

Forelesning, 6. oktober

Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Parametrisere med $x = a \cos t$
 $y = b \sin t$
 $0 \leq t \leq 2\pi$

Hyperbel $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ Sette $x = \frac{a}{\cos t}$ $y = b \cdot \tan t$ $-\pi \leq t \leq \pi$
 $t \neq \pm \frac{\pi}{2}$

$$\left(\frac{a}{\cos t}\right)^2 - \frac{(b \tan t)^2}{b^2} = \frac{1}{\cos^2 t} - \frac{\sin^2 t}{\cos^2 t} = \frac{1 - \sin^2 t}{\cos^2 t} = \frac{\cos^2 t}{\cos^2 t} = 1$$

Alternativt $x = a \cosh u$, $y = b \sinh u$ $u \in \mathbb{R}$.

Når $t \rightarrow \pm \frac{\pi}{2}$, så $\frac{a}{\cos t}$, $b \tan t \rightarrow \infty$.

Men $\frac{\frac{a}{\cos t}}{b \frac{\sin t}{\cos t}} = \frac{a}{b \sin t} \xrightarrow{t \rightarrow \pm \frac{\pi}{2}} \pm \frac{a}{b}$ dus asymptoter $bx = \pm ay$

Tangentlinjer til ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Implisitt derivasjon $\frac{d}{dx} \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \right)$ gir

$$2x \frac{dx}{dx} + 2y \frac{dy}{dx} = 0 \quad | a^2 b^2$$

$$2b^2 x + 2a^2 y \cdot y' = 0$$

For et punkt (p, q) på ellipsen har vi

$$y' = -\frac{b^2 p}{a^2 q}$$

Som gi tangent linje

$$y - q = \frac{-b^2 p}{a^2 q} (x - p) \quad \text{eller} \quad \frac{px}{a^2} + \frac{qy}{b^2} = \frac{p^2}{a^2} + \frac{q^2}{b^2} = 1$$

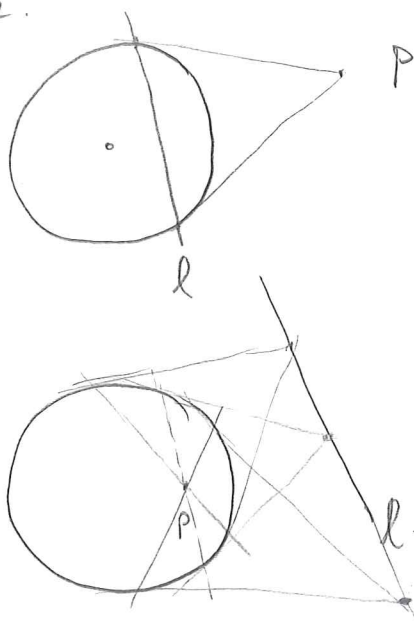
For hyperbler $\frac{px}{a^2} - \frac{qy}{b^2} = 1$.

For parabler $y^2 = 4cx$; $qy = 2c(x+p)$

Pol og polare

Pol er et punkt, polare er en linje.
Ved hjelp av en sirkel er de nært knytt.

Geometrisk.



$l(P)$ polaren til P .

$P(l)$ polen til l .

Analytisk

$$P = (x_0, y_0)$$

$$l(P): \frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = 1$$

skjæring med ellipse

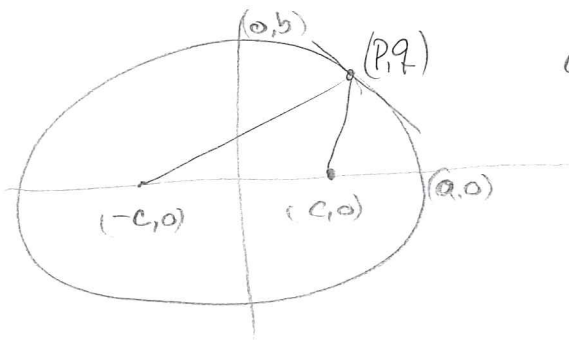
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{a^2} + \left(\left(1 - \frac{x_0 x}{a^2} \right) \frac{b^2}{y_0} \right)^2 \frac{1}{b^2} = 1$$

$$a=b=1$$

$$x^2 + \frac{1}{y_0^2} - 2 \frac{x_0 x}{y_0^2} + \frac{x_0^2 x^2}{y_0^2} = 1 \quad \Leftrightarrow \quad \frac{x^2}{a^2} + \frac{b^2}{y_0^2} - 2 \frac{x_0 x}{y_0^2} + \frac{x_0^2 x^2 b^2}{a^4 y_0^2} = 1$$

Spilingsregenskap



$$a^2 - b^2 = c^2$$

$$q^2 = b^2 \left(1 - \frac{p^2}{a^2}\right) = b^2 - \frac{b^2 p^2}{a^2}$$

Tangentlinje i (p, q) : $\frac{px}{a^2} + \frac{qy}{b^2} = 1$

Retningsvektor $(qa^2, -pb^2)$

$(-c, 0) \rightarrow (p, q)$ Retningsvektor $(p+c, q)$ Normert $\frac{1}{\sqrt{q^2 + (p+c)^2}} (p+c, q)$
 $(c, 0) \rightarrow (p, q)$ — " — $(p-c, q)$ — " — $\frac{1}{\sqrt{q^2 + (p-c)^2}} (p-c, q)$

$$(qa^2, -pb^2) \cdot \frac{1}{\sqrt{q^2 + (p \pm c)^2}} (p \pm c, q)$$

$$= \frac{1}{\sqrt{q^2 + (p \pm c)^2}} (qa^2 p \pm qa^2 c - pq b^2)$$

$$pq c^2 \pm qa^2 c = qc (pc \pm a^2)$$

$$= \frac{qc (pc \pm a^2)}{\sqrt{q^2 + (p \pm c)^2}} = \frac{qc (pc \pm a^2)}{\frac{1}{a} (a^2 \pm pc)} = \pm qac$$

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$$q^2 + (p \pm c)^2 = b^2 - \frac{b^2 p^2}{a^2} + p^2 \pm 2pc + c^2$$

$$= b^2 + c^2 + \frac{p^2}{a^2} (a^2 - b^2) \pm 2pc$$

$$= a^2 + \frac{p^2}{a^2} c^2 \pm 2pc = \frac{1}{a^2} (a^4 \pm 2pca^2 + p^2 c^2)$$

$$= \frac{1}{a^2} (a^2 \pm pc)^2$$