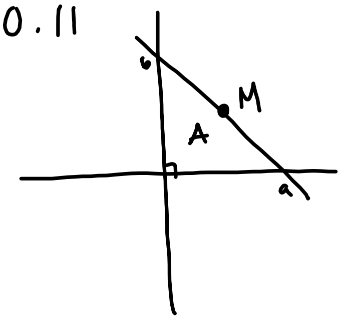
# MAT 2500 06.10.2020

5.10.11



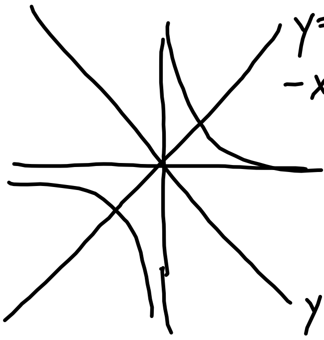

$A = \frac{a \cdot b}{2}$ $M = \left(\frac{1}{2}a, \frac{1}{2}b\right)$ $x \cdot y = \frac{1}{2}A$

ok for $a, b > 0$ og $a, b < 0$

$A = \frac{|a \cdot b|}{2}$ $|xy| = \frac{1}{2}A$

$xy = 1$ er en hyperbel :

Koordinatskifte




$$\left.\begin{array}{l} Y = -x + y \\ X = x + y \end{array}\right\} \quad \begin{array}{l} \frac{1}{2}(X - Y) = x \\ \frac{1}{2}(X + Y) = y \end{array}$$

$$1 = \frac{1}{2}(X - Y) \cdot \frac{1}{2}(X + Y)$$
$$= \frac{1}{4}(X^2 - Y^2)$$ som er standardform

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$$g(x,y) = ax^2 + bxy + cy^2 + dx + ey + f$$

$$A = \begin{bmatrix} a & \frac{1}{2}b & \frac{1}{2}d \\ \frac{1}{2}b & c & \frac{1}{2}e \\ \frac{1}{2}d & \frac{1}{2}e & f \end{bmatrix}$$

a) Vi så $C : g(x,y) = 0$ to linjer $\Rightarrow g(x,y) = (\alpha_1 x + \beta_1 y + \gamma_1)(\alpha_2 x + \beta_2 y + \gamma_2)$

$A = \frac{1}{2}(B + B^t)$ der $B = \begin{bmatrix} \alpha_1 \\ \beta_1 \\ \gamma_1 \end{bmatrix} \cdot \begin{bmatrix} \alpha_2 & \beta_2 & \gamma_2 \end{bmatrix}$

b) Anta at linjene har felles punkt $(x_0, y_0)$

Skal vise $A \begin{bmatrix} x_0 \\ y_0 \\ 1 \end{bmatrix} = \underline{0}$ :

Regner ut v.s.:

$$\frac{1}{2}(B + B^t) \cdot \begin{bmatrix} x_0 \\ y_0 \\ 1 \end{bmatrix} = \frac{1}{2}\Bigg(\overbrace{\begin{bmatrix} \alpha_1 \\ \beta_1 \\ \gamma_1 \end{bmatrix} \cdot \begin{bmatrix} \alpha_2 & \beta_2 & \gamma_2 \end{bmatrix}}^{B} \cdot \begin{bmatrix} x_0 \\ y_0 \\ 1 \end{bmatrix} + \overbrace{\begin{bmatrix} \alpha_2 \\ \beta_2 \\ \gamma_2 \end{bmatrix} \begin{bmatrix} \alpha_1 & \beta_1 & \gamma_1 \end{bmatrix}}^{B^t} \begin{bmatrix} x_0 \\ y_0 \\ 1 \end{bmatrix}\Bigg)$$

$$= \frac{1}{2}\Bigg(\begin{bmatrix} \alpha_1 \\ \beta_1 \\ \gamma_1 \end{bmatrix} \cdot \underbrace{(\alpha_2 x_0 + \beta_2 y_0 + \gamma_2)}_{= 0} + \begin{bmatrix} \alpha_2 \\ \beta_2 \\ \gamma_2 \end{bmatrix} \underbrace{(\alpha_1 x_0 + \beta_1 y_0 + \gamma_1)}_{= 0}\Bigg)$$

siden $(x_0, y_0)$ er felles pkt

$$= \underline{0}$$

$A \cdot \underline{v} = \underline{0}$ for $\underline{v} \neq \underline{0}$ $\quad \underline{v} = \begin{bmatrix} x_0 \\ y_0 \\ 1 \end{bmatrix}$

En ikke-triviell løsning betyr at vi har uendelig mange løsninger, og det er ekvivalent med $\det A = 0$.

c) Anta linjene er parallelle:

Kan derfor anta $\underline{\alpha_1 = \alpha_2 \text{ og } \beta_1 = \beta_2}$

$\alpha_1 x + \beta_1 y + \gamma_1 = 0 \qquad \alpha_2 x + \beta_2 y + \gamma_2 = 0$

$\beta_1 \text{ og } \beta_2 \neq 0: \quad k = \frac{-\alpha_1}{\beta_1} = \frac{-\alpha_2}{\beta_2}$, da må $\alpha_2 = m \cdot \alpha_1$ og $\beta_2 = m \cdot \beta_1$, $m \neq 0$

$m\alpha_1 x + m\beta_1 y + \gamma_2 = 0$

$\alpha_1 x + \beta_1 y + \frac{\gamma_2}{m} = 0$

$\beta_1 = \beta_2 = 0$ samme!

Kan skrive $g(x,y) = (\alpha x + \beta y + \gamma_1)(\alpha x + \beta y + \gamma_2)$

La $\underline{n} = (\beta, -\alpha)$

Skal vise $A \cdot \begin{bmatrix} \beta \\ -\alpha \\ 0 \end{bmatrix} = \underline{0}$

$$\frac{1}{2}(B + B^t)\begin{bmatrix} \beta \\ -\alpha \\ 0 \end{bmatrix} = \frac{1}{2}\left( \begin{bmatrix} \alpha \\ \beta \\ \gamma_1 \end{bmatrix} \underbrace{[\alpha \ \beta \ \gamma_2] \cdot \begin{bmatrix} \beta \\ -\alpha \\ 0 \end{bmatrix}}_{\substack{\alpha\beta - \beta\alpha + 0 \cdot \gamma_2 \\ \| \\ 0}} + \begin{bmatrix} \alpha \\ \beta \\ \gamma_2 \end{bmatrix} \underbrace{[\alpha \ \beta \ \gamma_1] \cdot \begin{bmatrix} \beta \\ -\alpha \\ 0 \end{bmatrix}}_{\substack{\| \\ 0}} \right)$$

$$= \underline{0}$$

Som i b), må da $\det A = 0$.

(2) $\det A = 0 \Rightarrow C : g(x,y)=0$ er degenerert

Anta at $\det A \neq 0$

Tangentlinje (skal vise) til $C$ i $(x_0,y_0)$: $[x\ y\ 1] \cdot A \cdot \begin{bmatrix} x_0 \\ y_0 \\ 1 \end{bmatrix} = 0$

Se på $y=f(x)$ og $C$ som grafen til $f$ på et lite område.

$$g(x,y) = ax^2 + bxy + cy^2 + dx + ey + f = 0$$

implisitt derivasjon:

$$2ax + b(y + xy') + 2cy \cdot y' + d + ey' = 0$$

Løser for $y'$: $y'(bx + 2cy + e) = -d - 2ax - by$

$$y' = -\frac{(d + 2ax + by)}{(bx + 2cy + e)}$$

$$T_{C,(x_0,y_0)}: \quad y - y_0 = -\frac{(d + 2ax_0 + by_0)}{(bx_0 + 2cy_0 + e)}(x - x_0)$$

$$bx_0(y-y_0) + 2cy_0(y-y_0) + e(y-y_0) + d(x-x_0) + 2ax_0(x-x_0) + by_0(x-x_0) = 0$$

$$[x\ y\ 1] \cdot \begin{bmatrix} a & \frac{1}{2}b & \frac{1}{2}d \\ \frac{1}{2}b & c & \frac{1}{2}e \\ \frac{1}{2}d & \frac{1}{2}e & f \end{bmatrix} \cdot \begin{bmatrix} x_0 \\ y_0 \\ 1 \end{bmatrix} = 0$$

$$[x\ y\ 1] \cdot \begin{bmatrix} ax_0 + \frac{1}{2}by_0 + \frac{1}{2}d \\ \frac{1}{2}bx_0 + cy_0 + \frac{1}{2}e \\ \frac{1}{2}dx_0 + \frac{1}{2}ey_0 + f \end{bmatrix} = 0$$

$$ax_0x + \frac{1}{2}by_0x + \frac{1}{2}dx + \frac{1}{2}bx_0y + cy_0y + \frac{1}{2}ey + \frac{1}{2}dx_0 + \frac{1}{2}ey_0 + f = 0$$

$g(x_0,y_0)=0$: $\quad \frac{1}{2}dx_0 + \frac{1}{2}ey_0 + f = -ax_0^2 - bx_0y_0 - cy_0^2 - \frac{1}{2}dx_0 - \frac{1}{2}ey_0$

$$\boxed{0 = ax_0(x-x_0) + \frac{1}{2}by_0(x-x_0) + \frac{1}{2}bx_0(y-y_0) + cy_0(y-y_0) + \frac{1}{2}d(x-x_0) + \frac{1}{2}e(y-y_0)}$$

● $= \frac{1}{2}$ ● så tangentlinja er som oppgitt.

12 1 $\mathbb{P}^2_{\mathbb{R}}$ $(x_0 : x_1 : x_2) \sim (y_0 : y_1 : y_2)$ hvis $y_i = t x_i$, $t \neq 0$

$(0:0:0) \notin \mathbb{P}^2_{\mathbb{R}}$

$\mathbb{P}^2_{\mathbb{R}}$ kan deles av tre mengder

$U_0$ $U_1$ $U_2$

$x_0 \neq 0$ $x_1 \neq 0$ $x_2 \neq 0$

Identifiserer $\mathbb{R}^2 \simeq U_2$

$(x,y) \longmapsto (x : y : 1)$

$\left(\frac{x_0}{x_2}, \frac{x_1}{x_2}\right) \longleftarrow (x_0 : x_1 : x_2)$ $x_2 \neq 0$

a) $-2x_0 + 4x_1 - x_2 = 0$ $x_0 - 2x_1 + 3x_2 = 0$

$(-2, 4, -1) \times (1, -2, 3) = (4 \cdot 3 - (-2)(-1), -(-2) \cdot 3 - (-1) \cdot 1, (-2)(-2) - 4 \cdot 1)$

$= (10, 5, 0)$

$P = (10 : 5 : 0) \sim (2 : 1 : 0)$

$x_2 = 0$, så $P \notin U_2$.