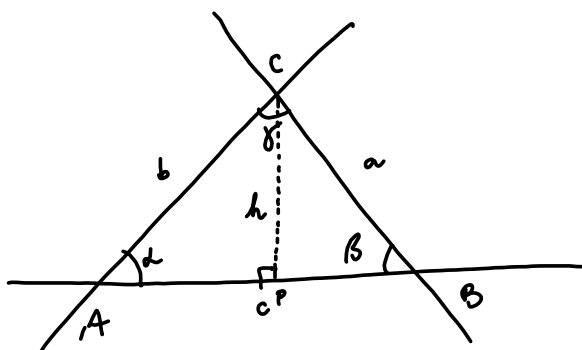


MAT 2500

24.08.2022

VIKTIGE TREKANTSETNINGER



SINUSSETNINGEN

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

BEVIS $\sin \alpha = \frac{h}{b}$ $\sin \beta = \frac{h}{a}$

$$h = b \sin \alpha = a \sin \beta$$

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} \quad \square$$

COSINUSSETNINGEN

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

BEVIS

① $\cos \alpha = \frac{AP}{b} \quad \cos \beta = \frac{BP}{a}$

$\gamma = \frac{\pi}{2}$
 $\downarrow$
 $\cos \gamma = 0$
 Pythagoras

$$c = AP + BP = b \cos \alpha + a \cos \beta$$

$$c^2 = \underline{bc \cdot \cos \alpha + ac \cdot \cos \beta}$$

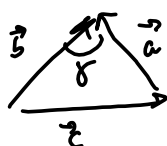
Tilsvarende:

$$a^2 = ab \cos \gamma + ac \cdot \cos \beta$$

$$b^2 = ab \cdot \cos \gamma + bc \cdot \cos \alpha$$

$$a^2 + b^2 = 2ab \cos \gamma + \underbrace{ac \cdot \cos \beta + bc \cdot \cos \alpha}_{c^2}$$

②



$$\vec{c} + \vec{a} = \vec{b}$$

$$\vec{c} = \vec{b} - \vec{a}$$

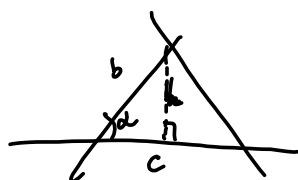
$$\vec{c} \cdot \vec{c} = (\vec{b} - \vec{a}) \cdot (\vec{b} - \vec{a})$$

$$= \vec{b} \cdot \vec{b} + \vec{a} \cdot \vec{a} - 2\vec{a} \cdot \vec{b}$$

$$\cos \gamma = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|}$$

$$\|\vec{c}\|^2 = \|\vec{b}\|^2 + \|\vec{a}\|^2 - 2 \|\vec{a}\| \|\vec{b}\| \cdot \cos \gamma$$

AREALSÆNINGEN



$$A_{\text{real}} = \frac{1}{2} \cdot c \cdot h$$

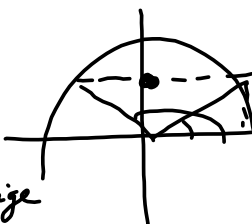
$$h = b \sin \alpha$$

$$A_{\text{real}} = \frac{1}{2} bc \sin \alpha$$

OBS $\alpha, \beta, \gamma \in [0, \pi)$

Cossetning kan brukes både!

Sinus/arealsetn kan gi to mulige
vedier både!



TRIGONOMETRISKE IDENTITETER

①  $\sin^2 \theta + \cos^2 \theta = 1$

② $\sin(u+v) = \sin u \cdot \cos v + \sin v \cdot \cos u$
 $\cos(u+v) = \cos u \cdot \cos v - \sin u \cdot \sin v$

$$e^{i(u+v)} = \cos(u+v) + i \sin(u+v)$$

$$\begin{aligned} e^{iu} \cdot e^{iv} &= (\cos u + i \sin u)(\cos v + i \sin v) \\ &= \cos u \cdot \cos v + (-1) \sin u \cdot \sin v \\ &\quad + i \sin u \cos v + i \sin v \cdot \cos u \end{aligned}$$

③ $\sin(u+v)$

$$\int_0^{x_1} \frac{dt}{\sqrt{1-t^2}} + \int_0^{x_2} \frac{dt}{\sqrt{1-t^2}} = \int_0^{x_3} \frac{dt}{\sqrt{1-t^2}}$$

$$\arcsin x_1 - 0 + \arcsin x_2 - 0 = \arcsin x_3 - 0$$

$$\sin(\arcsin x_1 + \arcsin x_2) = x_3$$

$$u = \arcsin x_1 \quad v = \arcsin x_2$$

$$\sin u = x_1 \quad \sin v = x_2$$

$$\begin{aligned} x_3 &= \sin(u+v) \stackrel{②}{=} \sin u \cdot \cos v + \cos u \cdot \sin v \\ &= x_1 \cdot \sqrt{1-x_2^2} + x_2 \cdot \sqrt{1-x_1^2} \end{aligned}$$

∴

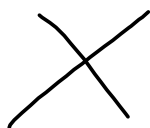
ABELS ADDISJONSTEOREM

Kommer tilbake til dette

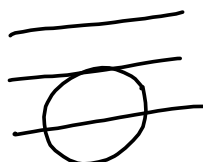
KONSTRUKSJON

- ① Blankt ark
- ② Marker punkter
- ③ Rett linje gjennom to punkter
- linjalen har ingen målestokk
- ④ Vi kan tegne sirkler om et punkt
- KLASSISK: kan ikke huske avstand
- VI: husker avstand

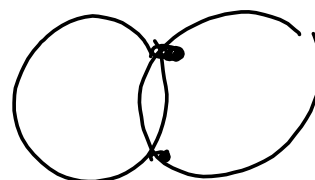
OBS:



logisk

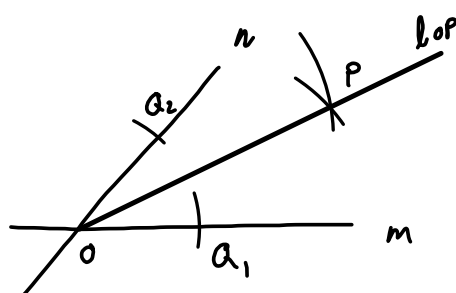


logisk



intuitiv

4.3.A HALVERE EN VINKEL

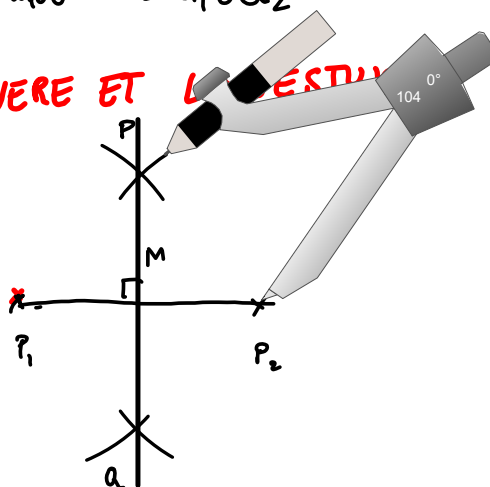


$$OQ_1 = OQ_2$$

$$PQ_1 = PQ_2$$

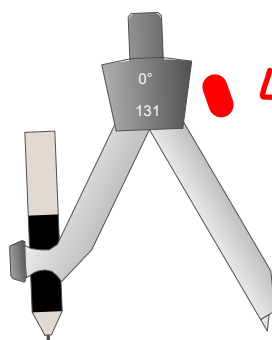
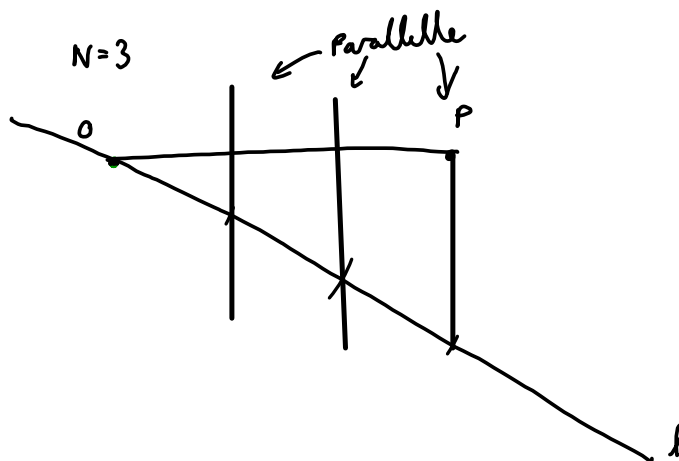
l_{OP} halverer $\angle Q_1 O Q_2$

4.3.B HALVERE ET LØSSTU

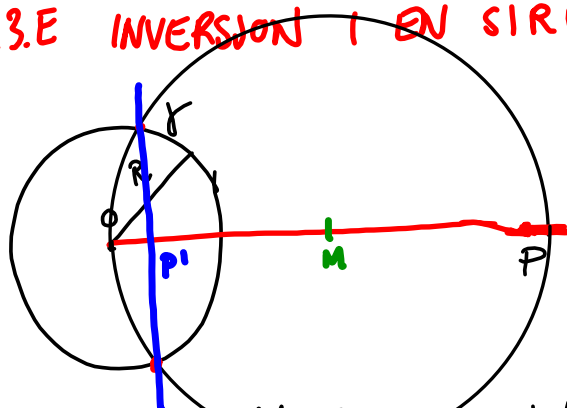


$M = l_{P_1 P_2} \cap l_{Pa}$ der P_1, P_2 i
to blå store deler.

4.3.0 Deler af linjestykker i N dele

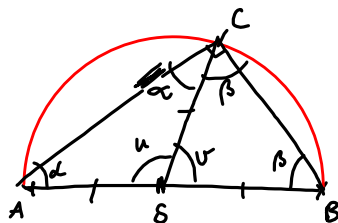


4.3.E INVERSION I EN SIRKEL



Inversionen af et punkt P over sirkelen $\gamma(O, R)$
 er et punkt P' på OP s.a.
 $OP \cdot OP' = R^2$

THALES SETNING



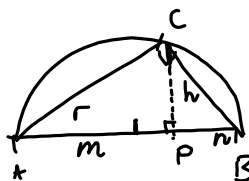
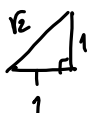
I en $\triangle ABC$ der
 AB er diameter i
 en sirkel og C
 er på sirkelbuen,
 så er $\angle C$ en rett vinkel

BEVIS

• For to likebeinte trekanter
 med par av like vinkler

$$\left. \begin{array}{l} 2\alpha + u = \pi \\ 2\beta + v = \pi \\ \underline{u + v = \pi} \end{array} \right\} \begin{array}{l} 2(\alpha + \beta) + \underline{u + v} = 2\pi \\ 2(\alpha + \beta) = \pi \\ \angle C = \alpha + \beta = \frac{\pi}{2} \end{array}$$

4.3.F. KVADRATETTER



$\triangle APC$ formlike $\triangle CPB$

$$\frac{CP}{AP} = \frac{PB}{CP}$$

$$CP^2 = AP \cdot PB$$

$$h = \sqrt{m \cdot n}$$

MELLOMPROPORTJONALEN

$$h = (m \cdot n)^{\frac{1}{2}}$$

geometrisk gjennomsnitt

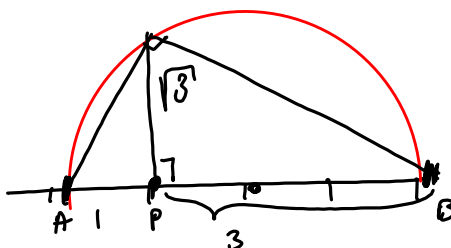
$$r = \frac{(m+n)}{2}$$

aritmetisk gjennomsnitt

$$\sqrt{3} = \sqrt{3 \cdot 1}$$

$$\begin{aligned} \sqrt{6} &= \sqrt{2 \cdot 3} \\ &= \sqrt{1 \cdot 6} \end{aligned}$$

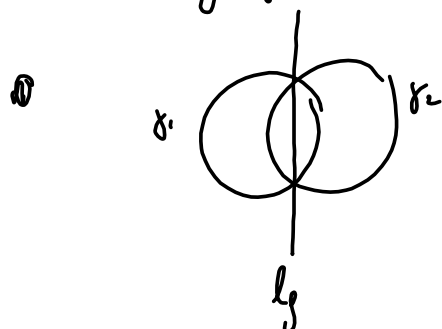
LEKSE



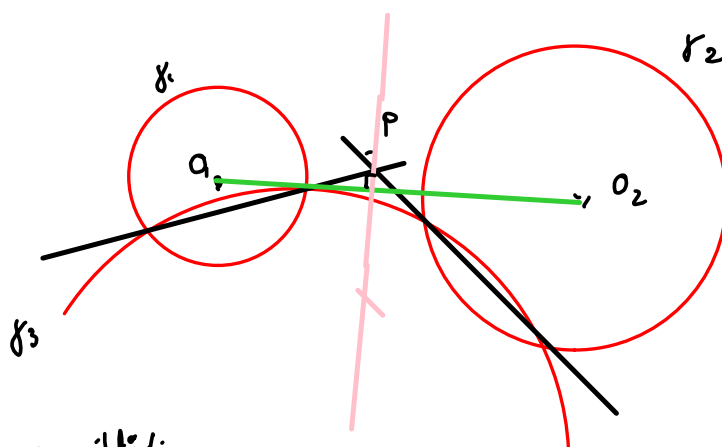
4.3.G. RADIKALAKSEN

- Radikalaksen l_g til to sirkler γ_1 og γ_2 er mengden av punkter som har samme potens mht. de to sirklene
- Potens: $p(P, \gamma)$ til et punkt P mht sirkel $\gamma(O, r)$:

$$p(P, \gamma) = OP^2 - r^2$$



②



- γ_3 vilkårlig
- Tegn radikalaksene $\gamma_1\gamma_3$ og $\gamma_2\gamma_3$.
- $l_{o_1o_2}$
- Normalen fra P på $l_{o_1o_2}$ er radikalaksen til γ_1 og γ_2 .