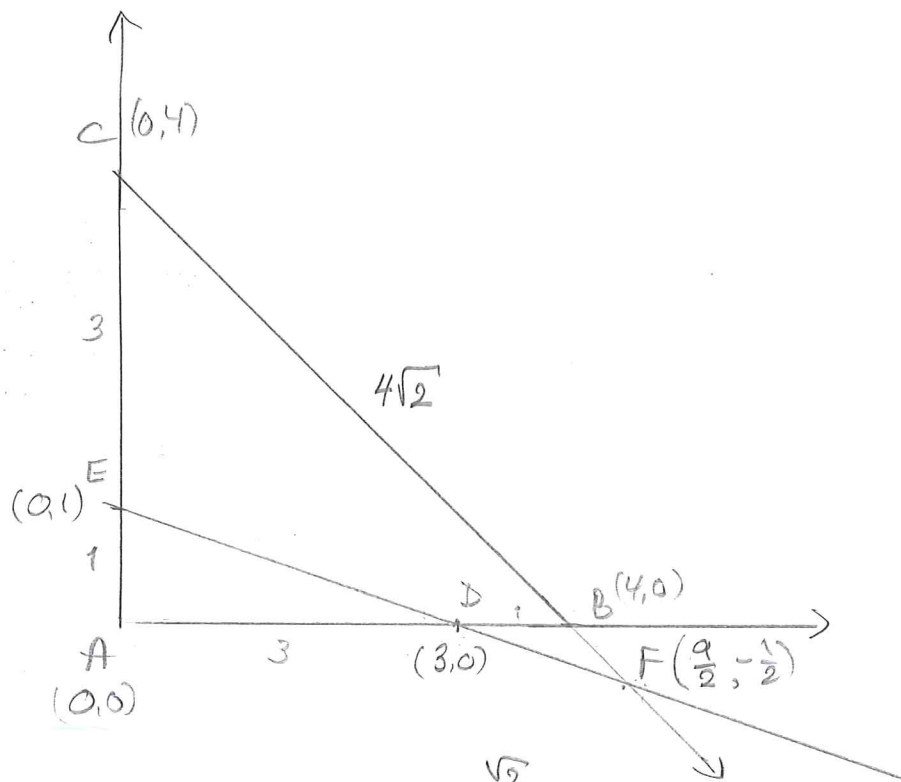


Menelaos setzung - et eksempel.



$$\frac{\overline{AD}}{\overline{DB}} \cdot \frac{\overline{BF}}{\overline{FC}} \cdot \frac{\overline{CE}}{\overline{EA}} = \frac{3}{1} \cdot \frac{\frac{\sqrt{2}}{2}}{-\frac{9\sqrt{2}}{2}} \cdot \frac{-3}{-1} = \underline{\underline{-1}}$$

$$\begin{aligned} \overline{BF} &= \sqrt{\left(\frac{9}{2} - 4\right)^2 + \left(-\frac{1}{2} - 0\right)^2} \\ &= \sqrt{\frac{1}{4} + \frac{1}{4}} = \frac{\sqrt{2}}{2} \end{aligned}$$

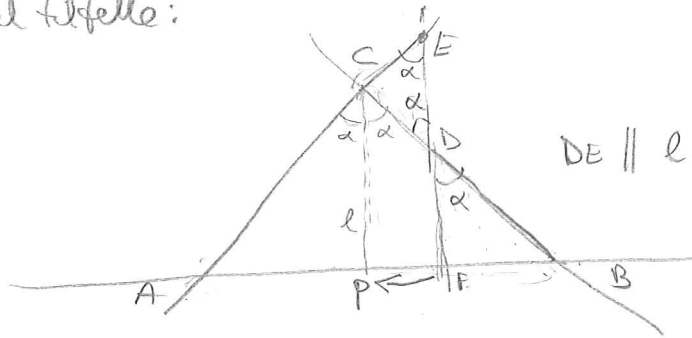
$$\begin{aligned} \overline{FC} &= -\frac{\sqrt{2}}{2} - 4\sqrt{2} \\ &= -\frac{9\sqrt{2}}{2} \end{aligned}$$

Spezial fälle:

$$\angle ACP = \angle AEF$$

$$\angle PCB = \angle FDB$$

4.



$DE \parallel l \Rightarrow CE = CD$ (Hörner?)

$$\frac{BD}{DC} \cdot \frac{AF}{FB} \cdot \frac{CE}{AE} = 1$$

La $ED \rightarrow C$. Grenze fälle $E=C=D$, ger (F halbiert AB)
 $\Downarrow$
 $\frac{AF}{FB} = 1$

$$\frac{BD}{DC} \cdot \frac{CE}{EA} = \frac{BD}{EA} \rightarrow \frac{BC}{CA}$$

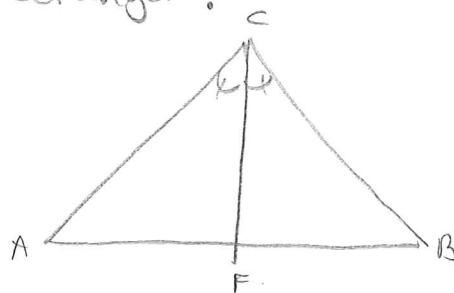
Kombiniert mod

$$\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = 1 \quad (\text{abs. Verdi})$$

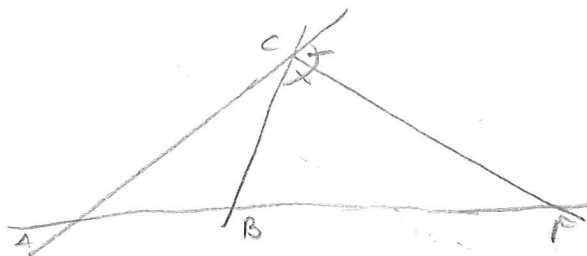
zu $\frac{BC}{CA} \cdot \frac{AF}{FB} \rightarrow 1$

oder i
grenze $\frac{BC}{CA} \Rightarrow \frac{FB}{AF} = \frac{PB}{AB}$

Halbierungse - setzungen:

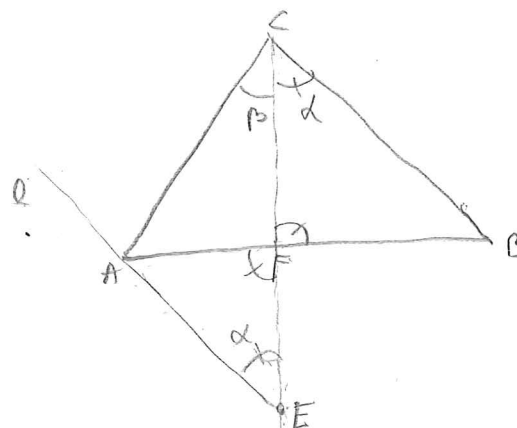


Inwendig $\frac{AC}{CB} = \frac{AF}{FB}$



Uwendig $\frac{AF}{FB} = -\frac{AC}{CB}$

Beis



$e \parallel BC$

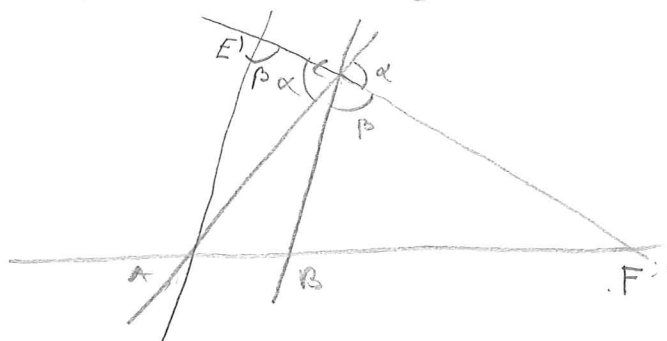
$$\triangle AFE \sim \triangle BFC$$

$$\Rightarrow \frac{AF}{BF} = \frac{AE}{BC}$$

Vi har $AE = AC \Leftrightarrow \alpha = \beta$.

dvs $\frac{AF}{BF} = \frac{AC}{BC} \Leftrightarrow \alpha = \beta$ (dvs halvering av $\angle C$.)

Tilsvarende for utvendig:

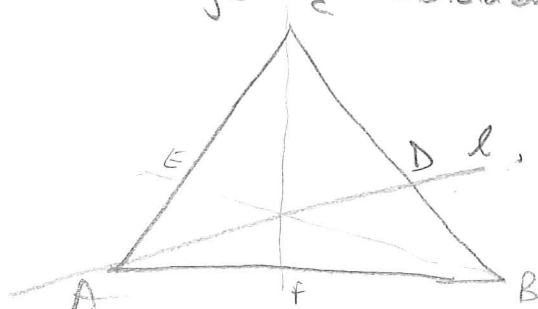


$$\triangle AFE' \sim \triangle BFC \Rightarrow \frac{AF}{BF} = \frac{AE'}{BC}$$

Vi har $AE' = AC \Leftrightarrow \alpha = \beta$

dvs $\frac{AF}{BF} = \frac{AC}{BC} \Leftrightarrow \alpha = \beta$ (havens av ytre vinkel i C)

Definisjon Ceva-linje (Linje gjennom Menelaos-punkt og motsatte hjørner)



$A \in l, B, C \notin l$

}
Ceva-linje

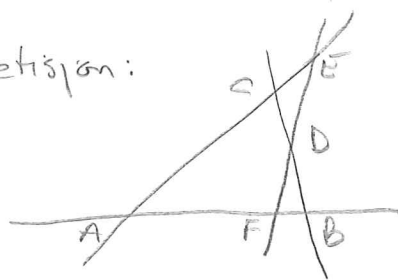
Cevas setning:

Dersom tre Ceva-linjer AD, BE, CF er konkurrente (som betyr at møtes i ett punkt), så er

$$\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = 1$$

Motsatt, hvis relasjonen holder, så er Ceva-linjen parallell eller konkurrente.

Repetisjon:



Menelaos-punkter

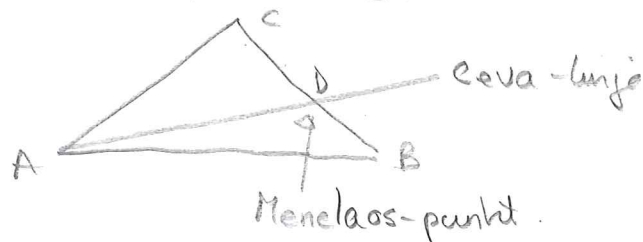
D for BC

E for CA

F for AB

$$\text{Kolineære} \Leftrightarrow \frac{\overline{BD}}{\overline{DC}} \cdot \frac{\overline{CE}}{\overline{EA}} \cdot \frac{\overline{AF}}{\overline{FB}} = -1.$$

Definisjon: En linje gjennom et hjørne i $\triangle ABC$ som ikke faller sammen med noen av linjene i trekanten kalles en Ceva-linje



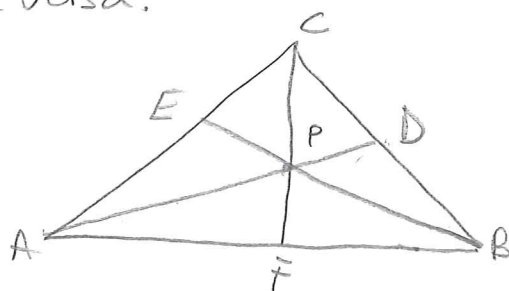
Cevas setning

Dersom tre Ceva-linjer AD, BE og CF i $\triangle ABC$ er KONKURRENTE, så er

Møtes i ett punkt.

$$\frac{\overline{BD}}{\overline{DC}} \cdot \frac{\overline{CE}}{\overline{EA}} \cdot \frac{\overline{AF}}{\overline{FB}} = 1.$$

Og vice versa.



Bevis. ($\Rightarrow$)

Menelaos' setning på $\triangle ABD$ og $\triangle ADC$

$$\left(\frac{\overline{BC}}{\overline{CD}} \cdot \frac{\overline{DP}}{\overline{PA}} \cdot \frac{\overline{AF}}{\overline{FB}} \right) \left(\frac{\overline{DB}}{\overline{BC}} \cdot \frac{\overline{CE}}{\overline{EA}} \cdot \frac{\overline{AP}}{\overline{PD}} \right) = (-1) \cdot (-1) = 1.$$

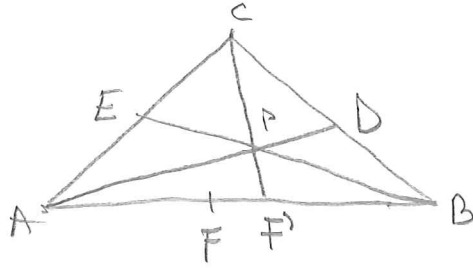
$$\frac{\overline{DB}}{\overline{CD}} \cdot \frac{\overline{CE}}{\overline{EA}} \cdot \frac{\overline{AF}}{\overline{FB}} = \frac{\overline{BD}}{\overline{DC}} \cdot \frac{\overline{CE}}{\overline{EA}} \cdot \frac{\overline{AF}}{\overline{FB}} = 1.$$

($\Leftarrow$)

Anta

$$\frac{\overline{BD}}{\overline{DC}} \cdot \frac{\overline{CE}}{\overline{EA}} \cdot \frac{\overline{AF}}{\overline{FB}} = 1$$

(og anta at ingen av Ceva-linjene er parallelle)



Det betyr at CF er en Ceva-linje og de tre linjene AD , BE og CF er konkurrerte.

Første del av beviset gir

$$\frac{\overline{BD}}{\overline{DC}} \cdot \frac{\overline{CE}}{\overline{EA}} \cdot \frac{\overline{AF'}}{\overline{F'B}} = 1$$

Antagelsen er at

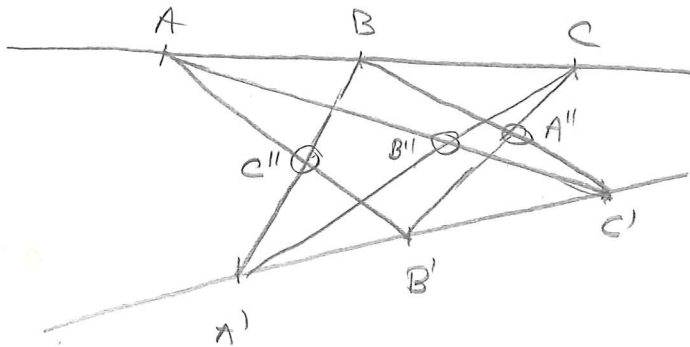
$$\frac{\overline{BD}}{\overline{DC}} \cdot \frac{\overline{CE}}{\overline{EA}} \cdot \frac{\overline{AF}}{\overline{FB}} = 1$$

Det gir

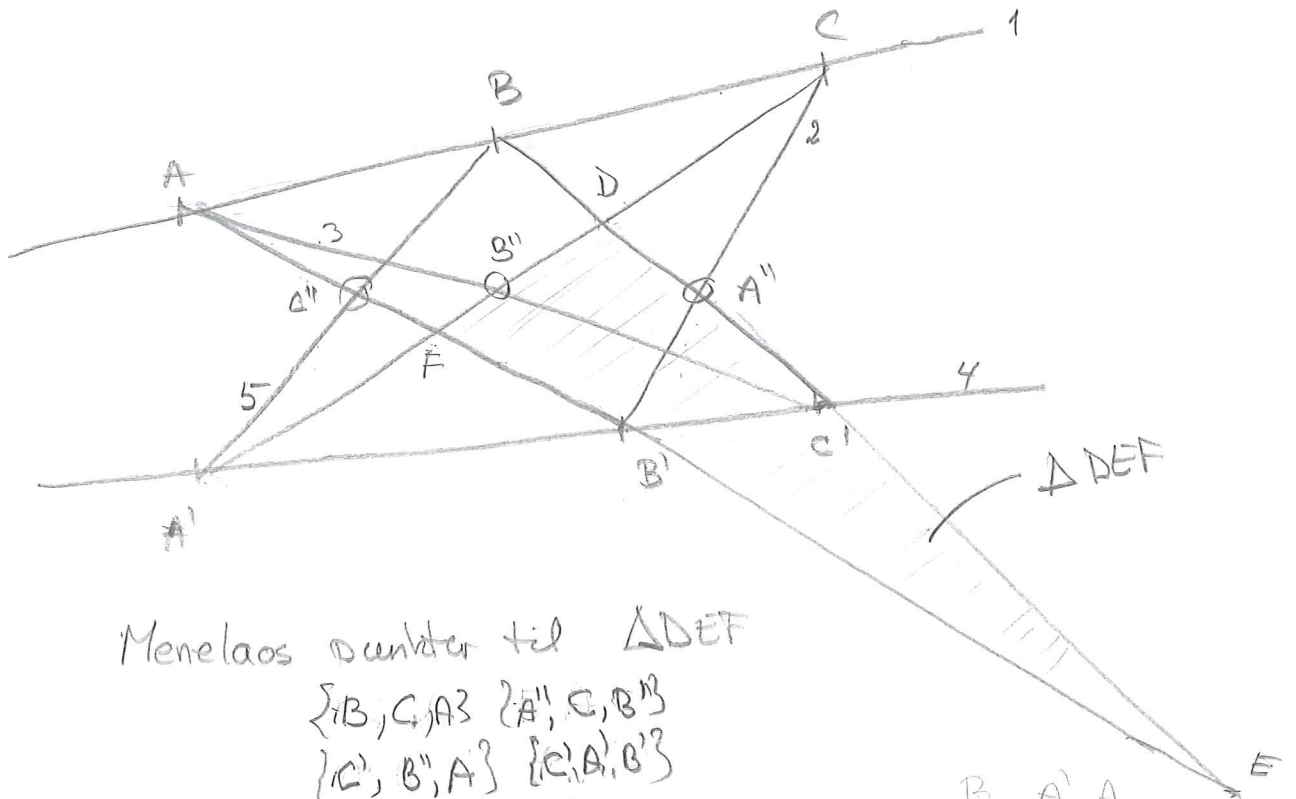
$$\frac{\overline{AF'}}{\overline{F'B}} = \frac{\overline{AF}}{\overline{FB}}$$

Delingsforholdet bestemmer delingspunktet entydig og det følger at $F = F'$.

Pappos setning



A'' , B'' og C'' er
kollineære.



Menelaos punkter til $\triangle DEF$

$$\{B, C, A\} \{A', C, B'\}$$

$$\{C', B', A\} \{C, A', B'\}$$

$$\{B, A', C'\}$$

$$\begin{array}{ccc} B & A' & A \\ A' & B' & C' \\ C' & C & B \end{array}$$

Menelaos setning gir

$$\frac{EB}{BD} \cdot \frac{DC}{CF} \cdot \frac{FA}{AE} = -1$$

$$\frac{EA''}{A''D} \cdot \frac{DC}{CF} \cdot \frac{FB'}{B'E} = -1$$

$$\frac{EC'}{C'D} \cdot \frac{DB''}{B''F} \cdot \frac{FA}{AE} = -1$$

$$\frac{EC'}{C'D} \cdot \frac{DA'}{A'F} \cdot \frac{FB'}{B'E} = -1$$

$$\frac{EB}{BD} \cdot \frac{DA'}{A'F} \cdot \frac{FC''}{C''E} = -1$$

Som gitt:

$$\left(\frac{EB}{BD} \cdot \frac{DC}{CF} \cdot \frac{FA}{AE}\right) \left(\frac{EA}{AD} \cdot \frac{BC}{CF} \cdot \frac{FB}{BE}\right) \left(\frac{EC}{CD} \cdot \frac{DB}{BF} \cdot \frac{DA}{AF}\right) \left(\frac{ED}{DB} \cdot \frac{BF}{FD} \cdot \frac{FC}{CE}\right) = -1$$

eller senere et

$$\frac{\cancel{BD}}{\cancel{EB}} \cdot \frac{\cancel{CF}}{\cancel{DC}} \cdot \frac{\cancel{AE}}{\cancel{FA}} \cdot \frac{EA}{AD} \cdot \frac{BC}{CF} \cdot \frac{FB}{BE} \cdot \frac{EC}{CD} \cdot \frac{DB}{BF} \cdot \frac{DA}{AF} \cdot \frac{ED}{DB} \cdot \frac{BF}{FD} \cdot \frac{FC}{CE} = -1$$

$$\frac{EA}{AD} \cdot \frac{DB}{BF} \cdot \frac{FC}{CE} = -1$$

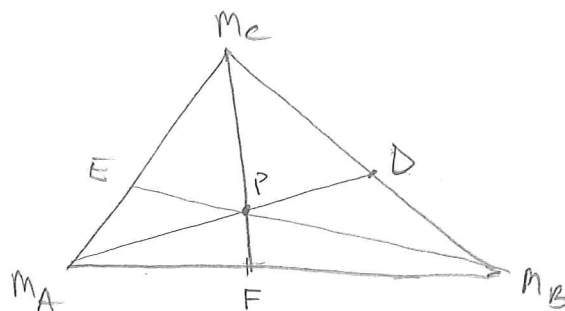
$\Rightarrow A'', B'', C''$ er kollinear.

Menelaos av Alexandria

Levde trolig 70-140 e.Kr

Giovanni Ceva 1647-1734. Italienisk

Cevas mekanisk bevis



Tyngdepunkt för A og B

$$AF \cdot m_A = FB \cdot m_B$$

$$\frac{AF}{FB} = \frac{m_B}{m_A}$$

— " — Bog C

$$BD \cdot m_B = CD \cdot m_C$$

$$\frac{BD}{CD} = \frac{m_C}{m_B}$$

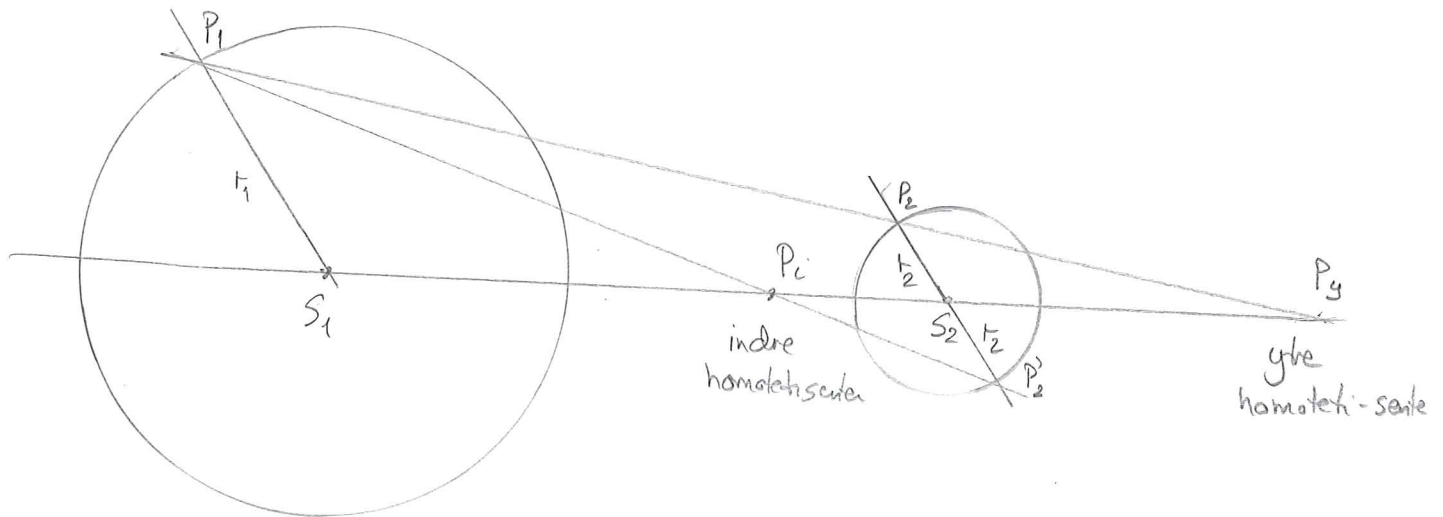
— " — A og C

$$AE \cdot m_A = EC \cdot m_C$$

$$\frac{AE}{EC} = \frac{m_C}{m_A}$$

Alle linjerne skærer hinanden i P : er ekvivalent med

$$\frac{m_B}{m_A} \cdot \frac{m_A}{m_C} \cdot \frac{m_C}{m_B} = \frac{AF}{FB} \cdot \frac{EC}{AE} \cdot \frac{BD}{CD} = 1$$



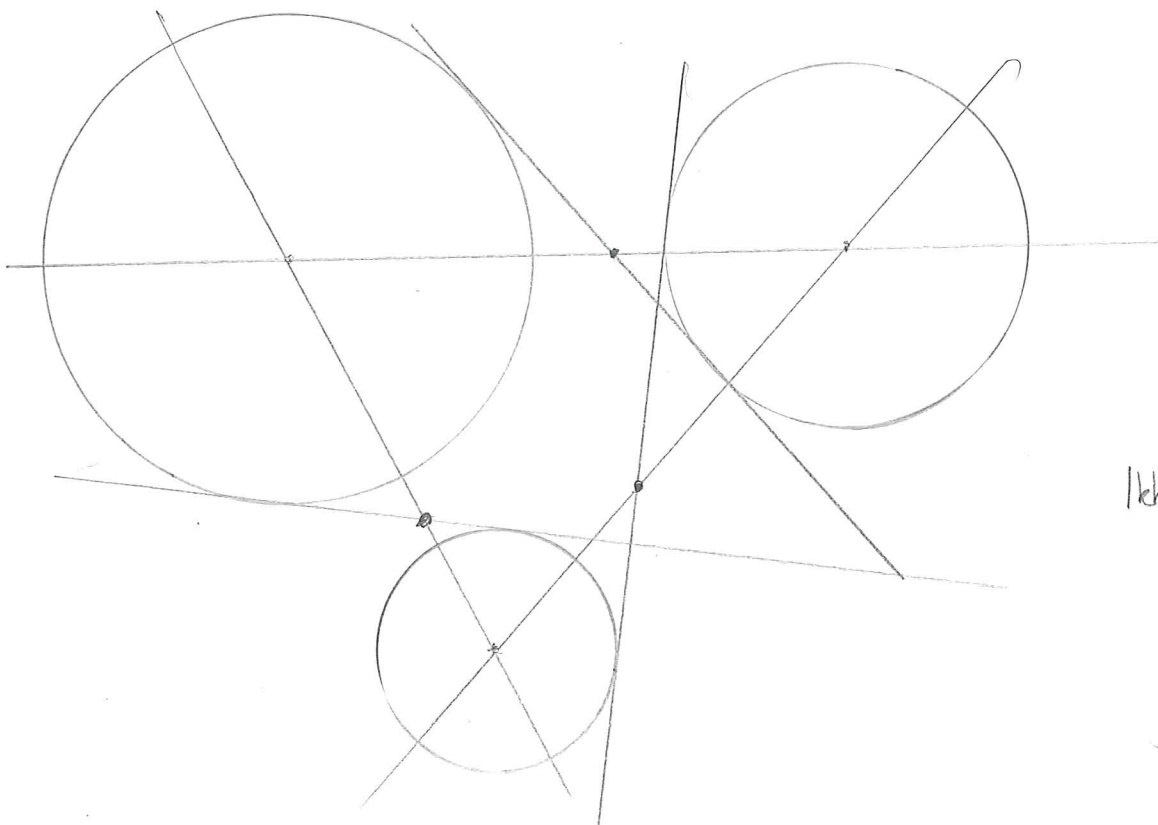
$$\triangle S_1 P_1 P_y \sim \triangle S_2 P_2 P_y \Rightarrow \frac{S_1 P_1}{S_1 P_y} = \frac{S_2 P_2}{S_2 P_y}$$

$$\Rightarrow \frac{S_1 P_y}{S_2 P_y} = \frac{r_1}{r_2}$$

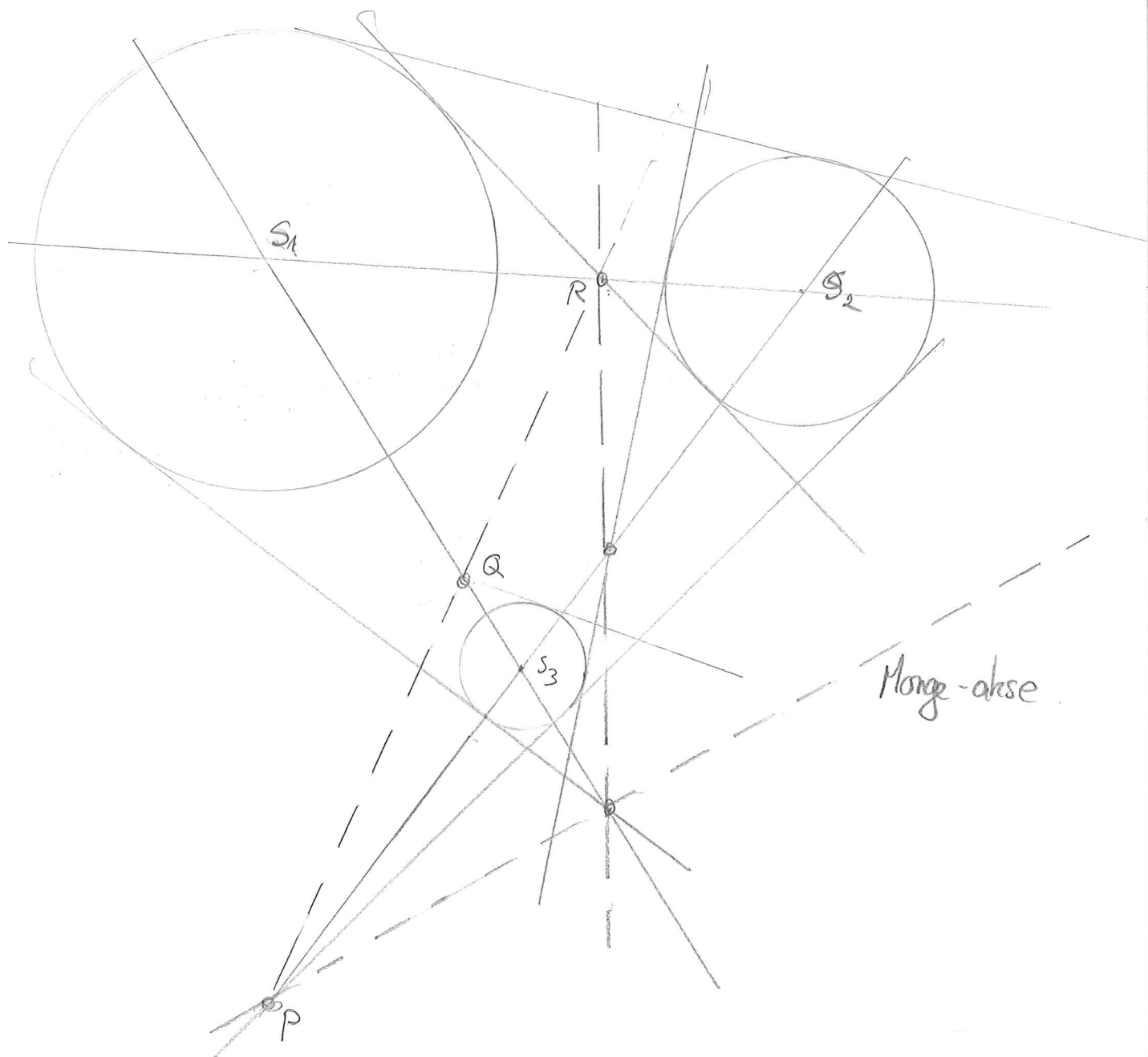
uavhengig av valget av P_1 (og P_2)

$$\triangle S_1 P_1 P_i \sim \triangle S_2 P_2' P_i$$

$$\Rightarrow \frac{S_1 P_1}{S_1 P_i} = \frac{S_2 P_2'}{S_2 P_i} \Rightarrow \frac{S_1 P_i}{S_2 P_i} = \frac{r_1}{r_2}$$



Ikke te indre !



Monge-akse : Linje gjennom 3 ytre homotetisentre
 eller 2 indre og 1 indre homotetisentr
 → Kolliare

Bevis: Menelaos på $\triangle S_1 S_2 S_3$

$$\frac{\overline{S_1 R}}{\overline{R S_2}} \cdot \frac{\overline{S_2 P}}{\overline{P S_3}} \cdot \frac{\overline{S_3 Q}}{\overline{Q S_1}} = -1$$

$$= \frac{r_1}{r_2} \cdot \frac{r_2}{r_3} \cdot \frac{-r_3}{-r_1} = \underline{\underline{-1}}$$