1-z^{n+1}}{1-z}}$$

$$= \frac{r}{n}(z + z^2 + \dots + z^n)$$

$$= \frac{r}{n}\left(\sum_{k=0}^{n} z^k - 1\right)$$

$$= \frac{r}{n}\left(\frac{1-z^{n+1}}{1-z} - 1\right)$$

$$= \frac{r}{n}\left(\frac{1-z^{n+1}-(1-z)}{1-z}\right)$$

$$= \frac{r}{n}\left(\frac{z-z^{n+1}}{1-z}\right)$$

$$= \frac{r\cdot z}{n}\left(\frac{\overbrace{1-z^n}^{0}}{1-z}\right)$$

$$z = e^{\frac{2\pi i}{n}}$$

$$z^n = \left(e^{\frac{2\pi i}{n}}\right)^n = \underline{1}$$

$$= \underline{\underline{0}}$$

(6) 2.4.15.

a) $\underline{x} = (1,0)$ $\quad \underline{y} = (0,1)$

$\underline{x}$

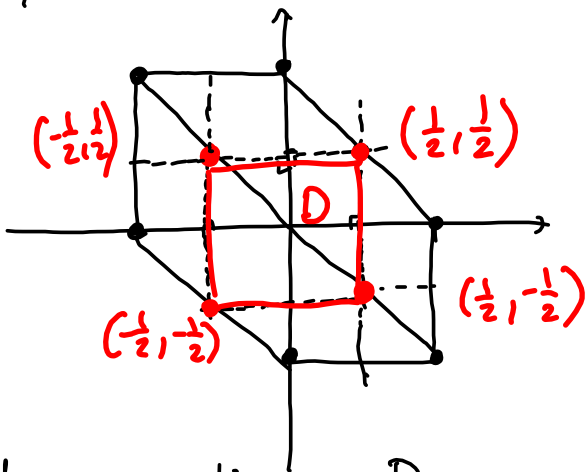
$\underline{y}$

$-\underline{x}+\underline{y}$

$-\underline{x}$

$-\underline{y}$

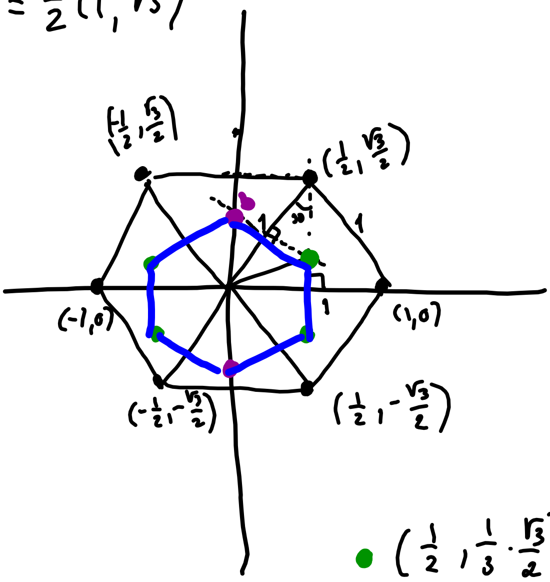
$\underline{x}-\underline{y}$

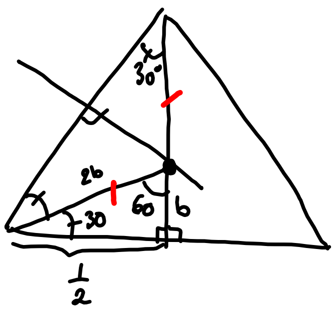


D er et kvadrat, så symmetrigruppe D_4.

b) $\underline{x} = (1,0)$ $\quad \underline{y} = \frac{1}{2}(1,\sqrt{3})$

$$\|\underline{y}\| = \sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = 1$$





$$(2b)^2 = \frac{1}{2}^2 + b^2$$

$$3b^2 = \frac{1}{4}$$

$$b^2 = \frac{1}{12}$$

$$b = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6} = \frac{1}{3}\cdot\frac{\sqrt{3}}{2}$$

• $\left(\frac{1}{2}, \frac{1}{3}\cdot\frac{\sqrt{3}}{2}\right)$

$\left(\frac{1}{2}, -\frac{1}{3}\cdot\frac{\sqrt{3}}{2}\right)$

$\left(-\frac{1}{2}, \frac{1}{3}\cdot\frac{\sqrt{3}}{2}\right)$

$\left(-\frac{1}{2}, -\frac{1}{3}\cdot\frac{\sqrt{3}}{2}\right)$

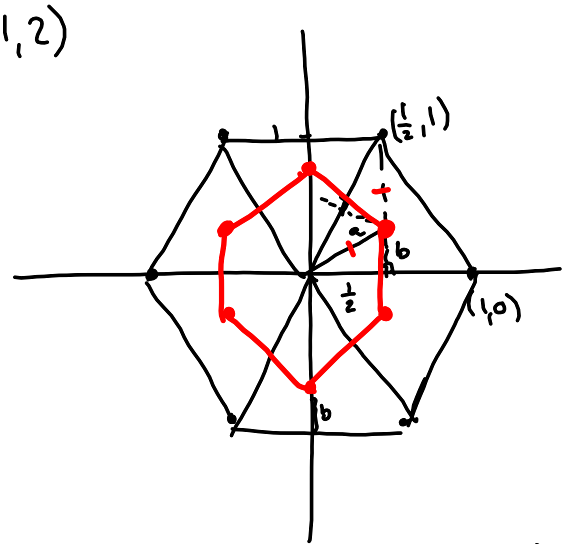
• $\left(0, \frac{\sqrt{3}}{3}\right)$

$\left(0, -\frac{\sqrt{3}}{3}\right)$

D er en regulær 6-kant

Symmetrigruppa er D_6.

c) $\underline{x} = (1,0)$ $\quad\quad$ $\underline{y} = \frac{1}{2}(1,2)$



$$a + b = 1$$
$$a^2 = b^2 + \left(\frac{1}{2}\right)^2$$
$$(1-b)^2 = b^2 + \frac{1}{4}$$
$$1 - 2b + b^2 = b^2 + \frac{1}{4}$$
$$\frac{3}{4} = 2b$$
$$\underline{b = \frac{3}{8}}$$

$\bullet$ $(\frac{1}{2}, \frac{3}{8})$ $\quad$ $(-\frac{1}{2}, \frac{3}{8})$

$(\frac{1}{2}, -\frac{3}{8})$ $\quad$ $(-\frac{1}{2}, -\frac{3}{8})$

$(0, \frac{5}{8})$ $\quad$ $(0, -\frac{5}{8})$

D er ikke-regulær

Symmetrier
- id
- ρ_π
- s_1 $\quad$ l_1 er y-aksen
- s_2 $\quad$ l_2 er x-aksen