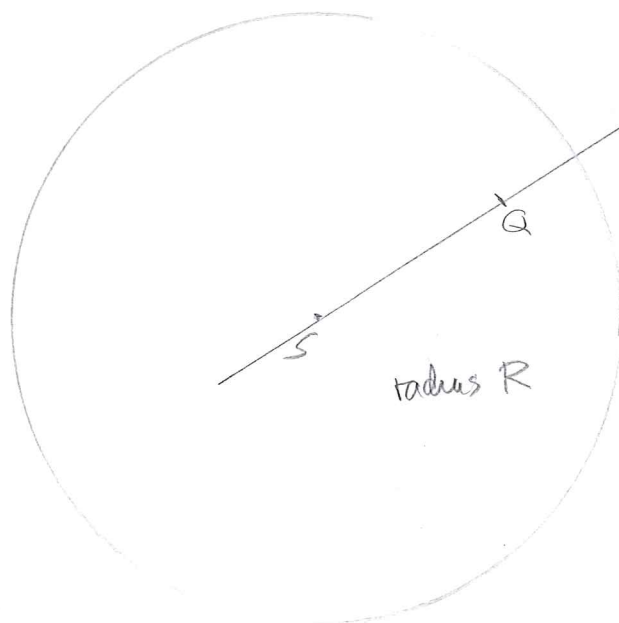
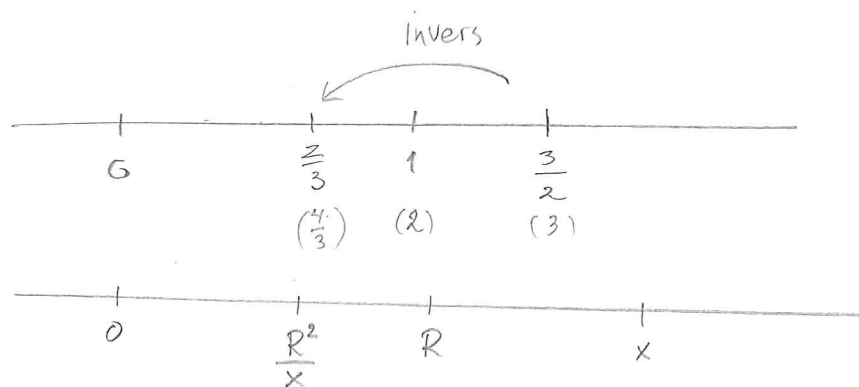
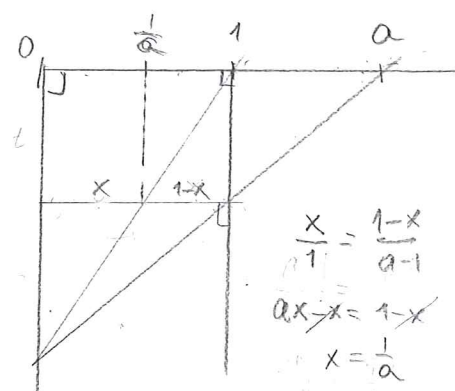


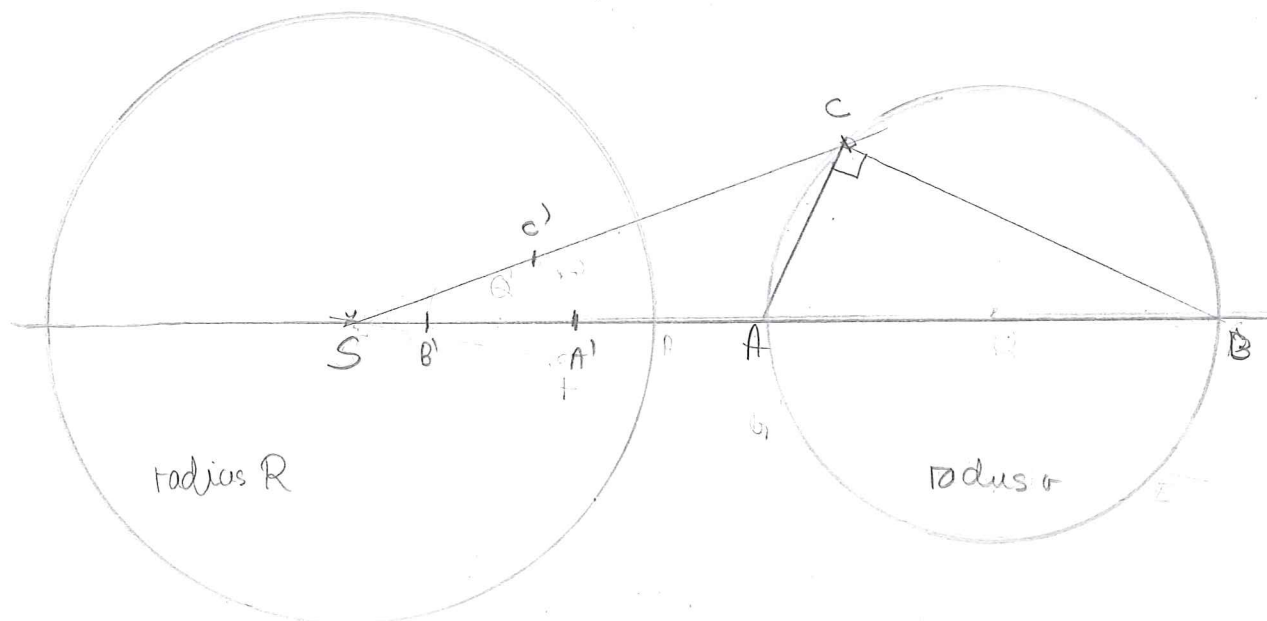
Inversjon over en sirkel.



$$SP \cdot SQ = R^2$$



Inversen til en sirkel er en sirkel



$$SP \cdot SQ = R^2$$

La C være et punkt på en sirkel, Ved Thales setning er $\angle ACB$ rett. La Inverteringen av A, B og C være A', B', C' ,

Vi har $SA \cdot SA' = SB \cdot SB' = SC \cdot SC' = R^2$.

Det følger at.

$$\frac{SA}{SC} = \frac{SC'}{SA'} \Rightarrow \triangle SAC \sim \triangle SC'A' \quad (\text{felles vinkel})$$

$$\frac{SB}{SC} = \frac{SC'}{SB'} \Rightarrow \triangle SBC \sim \triangle SC'B'$$

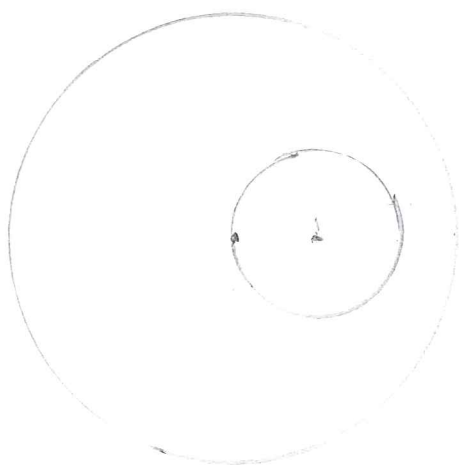
$$\Rightarrow \begin{aligned} \angle CAB &= \pi - \angle CAS \\ \angle CBA &= \pi - \angle A'C'S \\ \angle CBA &= \angle CBS = \angle B'C'S \end{aligned}$$

$$\begin{aligned} \text{som gir } \frac{\pi}{2} &= \angle CAB + \angle CBA \\ &= \pi - \angle A'C'S + \angle B'C'S \\ &= \pi - \angle A'C'B' \end{aligned}$$

$$\text{dvs. } \angle A'C'B' = \frac{\pi}{2}$$

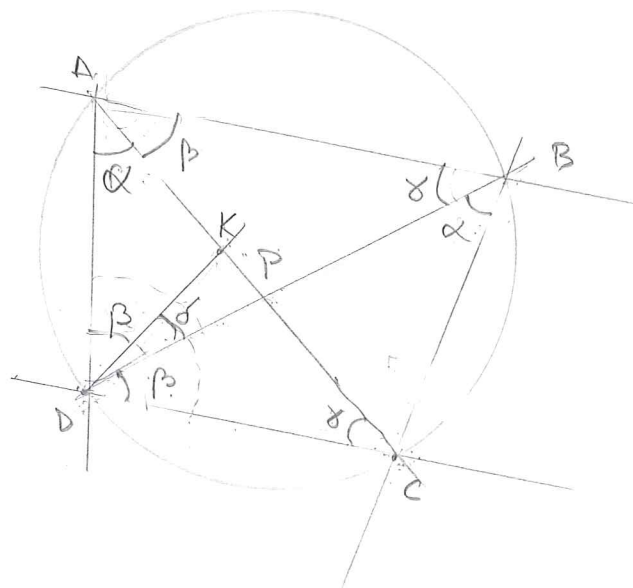
og ved Thales setning ligger C' på en sirkel gjennom A' og B' .

Konstruksjon av en rett linje:



Sirkel gjennom ∞

Ptolemaus setning.



K satt slik at
 $\angle ADK = \angle BDC$

□ ABCD
 Syklisk

$$\Leftrightarrow AC \cdot BD = AB \cdot CD + AD \cdot BC$$

Bevis ($\Rightarrow$)

$$\angle DAB = \angle DBC = \alpha$$

$$\angle ADB = \angle KDC$$

$$\angle BDC = \angle BAC = \beta$$

$$\angle ADK = \angle BDC = \beta$$

$$\angle ADB = \angle ACB$$

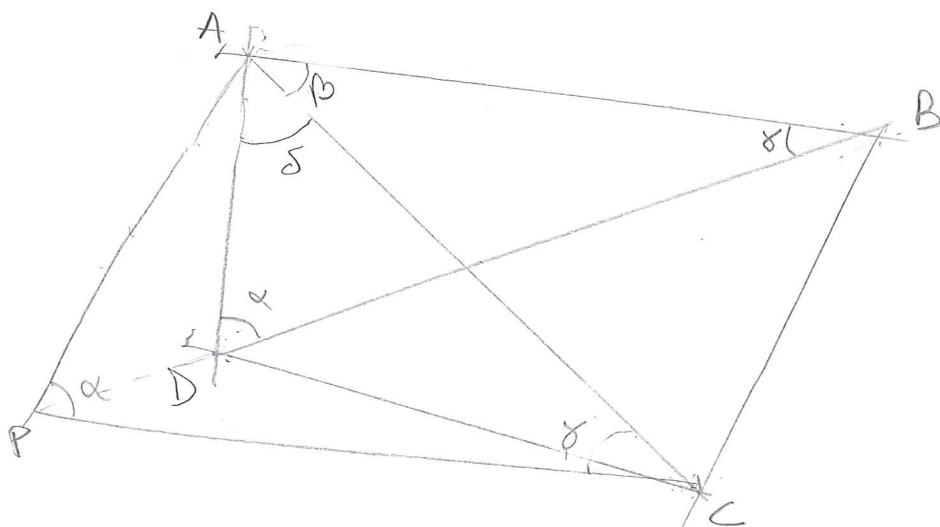
$$\Rightarrow \triangle ADK \sim \triangle BDC \quad \text{og} \quad \triangle ADB \sim \triangle KDC$$

(α, β) ($\delta, \beta + \delta$)

$$\Rightarrow \frac{AK}{AD} = \frac{BC}{BD} \quad \frac{CK}{DC} = \frac{AB}{BD}$$

$$\Rightarrow AC \cdot BD = (AK + KC) BD = AK \cdot BD + KC \cdot BD$$

$$= AD \cdot BC + AB \cdot CD$$



Antag $AC \cdot BD = AB \cdot CD + AD \cdot BC$

($\Leftarrow$)

Velg P slik at $\angle ACP = \angle ABD = \gamma$ og $\angle PAC = \angle ABD = \alpha$

$$\Rightarrow \triangle ABD \sim \triangle ACP$$

(α, γ)

som betyr $\frac{AB}{AC} = \frac{AD}{AP} = \frac{BD}{CP}$

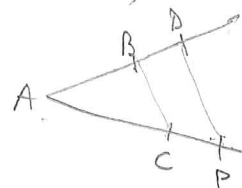
$$\Rightarrow CP = \frac{AC \cdot BD}{AB} \quad \frac{AB}{AD} = \frac{AC}{AP}$$

Vi har $\angle BAC = \beta = \pi - \gamma - \alpha - \delta$

$$\angle DAP = \pi - \alpha - \gamma - \delta = \beta$$

dvs $\triangle BAC \sim \triangle DAP$ (elles vinkel, samme forhold på motsatt side)

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AP} = \frac{BC}{DP}$$



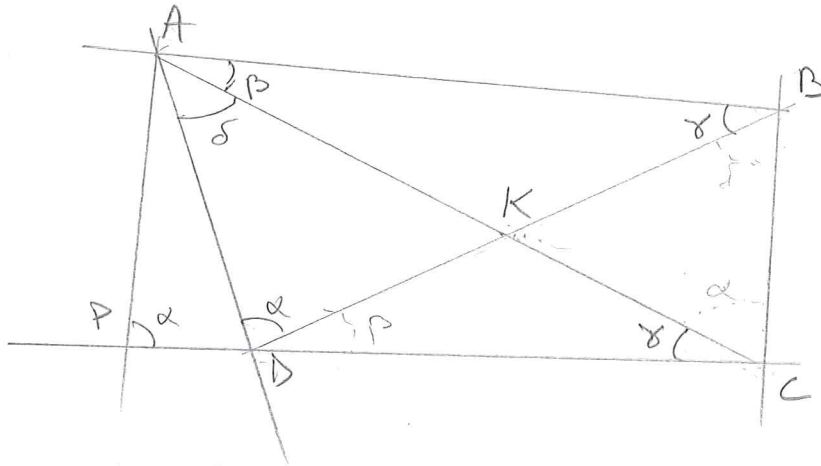
og dermed

$$CD + DP = CD + \frac{BC \cdot AD}{AB}$$

$$= \frac{AB \cdot CD + BC \cdot AD}{AB} = \frac{AC \cdot BD}{AB} = CP$$

Maa C, D, P er kollineære.

Vi må endre tegningen



K : skjærings mellom AC og BD .

$$\triangle ABK \sim \triangle DCK$$

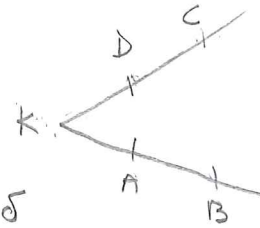
β, δ

$$\Rightarrow \frac{AB}{CD} = \frac{AK}{DK} = \frac{BK}{CK}$$

$$\Rightarrow \frac{DK}{CK} = \frac{AK}{BK}$$

$$\Rightarrow \triangle ADK \sim \triangle BCK$$

$$\Rightarrow \angle CBD = \angle DAC = \delta$$



$$\Rightarrow \delta + \beta + \delta + \delta + \alpha + \delta + \beta + \alpha = 2(\alpha + \beta + \delta + \delta) = 2\pi$$

$$\Rightarrow (\beta + \delta) + (\alpha + \delta) = \pi \quad \text{Sirkulær.}$$

BEED.