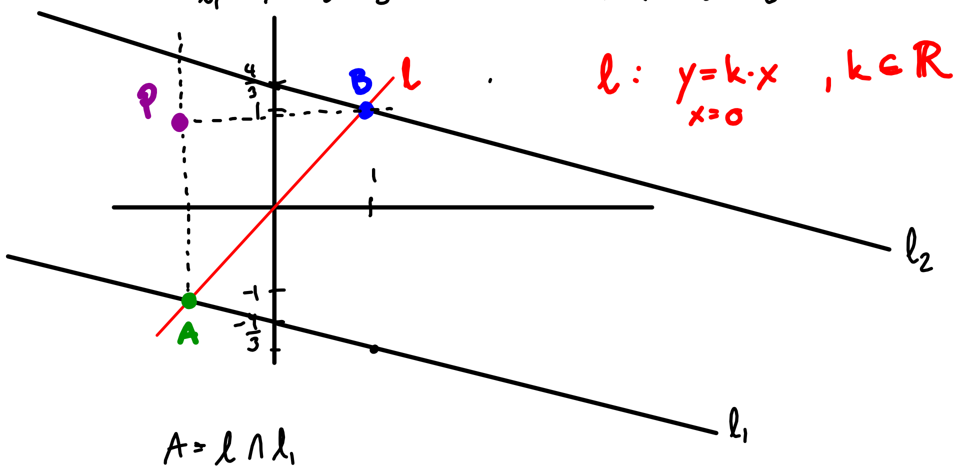
② 5.10.6

$x + 3y + 4 = 0$ $\qquad\qquad$ $x + 3y - 4 = 0$

$l_1: \ y = -\frac{1}{3}x - \frac{4}{3}$ $\qquad\qquad$ $l_2 \ \ y = -\frac{1}{3}x + \frac{4}{3}$

$l: \ \underset{x=0}{y = k \cdot x}, \ k \in \mathbb{R}$



$A = l \cap l_1$

$B = l \cap l_2$

$P = (A_x, B_y)$

Finner A_x:

$y = kx \qquad y = -\frac{1}{3}x - \frac{4}{3}$

$kx = -\frac{1}{3}x - \frac{4}{3}$

$x(k + \frac{1}{3}) = -\frac{4}{3}$

$$x = \frac{-4}{3(k+\frac{1}{3})} = \frac{-4}{3k+1}$$

Finner B_y

$y = kx \qquad y = -\frac{1}{3}x + \frac{4}{3}$

$x = \frac{1}{k} \cdot y$

$y = -\frac{1}{3}\left(\frac{1}{k} \cdot y\right) + \frac{4}{3}$

$\left(1 + \frac{1}{3k}\right)y = \frac{4}{3}$

$y = \frac{4}{3(1+\frac{1}{3k})}$

$y = \frac{4}{3 + \frac{1}{k}}$

$$y = \frac{4k}{3k+1}$$

Parametrisering av alle punkter P på det geom. stedet | L er linja $x=0$

$$P = \left\{\left(-\frac{4}{3k+1}, \frac{4k}{3k+1}\right) : k \in \mathbb{R}\right\} \cup \left\{\left(0, \frac{4}{3}\right)\right\}$$

Vi må eliminere k:

$x = \frac{-4}{3k+1} \qquad k = -\frac{1}{3} - \frac{4}{3x}$

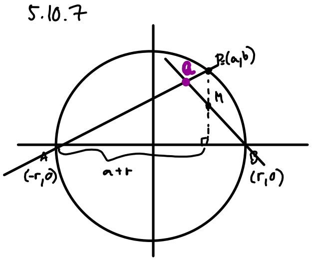
Setter det inn i $y = \frac{4k}{3k+1} = \frac{4(-\frac{1}{3} - \frac{4}{3x})}{3(-\frac{1}{3} - \frac{4}{3x}) + 1}$

$$y = \frac{-\frac{4}{3}(1 + \frac{4}{x})}{\cancel{-1} - \frac{4}{x} \cancel{+1}}$$

$$y = \frac{x}{3}\left(1 + \frac{4}{x}\right)$$

$$y = \frac{1}{3}x + \frac{4}{3}$$ som er det geometriske stedet vi er ute etter!

(3) 5.10.7



$a^2+b^2=r^2$

$M=(a,\frac{1}{2}b)$

Vi skal la P gå rundt på sirkelen og finne det geometriske stedet som Q gir oss.

Linja gjennom A og P:

$y-y_0=k(x-x_0)$

$y-0=\frac{b}{a+r}(x+r)$

$y=\frac{b}{a+r}(x+r)$

Linja gjennom BM

$y=cx+d$

B: $0=c\cdot r+d$ M: $\frac{1}{2}b=c\cdot a+d$

$d=-cr$ $c=\frac{b}{2(a-r)}$

$y=\frac{b}{2(a-r)}(x-r)$

Setter dem lik hverandre:

$$\frac{\cancel{b}}{a+r}(x+r)=\frac{\cancel{b}}{2(a-r)}(x-r)$$

$$2(a-r)(x+r)=(a+r)(x-r)$$

$$x(2(a-r)-(a+r))=-(a+r)r-r2(a-r)$$

$$x=\frac{-3ar+r^2}{a-3r}=\frac{r(r-3a)}{a-3r}$$

Setter inn i én:

$$y=\frac{b}{a+r}\left(\frac{r(r-3a)}{a-3r}+r\right)$$

$$=\frac{b}{a+r}\left(\frac{r^2-3ar+r(a-3r)}{a-3r}\right)$$

$$=\frac{b}{a+r}\left(\frac{-2r^2-2ar}{a-3r}\right)$$

$$=\frac{-2br}{\cancel{(a+r)}}\cdot\frac{\cancel{(r+a)}}{(a-3r)}$$

$$y=\frac{-2br}{(a-3r)}$$

Param av $Q=\left\{\left(\frac{r(r-3a)}{a-3r},\frac{-2br}{a-3r}\right):a^2+b^2=r^2\right\}$

Kvadrerer $y=\frac{-2br}{a-3r}$

$$y^2=\frac{4b^2r^2}{(a-3r)^2}=\frac{4(r^2-a^2)r^2}{(a-3r)^2}$$

Løser $x=\frac{r(r-3a)}{a-3r}$ for a:

$$(a-3r)\cdot x=r^2-3ar$$

$$a(x+3r)=r^2+3rx$$

$$a=\frac{r^2+3rx}{x+3r}=\frac{r(r+3x)}{(x+3r)}$$

Setter inn i y^2:

$$y^2=\frac{4\left(r^2-\left(\frac{r(r+3x)}{(x+3r)}\right)^2\right)\cdot r^2}{\left(\frac{r(r+3x)}{(x+3r)}-3r\right)^2}$$

$$\left(\frac{r(r+3x)-3r(x+3r)}{(x+3r)}\right)^2y^2=4r^2\left(\frac{r^2(x+3r)^2-r^2(r+3x)^2}{(x+3r)^2}\right)$$

$$(r(r+3x)-3r(x+3r))^2y^2=4r^4((x+3r)^2-(r+3x)^2)$$

$$(r^2+\cancel{3rx}-\cancel{3rx}-9r^2)^2y^2=4r^4(x^2+\cancel{6xr}+9r^2-r^2-\cancel{6rx}-9x^2)$$

$$(-8)^2\cancel{r^4}y^2=4\cancel{r^4}(-8x^2+8r^2)$$

$$2\cdot\cancel{8}\cdot y^2=\cancel{8}(r^2-x^2)$$

$$x^2+2y^2=r^2 \quad |:r^2$$

$$\boxed{\frac{x^2}{r^2}+\frac{y^2}{\frac{r^2}{2}}=1}$$

Ellipse med halvakser r og $\frac{r}{\sqrt{2}}$.

Sentrum i $(0,0)$.