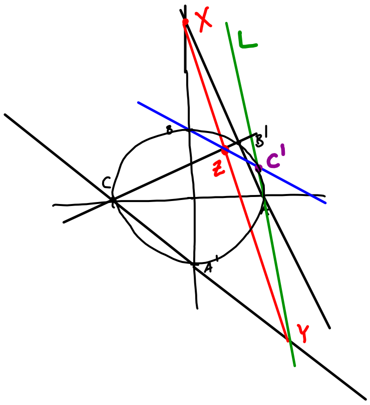
MAT 2500 10.11.2020

18-(1)

$A = (1:0:1)$
$B = (0:1:1)$
$C = (-1:0:1)$
$A' = (0:-1:1)$
$B' = (\frac{\sqrt{2}}{2} : \frac{\sqrt{2}}{2} : 1)$

$x_0^2 + x_1^2 = x_2^2$



$$AB': \quad 0 = \begin{vmatrix} x_0 & x_1 & x_2 \\ 1 & 0 & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 \end{vmatrix} = -\frac{\sqrt{2}}{2}x_0 + \left(\frac{\sqrt{2}}{2} - 1\right)x_1 + \frac{\sqrt{2}}{2}x_2$$

$$A'B: \quad 0 = \begin{vmatrix} x_0 & x_1 & x_2 \\ 0 & -1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = -2x_0 \quad : \quad x_0 = 0$$

$X = AB' \cap A'B$ $\qquad v_X = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} - 1, \frac{\sqrt{2}}{2}\right) \times (1,0,0)$

$\qquad = \left(0, \frac{\sqrt{2}}{2}, 1 - \frac{\sqrt{2}}{2}\right)$

$\underline{= \left(0 : \frac{\sqrt{2}}{2} : 1 - \frac{\sqrt{2}}{2}\right)}$

$A'C: \quad x_0 + x_1 + x_2 = 0$

$B'C: \quad -\frac{\sqrt{2}}{2}x_0 + \left(1 + \frac{\sqrt{2}}{2}\right)x_1 - \frac{\sqrt{2}}{2}x_2 = 0$

L, $A \in L \leadsto$ $\quad ax_0 + bx_1 + cx_2 = 0$

$\quad a + c = 0 \quad c = -a$

$L: \; ax_0 + bx_1 - ax_2 = 0 \qquad a \neq 0$

$\underline{L: \; x_0 + tx_1 - x_2 = 0} \qquad t = \frac{b}{a}$

$Y = A'C \cap L: \quad v_Y = (1,1,1) \times (1,t,-1)$

$\vdots$

$\underline{= (-1-t : 2 : t-1)}$

$$XY: \quad 0 = \begin{vmatrix} x_0 & x_1 & x_2 \\ 0 & \frac{\sqrt{2}}{2} & 1 - \frac{\sqrt{2}}{2} \\ -1-t & 2 & t-1 \end{vmatrix} = \left(\frac{\sqrt{2}}{2}t - 2 + \frac{\sqrt{2}}{2}\right)x_0 - (1+t)\left(1 - \frac{\sqrt{2}}{2}\right)x_1 + \frac{\sqrt{2}}{2}(1+t)x_2$$

$Z = B'C \cap XY \quad \leadsto \quad Z = (1+t : \sqrt{2} : 1 + \sqrt{2} - t)$

$$BZ: \quad 0 = \begin{vmatrix} x_0 & x_1 & x_2 \\ 0 & 1 & 1 \\ 1+t & \sqrt{2} & 1+\sqrt{2}-t \end{vmatrix} = (1-t)x_0 + (1+t)x_1 - (1+t)x_2$$

$C' = BZ \cap L \qquad v_{C'} = (1-t, 1+t, -1-t) \times (1, t, -1)$

$\qquad = (-1 + t^2, -2t, -1 - t^2)$

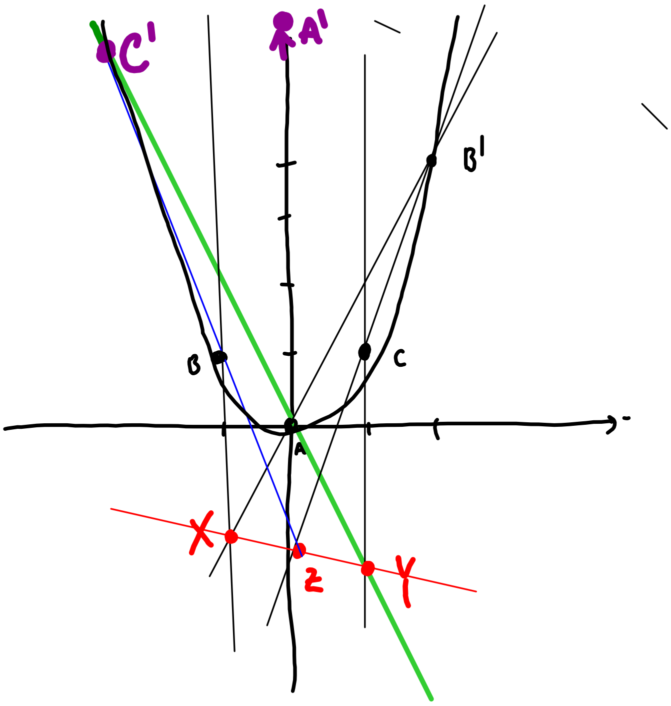
$\underline{\underline{C' = (1 - t^2 : 2t : 1 + t^2)}}$

$$\begin{aligned} x_0^2 + x_1^2 &= (1-t^2)^2 + (2t)^2 \\ &= 1 - 2t^2 + t^4 + 4t^2 \\ &= 1 + 2t^2 + t^4 \\ &= (1+t^2)^2 \\ &= x_2^2 \end{aligned}$$

$$\mathcal{C}: \quad \begin{array}{cccccc} x_0^2 & x_1^2 & x_2^2 & x_1x_2 & x_0x_2 & x_0x_1 \end{array}$$
$$\begin{vmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 1 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{1}{2} \end{vmatrix} \begin{pmatrix} A \\ B \\ C \\ A' \\ B' \end{pmatrix} = 0$$

18-1 b)

$A = (0:0:1)$
$B = (-1:1:1)$
$C = (1:1:1)$
$A' = (0:1:0)$
$B' = (2:4:1)$



$x_1 \cdot x_2 - x_0^2 = 0$

$y = x^2$

$$(4.2) \quad Q = [X, A, B] \cdot [X, C, A'] \cdot [B', A, A'] \cdot [B', B, C] - [X, A, A'] \cdot [X, B, C] \cdot [B', A, B] \cdot [B', C, A']$$

$$= (-x_0 - x_1) \cdot (-x_0 + x_2) \cdot (-2) \cdot 6 - (-x_0)(2x_1 - 2x_2)(-6)(-1)$$

$$= 12 x_1 x_2 - 12 x_0^2 = 0$$

$$\underline{x_1 x_2 - x_0^2 = 0}$$

18-2

$$\mathbb{P}^1 \simeq \mathbb{R} \cup \{\infty\}$$

$$(x:1) \longleftarrow\!\shortmid\ x$$

$$(1:0) \longleftarrow\!\shortmid\ \infty$$

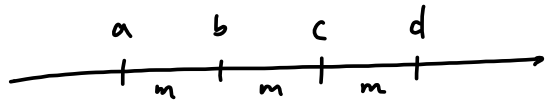
KRYSSFORHOLD:

$$(a,b;c,d) = \frac{(c-a)(b-d)}{(b-c)(d-a)}$$

med konvensjon $\frac{\infty}{\infty} = 1$

$-\frac{\infty}{\infty} = -1$

a)



$$c-a = 2m$$
$$b-d = -2m$$
$$b-c = -m$$
$$d-a = 3m$$

$$(a,b;c,d) = \frac{2m(-2m)}{-m \cdot 3m} = \underline{\underline{\frac{4}{3}}}$$

b)

$$c-a = m$$
$$b-d = 2m-d$$
$$b-c = 2m-m = m$$
$$d-a = d$$

$$(a,b;c,d) = \frac{\cancel{m}(2m-d)}{\cancel{m} \cdot d} = \frac{2m-d}{d} = \frac{2m}{d} - 1$$

$$\lim_{d\to\infty} \frac{2m}{d} - 1 = \underline{\underline{-1}}$$

3

$$r = (a,b;c,d) = \frac{(c-a)(b-d)}{(b-c)(d-a)}$$

- $$(a,b;d,c) = \frac{(d-a)(b-c)}{(b-d)(c-a)} = \underline{\underline{\frac{1}{r}}}$$

- $$(b,a;c,d) = \frac{(c-b)(a-d)}{(a-c)(d-b)} = \frac{\cancel{(-1)}(b-c)\cancel{(-1)}(d-a)}{\cancel{(-1)}(c-a)\cancel{(-1)}(b-d)}$$
$$= \underline{\underline{\frac{1}{r}}}$$

b) $$(a,c;b,d) = \frac{(b \overset{+0}{-} a)(c-d)}{(c-b)(d-a)}$$

$$= \frac{(b \underline{-d+d} -a)(c-d)}{(c-b)(d-a)}$$

$$= \frac{(b-d)(c \overset{+0}{-} d) + (d-a)(c-d)}{(c-b)(d-a)}$$

$$= \frac{(b-d)(c \underline{-a+a} - d) + (d-a)(c-d)}{(c-b)(d-a)}$$

$$= \frac{(b-d)(c-a) - (d-a)(b \cancel{-d} - (c \cancel{-d}))}{(c-b)(d-a)}$$

$$= \underline{\underline{-r+1}}$$

(19) EKSAMEN 2012 – 3

$\mathbb{P}^2 \qquad (x_0 : x_1 : x_2)$

① $\ell_a \qquad A = (1:0:2)$

$B_a = (0:2:a)$

$$\ell_a : \quad 0 = \begin{vmatrix} x_0 & x_1 & x_2 \\ 1 & 0 & 2 \\ 0 & 2 & a \end{vmatrix} = \underline{-4x_0 - a x_1 + 2x_2 = 0}$$

② $\ell_{-1} : \ -4x_0 + x_1 + 2x_2 = 0 \qquad x_1 = 4x_0 - 2x_2$

$C : \quad x_0 x_1 - x_2^2 = 0$

$$C \cap \ell_{-1} : \quad x_0(4x_0 - 2x_2) - x_2^2 = 0$$

$$4x_0^2 - 2x_0x_2 - x_2^2 = 0 \qquad \underline{x_2 = 1}$$

$$4x_0^2 - 2x_0 - 1 = 0$$

$$x_0 = \frac{2 \pm \sqrt{4 + 16}}{2 \cdot 4}$$

$$\underline{x_0 = \frac{1 \pm \sqrt{5}}{4}} \qquad \text{(to punkter)}$$

$$x_1 = 4x_0 - 2x_2 = 4\left(\frac{1 \pm \sqrt{5}}{4}\right) - 2 \cdot 1$$

$$= \underline{-1 \pm \sqrt{5}} \qquad \text{(svarer til hver sin } x_1\text{)}$$

$$\underline{P_1 = \left(\frac{1+\sqrt{5}}{4} : -1+\sqrt{5} : 1\right) \qquad P_2 = \left(\frac{1-\sqrt{5}}{4} : -1-\sqrt{5} : 1\right)}$$

③ $l_a: \; -4x_0 - a x_1 + 2x_2 = 0$

$$x_0 = -\frac{a}{4}x_1 + \frac{1}{2}x_2$$

$C \cap l_a: \; \left(-\frac{a}{4}x_1 + \frac{1}{2}x_2\right)x_1 - x_2^2 = 0$

$$-\frac{a}{4}x_1^2 + \frac{1}{2}x_1x_2 - x_2^2 = 0 \qquad (-4)$$

$$a x_1^2 - 2x_1x_2 + 4x_2^2 = 0 \qquad \underline{x_2 = 1}$$

$$a x_1^2 - 2x_1 + 4 = 0$$

$$D = (-2)^2 - 4\cdot a\cdot 4 = 0$$ (tangering → én løsning)

$$4 - 16a = 0$$

$$a = \frac{1}{4}$$

$$x_1 = \frac{-(-2)}{2a} = \frac{2}{2\cdot\frac{1}{4}} = 4$$

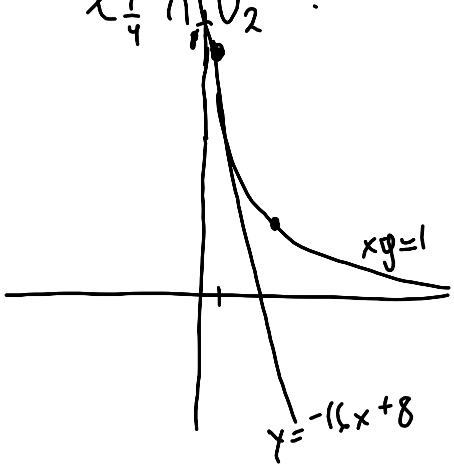
$$x_0 = -\frac{1}{4}\cdot\frac{1}{4}\cdot 4 + \frac{1}{2}\cdot 1 = \frac{1}{4}$$

$$P = \left(\frac{1}{4} : 4 : 1\right)$$

$C \cap U_2: \quad xy - 1 = 0 \qquad \underline{xy = 1}$

$l_{\frac{1}{4}} \cap U_2: \quad -4x - \frac{1}{4}y + 2 = 0$

$$y = -16x + 8$$



E 2011 - 3 $\quad \mathbb{P}^2_{\mathbb{R}} \quad (x_0 : x_1 : x_2)$

a) $L_1: \ x_0 + x_1 + 2x_2 = 0 \qquad P = L_1 \cap L_2$

$L_2: \ 3x_0 - x_1 + 4x_2 = 0$

$$v_P = (1,1,2) \times (3,-1,4)$$
$$= (6,2,-4)$$
$$P = (6:2:-4) \sim (3:1:-2) \sim \left(-\tfrac{3}{2} : -\tfrac{1}{2} : 1\right)$$

$Q = (0:0:1)$

$$l_{PQ}: \quad 0 = \begin{vmatrix} x_0 & x_1 & x_2 \\ 0 & 0 & 1 \\ 3 & 1 & -2 \end{vmatrix} = -x_0 + 3x_1$$

$$x_0 - 3x_1 = 0$$

b) $\mathcal{C}: \ x_0^2 + x_1^2 + 2x_0x_2 - 6x_1x_2 = 0$

$l_\infty: x_2 = 0$

6.13. $\quad D = 0^2 - 4 \cdot 1 \cdot 1 < 0$

C skjærer l_∞ i ingen reelle punkter

C er en ellipse i U_2 $(x_2 \neq 0)$

$$\mathcal{C} \cap U_2: \quad \frac{x_0^2}{x_2^2} + \frac{x_1^2}{x_2^2} + 2\frac{x_0}{x_2} \cdot \frac{x_2}{x_2} - 6\frac{x_1}{x_2} \cdot \frac{x_2}{x_2} = 0$$

$$x^2 + y^2 + 2x - 6y = 0$$
$$\underbrace{x^2 + 2x + 1}_{(x+1)^2} - 1 + \underbrace{y^2 - 2 \cdot y \cdot 3 + 3^2}_{(y-3)^2} - 3^2 = 0$$
$$(x+1)^2 + (y-3)^2 = 10$$

Sirkel med sentrum $(-1,3)$ og radius $\sqrt{10}$

c) $C \cap l_{PQ} \qquad l_{PQ}: x_0 = 3x_1$

$$C \cap l_{PQ}: \ (3x_1)^2 + x_1^2 + \underline{2(3x_1) \cdot x_2} \ \underline{- 6x_1x_2} = 0$$
$$10x_1^2 = 0$$
$$x_1 = 0 \quad \text{(dobbelt)}$$
$$x_0 = 0$$

$$C \cap l_{PQ} = (0:0:1) = Q$$