MAT 2500 09.10.2020

12 1 $\mathbb{P}^2_{\mathbb{R}}$ $(x_0 : x_1 : x_2) \sim (y_0 : y_1 : y_2)$

$y_i = t x_i \quad , t \neq 0.$

$(0:0:0) \notin \mathbb{P}^2$

$\mathbb{P}^2$ er dekket av tre mengder

U_0	U_1	$\boxed{U_2}$
$x_0 \neq 0$	$x_1 \neq 0$	$\boxed{x_2 \neq 0}$

Identifiserer $\mathbb{R}^2$ med U_2

$$\mathbb{R}^2 \simeq U_2$$

$$(x,y) \longmapsto (x : y : 1)$$

$$\left(\frac{x_0}{x_2}, \frac{x_1}{x_2}\right) \longleftarrow (x_0 : x_1 : x_2) \qquad x_2 \neq 0$$

a) $-2x_0 + 4x_1 - x_2 = 0 \qquad x_0 - 2x_1 + 3x_2 = 0$

$$(-2,4,-1) \times (1,-2,3) = (4\cdot 3 - (-2)(-1), (-1)\cdot 1 - (-2)\cdot 3, (-2)(-2) - 4\cdot 1)$$
$$= (10, 5, 0)$$

$P = (10:5:0) \sim (2:1:0) \qquad x_2 = 0$, så $P \notin U_2$

b) $2x_0 + 3x_1 - 6x_2 = 0 \qquad -x_0 + x_1 + 3x_2 = 0$

Gjelder på $P \in U_2$: $x_2 = 1$:

$$2\cdot\frac{x_0}{x_2} + 3\cdot\frac{x_1}{x_2} - 6\frac{x_2}{x_2} = 0$$

$2x + 3y - 6 = 0 \qquad -x + y + 3 = 0$

$$\left(\begin{array}{cc|c} 2 & 3 & 6 \\ -1 & 1 & -3 \end{array}\right) \begin{array}{c} \\ (2) \end{array} \sim \left(\begin{array}{cc|c} 0 & 5 & 0 \\ -1 & 1 & -3 \end{array}\right) \begin{array}{c} \\ (-1) \end{array} \sim \left(\begin{array}{cc|c} 0 & 1 & 0 \\ 1 & 0 & 3 \end{array}\right)$$

$y = 0$
$x = 3$

$P = (3,0) \in \mathbb{R}^2$

$P = (3:0:1) \in \mathbb{P}^2$

c) $6x_0 - 2x_1 + 4x_2 = 0 \qquad 3x_0 - x_2 = 0$

$(6,-2,4) \times (3,0,-1) = ((-2)(-1) - 4\cdot 0,\ 4\cdot 3 - 6(-1),\ 6\cdot 0 - 3(-2))$

$= (2, 18, 6)$

$P = (2:18:6) \sim (\frac{1}{3} : 3 : 1) \in \mathbb{P}^2$

$P = (\frac{1}{3}, 3) \in \mathbb{R}^2$

$(\frac{2}{6}, \frac{18}{6})$

d) $3x_0 + x_1 - 2x_2 = 0 \qquad 6x_0 + 2x_1 + 5x_2 = 0$

$(3,1,-2) \times (6,2,5) = (9, -27, 0)$

$\left(\begin{array}{ccc|c} 3 & 1 & -2 & 0 \\ 6 & 2 & 5 & 0 \end{array}\right)^{(-2)} \sim \left(\begin{array}{ccc|c} 3 & 1 & -2 & 0 \\ 0 & 0 & 9 & 0 \end{array}\right)_{\frac{1}{9}} \sim \left(\begin{array}{ccc|c} \underline{3} & 1 & 0 & 0 \\ 0 & 0 & \underline{1} & 0 \end{array}\right)$

$x_1 = t$
$x_0 = -\frac{1}{3}t$
$x_2 = 0$

$\begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} = t \begin{pmatrix} -\frac{1}{3} \\ 1 \\ 0 \end{pmatrix}, \quad t = -3$

$P = (1:-3:0) \sim t(-\frac{1}{3} : 1 : 0), \ t \in \mathbb{R} \setminus \{0\}.$

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a) $(4:3:2)$ og $(-2:5:1)$

Vi må finne a,b,c s.a. $ax_0+bx_1+cx_2=0$

$$\begin{pmatrix} 4 & 3 & 2 \\ -2 & 5 & 1 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 4 & 3 & 2 & 0 \\ -2 & 5 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 13 & 4 & 0 \\ -2 & 5 & 1 & 0 \end{array}\right)^{(-\frac{5}{13})}$$

$$\sim \left(\begin{array}{ccc|c} 0 & 13 & 4 & 0 \\ -2 & 0 & -\frac{7}{13} & 0 \end{array}\right)_{(-13)} \sim \left(\begin{array}{ccc|c} 0 & 13 & 4 & 0 \\ 26 & 0 & 7 & 0 \end{array}\right)$$

$c = t \quad , \; t \neq 0.$

$b = \frac{-4t}{13}$ $\qquad t = 26 \qquad \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -7 \\ -8 \\ 26 \end{pmatrix}$

$a = -\frac{7}{26}t$

$$t\left(-\frac{7}{26}x_0 - \frac{4}{13}x_1 + x_2\right) = 0 \qquad -7x_0 - 8x_1 + 26x_2 = 0$$

s. 83:

$$L: \quad 0 = \begin{vmatrix} x_0 & x_1 & x_2 \\ 4 & 3 & 2 \\ -2 & 5 & 1 \end{vmatrix} = x_0(3\cdot1 - 5\cdot2) - x_1(4\cdot1-(-2)\cdot2) + x_2(4\cdot5-(-2)\cdot6)$$

$$= \underline{-7x_0 - 8x_1 + 26x_2}$$

b)

$$L: \quad 0 = \begin{vmatrix} x_0 & x_1 & x_2 \\ 2 & 5 & 1 \\ 6 & 1 & 3 \end{vmatrix} = x_0(5\cdot3-1\cdot1) - x_1(2\cdot3-6\cdot1) + x_2(2\cdot1-6\cdot5)$$

$$= 14x_0 - 28x_2$$

$$= 14(x_0 - 2x_2)$$

$$\underline{\underline{0 = x_0 - 2x_2}}$$

$(2:5:1)$: $\quad 2 - 2\cdot1 = 0$

$(6:1:3)$: $\quad 6 - 2\cdot3 = 0$

!

c)

$$L: \quad 0 = \begin{vmatrix} x_0 & x_1 & x_2 \\ 4 & 6 & -2 \\ 5 & 0 & 0 \end{vmatrix} = x_0(6\cdot0 - 0\cdot(-2)) - x_1(4\cdot0-5(-2)) + x_2(4\cdot0-5\cdot6)$$

$$= -10x_1 - 30x_2$$

$$= -10(x_1 - 3x_2)$$

$$\underline{\underline{0 = x_1 - 3x_2}}$$

3

6.6.1. $P=(1:0:0)$ $Q=(1:1:0)$

$L_{PQ}: \quad x_2=0$

$R=(1:0:1)$ $S=(1:1:1)$

$L_{RS}: \quad x_0-x_2=0$

$L_{PQ}\cap L_{RS}: \quad \begin{matrix} x_2=0 \\ x_0=0 \end{matrix} \quad x_1 \text{ fri}: \quad \underline{\underline{(0:\overset{t}{1}:0)}}$

$$L_{PQ}: \quad 0=\begin{vmatrix} x_0 & x_1 & x_2 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix} = \underline{x_2}$$

$$L_{RS}: \quad 0=\begin{vmatrix} x_0 & x_1 & x_2 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} = x_0(0\cdot1-1\cdot1)-x_1(1\cdot1-1\cdot1)+x_2(1\cdot1-1\cdot0)$$

$$= \underline{-x_0+x_2}$$

$$(0,1,0)\times(-1,0,1) = (0,-1,0)$$

$$p=(0:-1:0)\sim\underline{(0:1:0)}$$