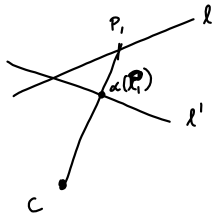
MAT 2500 20.10.2020

6.6.6. Gitt $\alpha : \ell \to \ell'$ med senter C

Skal vise at det fins $T : \mathbb{P}^2 \to \mathbb{P}^2$

med C som fikspunkt og $T|_\ell = \alpha$

$R \in \ell$ vil $T(R) = \alpha(R)$.



Kan anta $C = (0:0:1)$

$\ell : x_2 = 0$

$\ell' : Ax_0 + Bx_1 + Cx_2 = 0$

$$T = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

- C fikspunkt

$T(C) = \lambda \cdot C$

$$\begin{pmatrix} c \\ f \\ i \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \lambda \end{pmatrix} \qquad \underline{c = f = 0 \quad i = \lambda}$$

- $P_1 = (1:0:0) \in \ell$

Linja P_1C : $x_1 = 0$

$T(P_1) = \alpha(P_1) = P_1C \cap \ell'$

$$\left.\begin{aligned} Ax_0 + Bx_1 + Cx_2 &= 0 \\ x_1 &= 0 \end{aligned}\right\} \quad \begin{aligned} Ax_0 + Cx_2 &= 0 \\ x_1 &= 0 \end{aligned}$$

$$\begin{pmatrix} a \\ d \\ g \end{pmatrix} = T(P_1) = \alpha(P_1) = \mu \begin{pmatrix} -C \\ 0 \\ A \end{pmatrix}$$

$a = -\mu C$

$d = 0$

$g = \mu A$

- $P_2 = (0:1:0) \in \ell$

Linja P_2C : $x_0 = 0$

$T(P_2) = \alpha(P_2) = P_2C \cap \ell'$

$$\left.\begin{aligned} Ax_0 + Bx_1 + Cx_2 &= 0 \\ x_0 &= 0 \end{aligned}\right\} \quad \begin{aligned} Bx_1 + Cx_2 &= 0 \\ x_0 &= 0 \end{aligned}$$

$$\begin{pmatrix} b \\ e \\ h \end{pmatrix} = T(P_2) = \alpha(P_2) = \nu \cdot \begin{pmatrix} 0 \\ -C \\ B \end{pmatrix}$$

$b = 0$

$e = -\nu C$

$h = \nu B$

$$\underline{T = \begin{pmatrix} -\mu C & 0 & 0 \\ 0 & -\nu C & 0 \\ \mu A & \nu B & \lambda \end{pmatrix}}$$

- Generelt punkt $P = (y_0 : y_1 : 0) \in \ell$

$$T(P) = \begin{pmatrix} -\mu C y_0 \\ -\nu C y_1 \\ \mu A y_0 + \nu B y_1 \end{pmatrix}$$ må ligge på ℓ'

$$PC : \begin{vmatrix} x_0 & x_1 & x_2 \\ y_0 & y_1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0$$

$\underline{y_1 \cdot x_0 - y_0 \cdot x_1 = 0}$

$T(P) \in PC$ gir

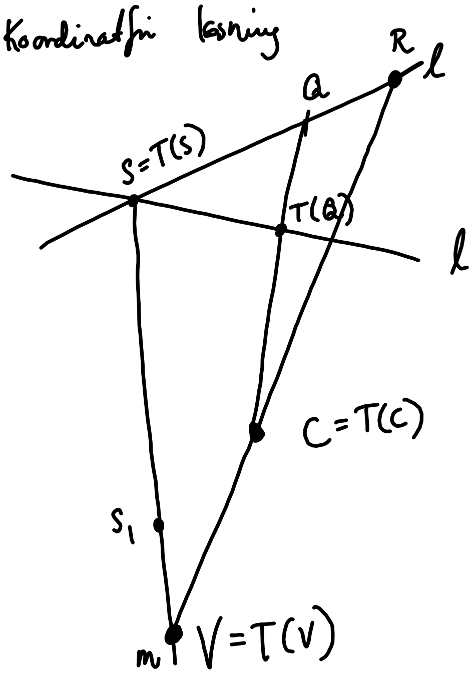
$$y_1(-\mu C y_0) - y_0(-\nu C y_1) = 0$$

$$C y_0 y_1 (-\mu + \nu) = 0 \qquad \text{så } \underline{\nu = \mu}$$

$$T = \begin{pmatrix} -\mu C & 0 & 0 \\ 0 & -\mu C & 0 \\ \mu A & \mu B & \lambda \end{pmatrix}$$ gir ønsket transformasjon

- Kan velge $\underline{\mu = \lambda = 1}$

Koordinatfri løsning



$Q \longmapsto T(Q)$

$C \longmapsto C = T(C)$

$S \longmapsto S = T(S)$

$S_1 \longmapsto S_1 = T(S_1)$

Fikserer m punktvis

$V = T(V)$

$C = T(C)$

$VC = RC = T(V)T(C)$

$T(R) = \alpha(R)$

6.6.7. $T: \mathbb{P}^2 \to \mathbb{P}^2$ $\qquad \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{pmatrix}$

Fikspunkt til T: $\quad T(P) = \lambda P \quad$ P egenvektor til egenverdi $\lambda \neq 0$.

$$0 = \begin{vmatrix} 2-\lambda & 1 & 0 \\ 0 & 1-\lambda & -1 \\ 0 & 2 & 4-\lambda \end{vmatrix} = (2-\lambda)\left[(1-\lambda)(4-\lambda) - 2(-1)\right]$$

$$= (2-\lambda)\left[4 - 4\lambda - \lambda + \lambda^2 + 2\right]$$

$$= (2-\lambda)(\underline{\lambda^2 - 5\lambda + 6})$$

$$= \underline{(2-\lambda)(\lambda-2)}(\lambda-3)$$

$\lambda_1 = 2 \qquad \lambda_2 = 3$

$m_1 = 2 \qquad m_2 = 1$

$\lambda_1 = 2$: $\left(\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 2 & 2 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & \underline{1} & 0 & 0 \\ 0 & 0 & \underline{1} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$$P_1 = (\lambda : 0 : 0) \sim \underline{\underline{(1 : 0 : 0)}}$$

$\lambda_2 = 3$: $\left(\begin{array}{ccc|c} -1 & 1 & 0 & 0 \\ 0 & -2 & -1 & 0 \\ 0 & 2 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$\overset{-2R_1 + R_2 \to R_1}{\sim} \left(\begin{array}{ccc|c} 2 & \underline{0} & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} \underline{1} & 0 & \frac{1}{2} & 0 \\ 0 & \underline{1} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$$P_2 = \left(-\tfrac{1}{2}t : -\tfrac{1}{2}t : t\right) \overset{t=-2}{\sim} \underline{\underline{(1 : 1 : -2)}}$$

P_1 og P_2 er fikspunktene til T.

II Finn alle linjer ℓ med $T(\ell) = \ell$

ℓ_{12}: P_1P_2 bør være en fiksert linje

$\ell: \; ax_0 + bx_1 + cx_2 = 0$

$T(\ell)$: Vilkårlig punkt $R = \begin{pmatrix} y_0 \\ y_1 \\ y_2 \end{pmatrix}$

$$T(R) = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{pmatrix} \cdot \begin{pmatrix} y_0 \\ y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 2y_0 + y_1 \\ y_1 - y_2 \\ 2y_1 + 4y_2 \end{pmatrix}$$

$R \in \ell \quad k\cdot(ay_0 + by_1 + cy_2) = 0$

$T(R) \in \ell \quad a(2y_0 + y_1) + b(y_1 - y_2) + c(2y_1 + 4y_2) = 0$

I $\quad y_0: \quad k\cdot a = 2\cdot a$

II $\quad y_1: \quad k\cdot b = a + b + 2c$

III $\quad y_2: \quad k\cdot c = -b + 4c$

(A) I: $a = 0$

II: $kb = b + 2c$

$(k-1)b - 2c = 0$

III $\quad kc = -b + 4c$

$b + (k-4)c = 0$

(B) $k = 2$

II $\quad 2b = a + b + 2c$

$0 = a - b + 2c$

III $\quad 2c = -b + 4c$

$0 = -b + 2c$

$\Big\} \; a = 0$

$$\left(\begin{array}{cc|c} k-1 & -2 & 0 \\ 1 & k-4 & 0 \end{array}\right)$$

$0 = (k-1)(k-4) - 1(-2)$

$= k^2 - 5k + 6$

$= (k-2)(k-3)$

$k=2$: $\left(\begin{array}{cc|c} 1 & -2 & 0 \\ 1 & -2 & 0 \end{array}\right) \sim \begin{array}{c} \begin{array}{cc} b & c \end{array} \\ \left(\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right) \end{array}$ $\quad a = 0$, $b = 2c$

$\ell: \; 2(0\cdot x_0 + 2c\cdot x_1 + c\cdot x_2) = 0$

$2c(2x_1 + x_2) = 0$

$2x_1 + x_2 = 0 \qquad P_1 = (1:0:0)$, $P_2 = (1:1:-2)$

$k=3$: $\left(\begin{array}{cc|c} 2 & -2 & 0 \\ 1 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right) \quad b = c,\; c \in \mathbb{R}$

$\ell: \; 3(cx_1 + cx_2) = 0$

$3c(x_1 + x_2) = 0$

$x_1 + x_2 = 0 \quad$ en linje gjennom P_1

Har med generaliserte egenvektorer å gjøre.

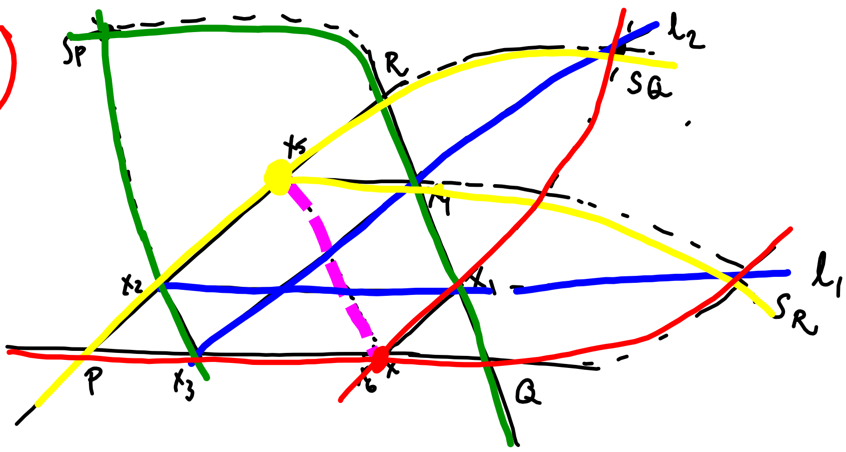
6.6.8 $T: \mathbb{P}^2 \longrightarrow \mathbb{P}^2$ har alltid minst ett fikspunkt.

- Nok å vise minst én ikke-null egenverdi
- Anta at alle egenverdier $= 0$

$$0 = \det(T - 0 \cdot I) = \det T$$

Umulig, for T er INVERTBEL matrise.
fordi T er en projektiv transformasjon

15. 1



$l_1 : \; X_1 \;\; X_2 \;\; S_R$

$l_2 : \; X_3 \;\; X_4 \;\; S_Q$

$X_1X_4 \cap X_2X_3$ — S_P

$X_2S_Q \cap X_4S_R$ — X_5

$X_3S_R \cap X_1S_Q$ — X

v/ Pappus' teorem er S_P, X_5 og X kollineære

$X \in X_5S_P$ og $X \in PQ$

v/ konstruksjon er X_6, S_P og X_5 kollineære

$X_6 \in X_5S_P$ og $X_6 \in PQ$

Så $X = X_6$

15 - 2 En mulig formulering:

La l_1 og l_2 være to linjer i $\mathbb{R}^2$.

La A_i, B_i, C_i være tre punkter på l_i.

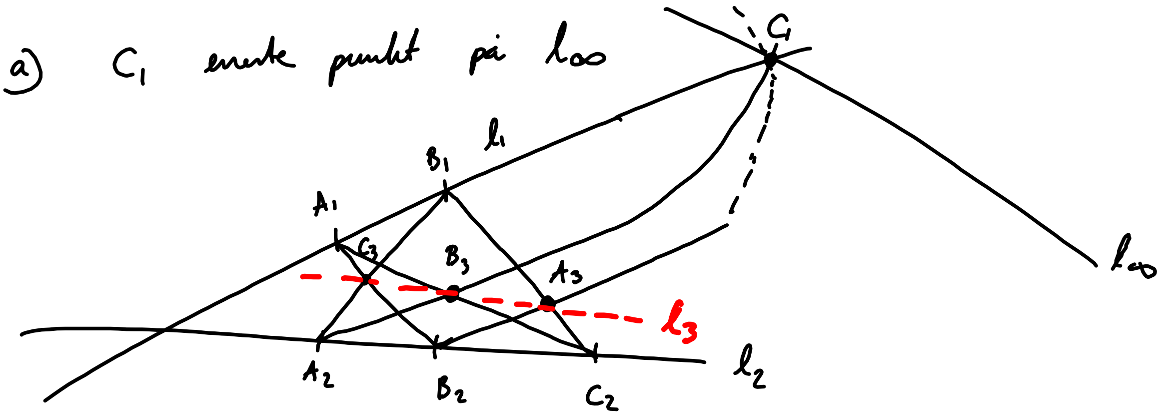
Hvis A_1B_2 snitter A_2B_1 i C_3

og B_1C_2 snitter B_2C_1 i A_3

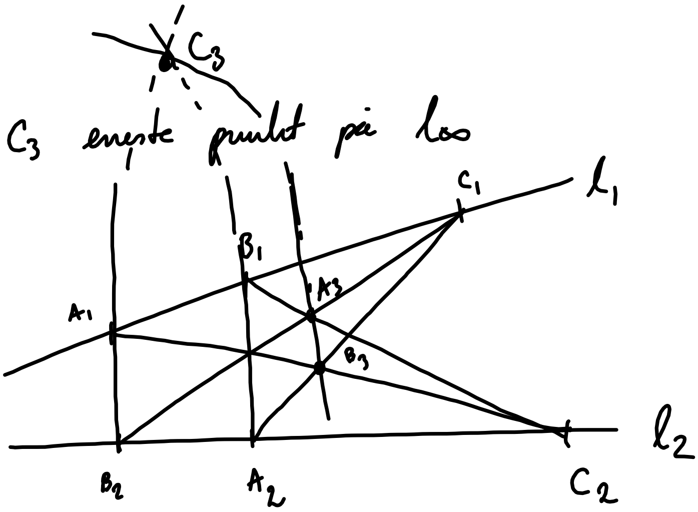
og C_1A_2 snitter C_2A_1 i B_3,

da er A_3, B_3, C_3 kollineære.

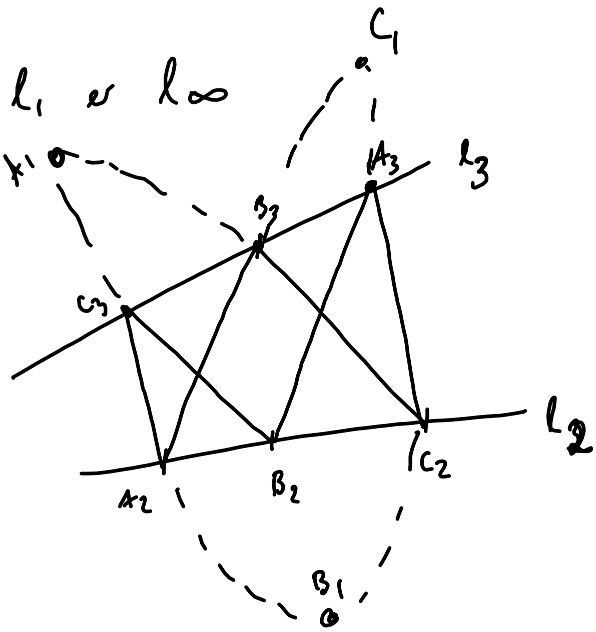
a) C_1 eneste punkt på l_∞



b) C_3 eneste punkt på l_∞



c) l_1 er l_∞



d) La A_1 B_2 og C_3 l_∞

$A_1B_2 \cap A_2B_1$