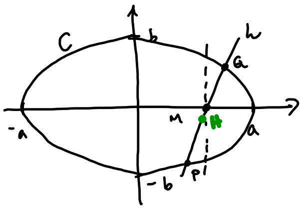
MAT 2500 29. 09. 2020

$C: \quad b^2x^2 + a^2y^2 = a^2b^2 \qquad a > b$

$M = (m, 0)$

$P, Q \in L \cap C$

$M \in L$



$H = \frac{P+Q}{2}$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$L: \quad y = cx + d \quad , c \in \mathbb{R}$

$M \in L: \quad 0 = c \cdot m + d \qquad d = -cm$

$\underline{y = c(x - m)} \quad , c \in \mathbb{R}$

$x = m$
Da er $H = M$

Setter inn for L i C:

$b^2x^2 + a^2c^2(x-m)^2 = a^2b^2$

$\underbrace{(b^2 + a^2c^2)}_{A}x^2 \underbrace{- 2a^2c^2m}_{B}x + \underbrace{a^2c^2m^2 - a^2b^2}_{C} = 0$

$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \qquad (P \text{ og } Q)$

Siden H er midt mellom P og Q, må H_x være midt mellom P_x og Q_x

$x_H = \frac{-B}{2A} = \frac{2a^2c^2m}{2(b^2+a^2c^2)} = \frac{a^2c^2m}{(b^2+a^2c^2)} = a^2c \cdot \frac{cm}{(b^2+a^2c^2)}$

$y_H = c \cdot x_H - cm$

$= \frac{c \cdot a^2c^2m - cm(b^2+a^2c^2)}{(b^2+a^2c^2)}$

$= -b^2 \frac{c \cdot m}{(b^2+a^2c^2)}$

. Finner uttrykk for c:

$\boxed{-b^2 \cdot x = a^2 \cdot c \cdot y} \qquad c = \frac{-b^2}{a^2} \cdot \frac{x}{y}$

. Setter inn i y:

$y = \frac{-b^2\left(-\frac{b^2}{a^2} \cdot \frac{x}{\cancel{y}}\right) \cdot m}{b^2 + a^2\left(-\frac{b^2}{a^2} \cdot \frac{x}{y}\right)^2}$

$y = \frac{\frac{b^4}{a^2} \cdot \frac{x}{y} \cdot m}{b^2 + \frac{b^4}{a^2} \cdot \frac{x^2}{y^2}} = \frac{b^{\cancel{4}\,2}xm}{a^2y\cancel{b^2}\left(1 + \frac{b^2x^2}{a^2y^2}\right)} = \frac{b^2xm}{a^2y + \frac{b^2x^2}{y}}$

$y\left(a^2y + \frac{b^2x^2}{y}\right) = b^2xm$

$a^2y^2 + \underbrace{b^2x^2 - b^2xm}\ = 0$

$+0 = b^2\left(\frac{m}{2}\right)^2 - b^2\left(\frac{m}{2}\right)^2$

$a^2y^2 + b^2\left(x - \frac{m}{2}\right)^2 = \frac{b^2m^2}{4} \qquad \Big| : \frac{b^2m^2}{4}$

$\frac{y^2}{\left(\frac{bm}{2a}\right)^2} + \frac{\left(x - \frac{m}{2}\right)^2}{\left(\frac{m}{2}\right)^2} = 1$

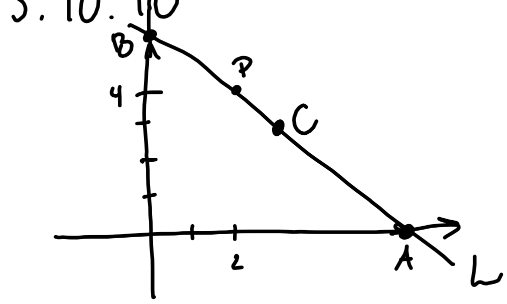
Ellipse $\quad S = \left(\frac{m}{2}, 0\right)$

Halvakser $\quad \frac{m}{2}$, $\frac{bm}{2a}$

$b^2x^2 - b^2xm = b^2(x^2 - x \cdot m)$

$= b^2\left(x^2 - x \cdot 2 \cdot \frac{m}{2} + \left(\frac{m}{2}\right)^2 - \left(\frac{m}{2}\right)^2\right)$

5.10.10



L dreier om $P=(2,4)$

$A=(a,0)$

$B=(0,b)$

$C=(\frac{1}{2}a, \frac{1}{2}b)$

$L: \quad y=k\cdot x+d \qquad k\in\mathbb{R}$

$x=2$ gir ingen B

$P\in L: \quad 4=k\cdot 2+d \qquad d=2(2-k)$

$L_k \quad y=k\cdot x+2(2-k) \qquad k\in\mathbb{R}$

A på L_k: $\quad 0=k\cdot a+2(2-k)$

$$a=\frac{2(k-2)}{k}$$

$$C=\left\{\left(\overset{x}{\frac{(k-2)}{k}}, \overset{y}{(2-k)}\right), k\in\mathbb{R}\right\}$$

B på L_k: $\quad b=k\cdot 0+2(2-k)$

$-2(2-k)=k\cdot a$

$$a=\frac{-2(2-k)}{k}=\frac{-2(-1)(k-2)}{k}=\frac{2(k-2)}{k}$$

Eliminerer k:

$y=2-k$

$k=2-y$

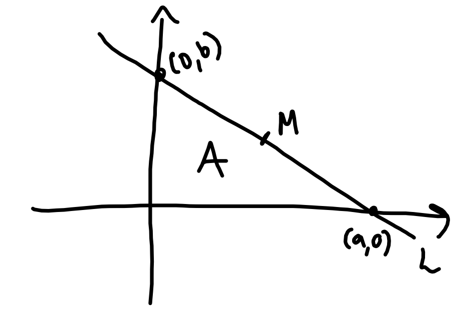
$$x=\frac{k-2}{k}=\frac{2-y-2}{2-y}=\frac{-1\cdot y}{-1(y-2)}$$

$$x=\frac{y}{y-2}$$

$xy-2x-y=0$

Er det geometriske stedet for C når L dreier om P.

5.10.11



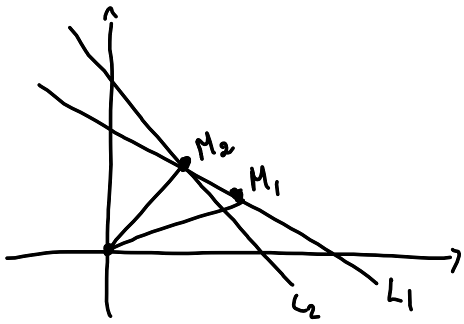
A fikset

M midtpunkt på hypotenus

$M = \left(\frac{a}{2}, \frac{b}{2}\right)$ $\quad x = \frac{a}{2} \quad y = \frac{b}{2}$

$A = \frac{ab}{2}$ $\quad x \cdot y = \frac{a \cdot b}{4} = \frac{1}{2}A$

$\underline{\underline{x \cdot y = \frac{1}{2}A}}$



$\min \|M - (0,0)\|^2$ når $a \cdot b = 2A$

$\min \left(\frac{1}{2}a\right)^2 + \left(\frac{1}{2}b\right)^2$ når $a \cdot b = 2A$

$\mathcal{L} = \frac{1}{4}(a^2 + b^2) - \lambda \cdot (ab - 2A)$

$\mathcal{L}'_a = \frac{a}{2} - \lambda \cdot b = 0 \quad \lambda = \frac{a}{2b}$

$\mathcal{L}'_b = \frac{b}{2} - \lambda \cdot a = 0 \quad \lambda = \frac{b}{2a}$

$C: \; \underline{ab = 2A} > 0$

$\frac{a}{2b} = \frac{b}{2a}$

$a^2 = b^2$

$\underline{a = b}$

$a^2 = 2A$

$\underline{a = \pm\sqrt{2A}}$

$\underline{b = \pm\sqrt{2A}}$

$\underline{M = \left(\pm\frac{\sqrt{2A}}{2}, \pm\frac{\sqrt{2A}}{2}\right)}$

$L: \quad y = cx + d$

$(a,0) \quad 0 = c \cdot a + d \;, \; d = -ca \rightarrow b = -c \cdot a \;, \text{ så } c = -\frac{b}{a}$

$(0,b) \quad b = d$

$y = -\frac{b}{a}x + b$

$\underline{y = -x \pm \sqrt{2A}}$

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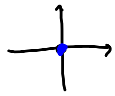
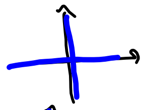
$$g(x,y) = ax^2 + bxy + cy^2 + dx + ey + f$$

$C = \{(x,y) \mid g(x,y) = 0\}$ kvadratisk kurve.

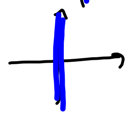
- Degenerert
  - tom x^2+y^2+1
  - ett punkt x^2+y^2



  - to ulike linjer $x \cdot y$



  - dobbel linje x^2



- Ikke degenerert $\Rightarrow$ kjeglesnitt

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$$A = \begin{bmatrix} a & \frac{1}{2}b & \frac{1}{2}d \\ \frac{1}{2}b & c & \frac{1}{2}e \\ \frac{1}{2}d & \frac{1}{2}e & f \end{bmatrix} \qquad g(x,y) = \underset{(1,3)}{[x, y, 1]} \cdot \underset{(3,3)}{A} \cdot \underset{(3,1)}{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}$$

C er to linjer eller dobbelt linje

a) $g(x,y) = (\alpha_1 x + \beta_1 y + \gamma_1)(\alpha_2 x + \beta_2 y + \gamma_2) \quad (\alpha_i, \beta_i) \neq (0,0)$

$$= \alpha_1\alpha_2 x^2 + (\alpha_1\beta_2 + \beta_1\alpha_2)xy + \beta_1\beta_2 y^2$$
$$+ (\alpha_1\gamma_2 + \alpha_2\gamma_1)x$$
$$+ (\beta_1\gamma_2 + \beta_2\gamma_1)y$$
$$+ \gamma_1\gamma_2$$

$$A = \begin{bmatrix} \alpha_1\alpha_2 & \frac{1}{2}(\alpha_1\beta_2+\beta_1\alpha_2) & \frac{1}{2}(\alpha_1\gamma_2+\alpha_2\gamma_1) \\ \bullet & \beta_1\beta_2 & \frac{1}{2}(\beta_1\gamma_2+\beta_2\gamma_1) \\ \bullet & \bullet & \gamma_1\gamma_2 \end{bmatrix}$$

Skal sjekke at $A = \frac{1}{2}(B + B^t)$

$$B = \begin{bmatrix} \alpha_1 \\ \beta_1 \\ \gamma_1 \end{bmatrix} \cdot [\alpha_2 \; \beta_2 \; \gamma_2] = \begin{bmatrix} \alpha_1\alpha_2 & \alpha_1\beta_2 & \alpha_1\gamma_2 \\ \beta_1\alpha_2 & \beta_1\beta_2 & \beta_1\gamma_2 \\ \gamma_1\alpha_2 & \gamma_1\beta_2 & \gamma_1\gamma_2 \end{bmatrix}$$

$$B^t = \begin{bmatrix} \alpha_1\alpha_2 & \beta_1\alpha_2 & \gamma_1\alpha_2 \\ \alpha_1\beta_2 & \beta_1\beta_2 & \gamma_1\beta_2 \\ \alpha_1\gamma_2 & \beta_1\gamma_2 & \gamma_1\gamma_2 \end{bmatrix} = \begin{bmatrix} \alpha_2 \\ \beta_2 \\ \gamma_2 \end{bmatrix} [\alpha_1 \; \beta_1 \; \gamma_1]$$

- $\frac{1}{2}(\alpha_1\beta_2 + \beta_1\alpha_2) = A_{1,2}$ og tilsvarende for alle andre elementer i matrisen

$$\underline{A = \frac{1}{2}(B + B^t)}$$