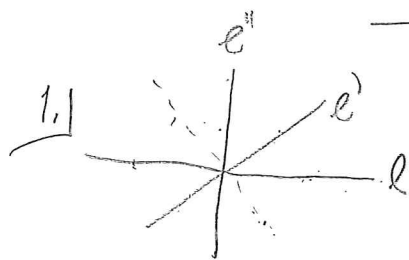


2018



a) $S_{e''} \circ S_l$: Flipspektel: omgevoerd
 Winkel $2 \cdot \frac{\pi}{2} = \pi$

$S_{e'} \circ S_{e''} \circ S_l$: Spiegeling om lijn g oriënt

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \xrightarrow{S_l} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \xrightarrow{S_{e''}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \xrightarrow{S_{e'}} \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\Rightarrow \text{Retraais } \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} : y = -x$$

Alternatief

$$\left(I - 2 \frac{\begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \end{pmatrix}}{\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}} \right) \left(I - 2 \frac{\begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix}}{\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}} \right) \left(I - 2 \frac{\begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix}}{\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}} \right)$$

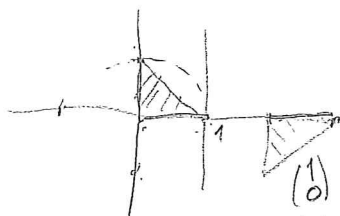
$$= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - 2 \frac{\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \begin{pmatrix} v_1 & v_2 \end{pmatrix}}{\begin{pmatrix} v_1 & v_2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}}$$

$$\text{Antw } v_1^2 + v_2^2 = 1$$

$$\begin{pmatrix} v_1^2 & v_1 v_2 \\ v_1 v_2 & v_2^2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$\Rightarrow v_1 = \pm \frac{1}{\sqrt{2}}, v_2 = \pm \frac{1}{\sqrt{2}} \text{ alle teg. } \underline{y = -x}$$

b) m: spiegeling om $x=1$, $g = S_m \circ S_{\pi}$



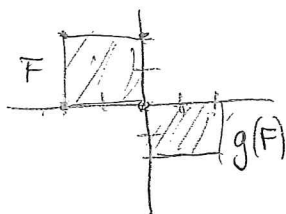
$$\begin{aligned} \begin{pmatrix} 1 \\ 0 \end{pmatrix} &\mapsto \begin{pmatrix} 2 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 1 \end{pmatrix} &\mapsto \begin{pmatrix} 2 \\ 1 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \end{pmatrix} &\mapsto \begin{pmatrix} 2 \\ 0 \end{pmatrix} \end{aligned}$$

$$g \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

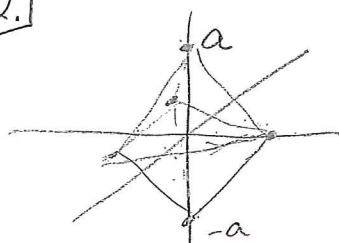
spiegeling om
x-aksen

c)



Symmetrie Rotaspi om oors S_{π}
 spiegeling om $y=x$

2.1



$$a) \|(0,0,a) - (1,0,0)\| = \|(1,0,0) - (\frac{1}{2}, \frac{\sqrt{3}}{2}, 0)\|$$

$$\sqrt{1+a^2} = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{3}$$

$$\Rightarrow a = \sqrt{2}$$

Noen har grad 3, andre 4.

b) Rotasjonsgrupper $\mathbb{Z}_3 \rightarrow 6 \rightarrow \mathbb{Z}_2$
 $\uparrow$
 S_3 (tel trekant)

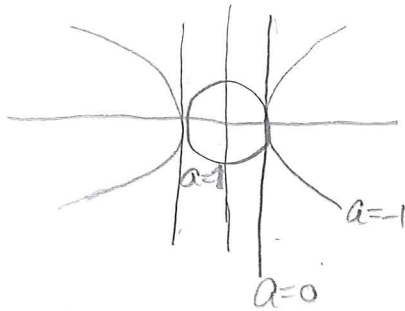
3.)

$$Q_a: x_0^2 + ax_1^2 - x_2^2 = 0$$

$$\mathbb{R}^2 \subset \mathbb{RP}^2$$

$$x_2 \neq 0.$$

a)



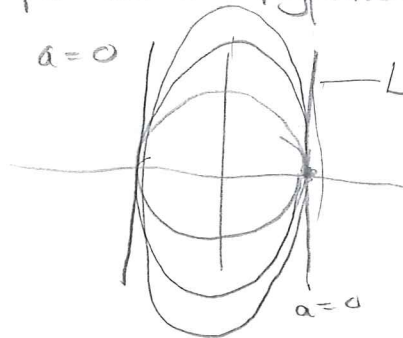
$$\left(\frac{x_0}{x_2}\right)^2 + a\left(\frac{x_1}{x_2}\right)^2 = 1 \quad \text{eller} \quad x^2 + ay^2 = 1$$

b)

Ellipse for $a > 0$ Hyperbel for $a < 0$.

c)

Deg. for $a = 0$



d)

$$P = (1:0:1) \quad Q = (1:1:1)$$

$$L: \begin{vmatrix} x_0 & x_1 & x_2 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -x_0 + x_2 = 0.$$

$$L \cap Q_a: \begin{cases} x_0^2 + ax_1^2 - x_2^2 = 0 \\ -x_0 + x_2 = 0 \end{cases} \quad \left. \begin{array}{l} ax_1^2 = 0 \\ x_2 = x_0 \\ x_0^2 - x_1^2 = 0 \end{array} \right\} \quad \begin{array}{l} \text{dvs. } a \neq 0 \quad x_1 = 0 \\ x_0 = x_2 \\ \text{ell punkt} \\ (1:0:1) = P \end{array}$$

$$\text{f. d. h. } a = 1, x_2 = 1: (1:0:1)$$

$$a = 1, x_2 = 1$$

e)

$$R: L \cap \infty\text{-ferne, dvs } x_2 = 0: R = (0:1:0)$$

$$L': \text{gerade } (0:1:0), \text{ tangente } Q_a, a \neq 0.$$

$$L' = \begin{vmatrix} x_0 & x_1 & x_2 \\ 0 & 1 & 0 \\ \alpha & \beta & \gamma \end{vmatrix} = \gamma x_0 - \alpha x_2 = 0$$

$$\text{Schneide } Q_a: \gamma = 0, \alpha \neq 0 \Rightarrow x_2 = 0 \quad x_0^2 + ax_1^2 = 0$$

$$\Delta = -4a = 0 \Rightarrow a = 0 \text{ degenerat}$$

$$\gamma \neq 0 \quad x_0 = \frac{\alpha}{\gamma} x_2 \quad \left(\frac{\alpha}{\gamma}\right)^2 + ax_1^2 - x_2^2 = 0$$

$$\left(\frac{\alpha^2}{\gamma^2} - 1\right)x_0^2 + ax_1^2 = 0$$

$$\Delta = -4a\left(\frac{\alpha^2}{\gamma^2} - 1\right) = 0 \Rightarrow \alpha = \gamma \text{ eller } \alpha = -\gamma$$

$$\alpha = \gamma \text{ giv } L, \quad \alpha = -\gamma \text{ giv } L' = x_0 + x_2 = 0.$$