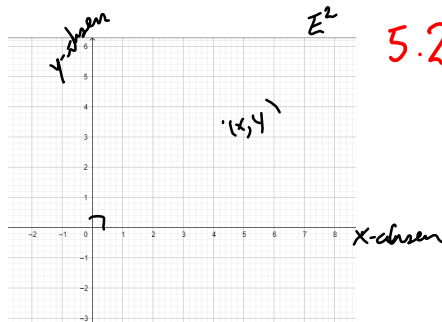


27.09.2022 MAT 2500

5.2. Koordinater



DEF 5.8. LINEAR LIKNING & LINJE

$$Ax + By = C$$

$$A, B, C \in \mathbb{R}$$

$$\text{A eller } B \neq 0$$

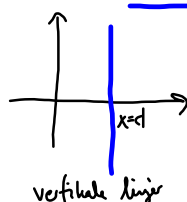
$$\text{OBS: } k \neq 0$$

$$kAx + kBy = kC \quad \text{gir samme linje}$$

$$\{(x, y) \in \mathbb{E}^2 \mid Ax + By = C\}$$

TO TILFELLER

① $B=0 \quad x = \frac{C}{A} = d$



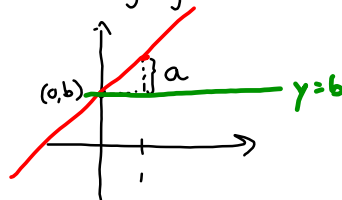
② $B \neq 0$

$$y = -\frac{A}{B}x + \frac{C}{B}$$

$$= ax + b$$

↑
stigningstall

skjæring med
y-aksen



$a=0$
Horisontale og skrå linjer

ILUSK: • linje l gjennom (x_1, y_1) og (x_2, y_2) der $x_1 \neq x_2$
 her $a = \frac{(y_2 - y_1)}{(x_2 - x_1)}$ og

likning

$$y - y_1 = a(x - x_1)$$

• Hvis $x_1 = x_2$, da er $x = x_1$ likning til l.

LEMMA 5.9. Linjene gitt ved

$$Ax + By = C \quad \text{og} \quad A'x + B'y = C'$$

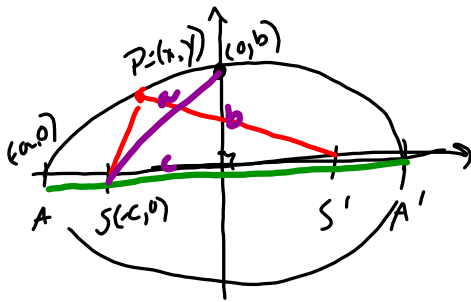
står normalt på hverandre $\Leftrightarrow AA' + BB' = 0$.

Tilsvarende l: $y = ax + b$ og l': $y = a'x + b'$

står normalt på hverandre $\Leftrightarrow a' = -\frac{1}{a}$

$$(1, a) \quad \text{og} \quad (1, a')$$

KJEGLESNITT



ELLIPSE:

- Sentrum i $(0,0)$
- x -aksen som store akse
- y -aksen som lille akse
- Toppunkt i $(\pm a, 0)$
- $S = (-c, 0)$ og $S' = (c, 0)$
der $c = ea$

✓ DEF 5.1: $\underline{PS} + \underline{PS'} = \overbrace{e \cdot \frac{2a}{1-e^2}}^{AA'} = \underline{AA'}$

$$\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a$$

$$\sqrt{(x-c)^2 + y^2} = 2a - \sqrt{(x+c)^2 + y^2}$$

$$(x-c)^2 + \underline{y^2} = 4a^2 - 4a\sqrt{(x+c)^2 + y^2} + (x+c)^2 + \underline{y^2}$$

$$\underline{x^2} - 2xc + \underline{c^2} = 4a^2 - 4a\sqrt{(x+c)^2 + y^2} + \underline{x^2} + 2xc + \underline{c^2}$$

$$4a\sqrt{(x+c)^2 + y^2} = 4a^2 + 4xc$$

$$a^2((x+c)^2 + y^2) = a^4 + 2a^2xc + x^2c^2$$

$$x^2 + \underline{2xc} + c^2 + y^2 = a^2 + \underline{2xc} + \frac{c^2}{a^2}x^2$$

$$x^2\left(1 - \frac{c^2}{a^2}\right) + y^2 = a^2 - c^2$$

$$\frac{x^2}{a^2}(a^2 - c^2) + y^2 = (a^2 - c^2)$$

$$\frac{x^2}{a^2} + \frac{y^2}{a^2 - c^2} = 1$$

oss:

$$c = ea \quad e \neq 1$$

$$x = 0$$

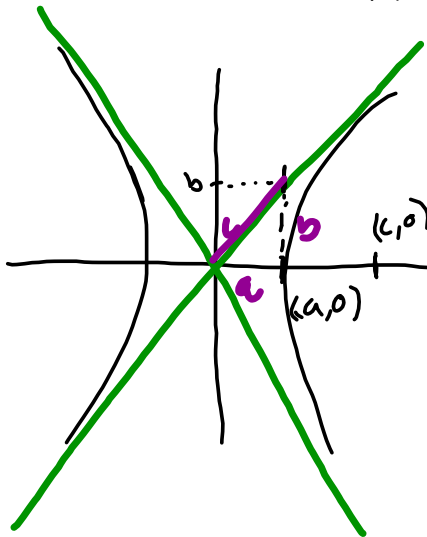
$$(0, \pm b)$$

$$b = \sqrt{a^2 - c^2}$$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

HYPARBEL: Helt firsvarnde med differanreformel (5.4)

$$\sqrt{(x-c)^2 + y^2} - \sqrt{(x+c)^2 + y^2} = 2a$$



$$\frac{x^2}{a^2} - \frac{y^2}{c^2 - a^2} = 1$$

Asymptoter!

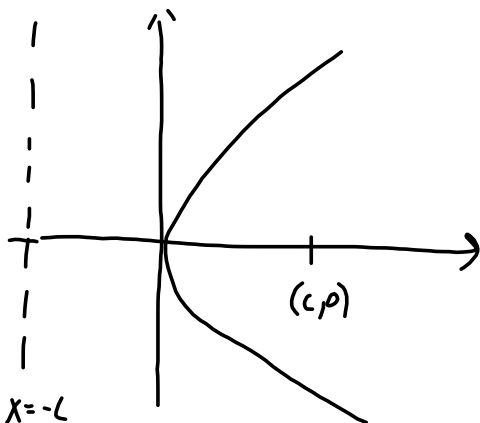
OBS

$$c = ea > a$$

$$\text{eller } b^2 = c^2 - a^2$$

PARABEL:

- sentrum (0,0)
- Brennpunkt (c,0)
- Styrelinge $x = -c$
- $P = (x,y)$ oppfyller $PM = PS$

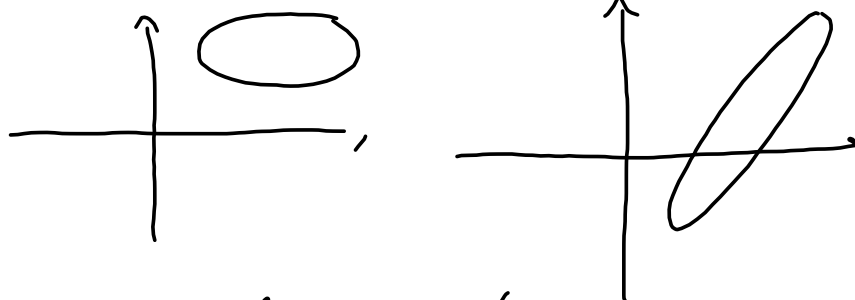


$$(x+c) = \sqrt{(x-c)^2 + y^2}$$

$$(x+c)^2 = (x-c)^2 + y^2$$

$$\underline{y^2 = 4cx}$$

GENERELT



Isometrier til standard form.

DEF 5.10. Den generelle andregningskurven
 Den generelle kjeglesnittet i planet har ligning

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

der minst én av A, B og $C \neq 0$.

DEGENERERT KJEGLESNITT

IKKE-DEGENERERT



Ingen punkter

} / 4 vet ikke helt

Ellipse
 Hyperbel
 Parabel

EKS: Skriv

$$16x^2 + 9y^2 - 32x + 36y - 92 = 0$$

$$B=0$$

på standardform

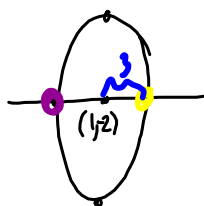
$$16x^2 - 32x + 9y^2 + 36y = 92$$

$$(4x)^2 - 2 \cdot 4x \cdot 4 + 4^2 + (3y)^2 + 2 \cdot 3y \cdot 6 + 6^2 = 92 + 4^2 + 6^2$$

$$(4x-4)^2 + (3y+6)^2 = 144$$

$$4^2(x-1)^2 + 3^2(y+2)^2 = 144 = 3^2 \cdot 4^2$$

$$\frac{(x-1)^2}{3^2} + \frac{(y+2)^2}{4^2} = 1 \Rightarrow \text{Ellipse}$$

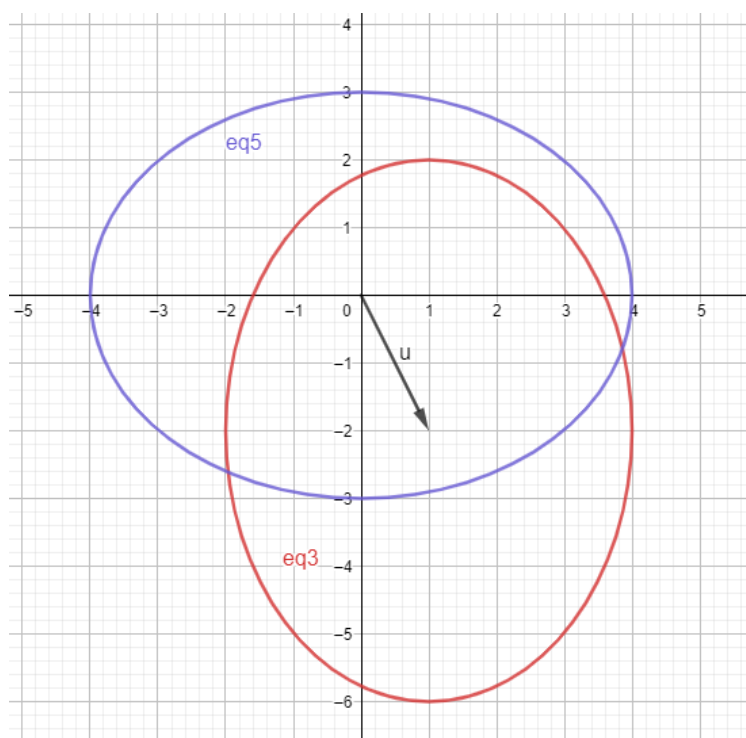


Leser av sentrum: $(1, -2)$
 Endepunkter ^{på akser} parallelt med x-aksen:

$$(-2, -2) \text{ og } (4, -2)$$

Endepunkter på akser parallelt med y-aksen:

$$(1, 2) \text{ og } (1, -6)$$



EKS: $\text{Vet } \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$

Vis at $xy = 1$ er en hyperbel!

$$xy - 1 = 0$$

$$xy = (x \ y) M \begin{pmatrix} x \\ y \end{pmatrix}$$

For isomorphism:

$$\begin{pmatrix} x \\ y \end{pmatrix} = Q^T \begin{pmatrix} x \\ y \end{pmatrix}$$

$$M = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$$

Substituer til å substituere

$$\begin{pmatrix} x \\ y \end{pmatrix} = Q \begin{pmatrix} x \\ y \end{pmatrix}$$

$$M = Q D Q^{-1}, \quad Q^T = Q^{-1}$$

↓
søke om
eigenverdier
(orthonormale)
↓
DIAGONAL
MED EIGENVERDIER

$$(x \ y) M \begin{pmatrix} x \\ y \end{pmatrix} - 1 = 0$$

$$(x \ y) \underbrace{Q^T M Q}_D \begin{pmatrix} x \\ y \end{pmatrix} - 1 = 0$$

$$D: \quad 0 = \det(M - \lambda I) = \begin{vmatrix} -\lambda & \frac{1}{2} \\ \frac{1}{2} & -\lambda \end{vmatrix}$$

$$\text{Eigenverdier } \lambda_1 = \frac{1}{2} \text{ og } \lambda_2 = -\frac{1}{2}$$

$$E_{\lambda_1}: \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{aligned} \vec{v}_1 &= \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ \vec{u}_1 &= \frac{\sqrt{2}}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{aligned}$$

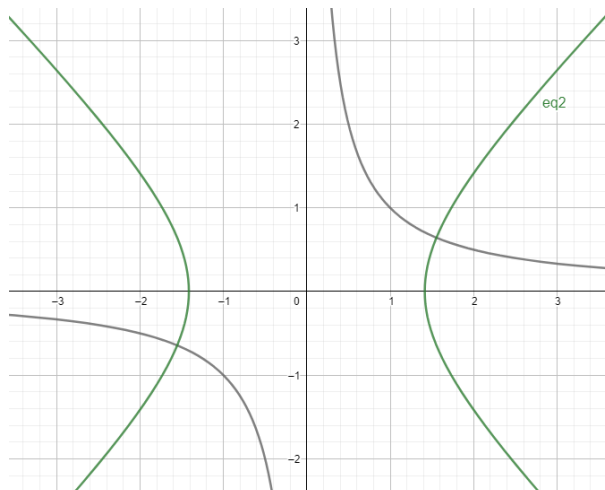
$$E_{\lambda_2}: \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{aligned} \vec{v}_2 &= \begin{pmatrix} 1 \\ -1 \end{pmatrix} \\ \vec{u}_2 &= \frac{\sqrt{2}}{2} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \end{aligned}$$

$$Q = \frac{\sqrt{2}}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \quad Q^T = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

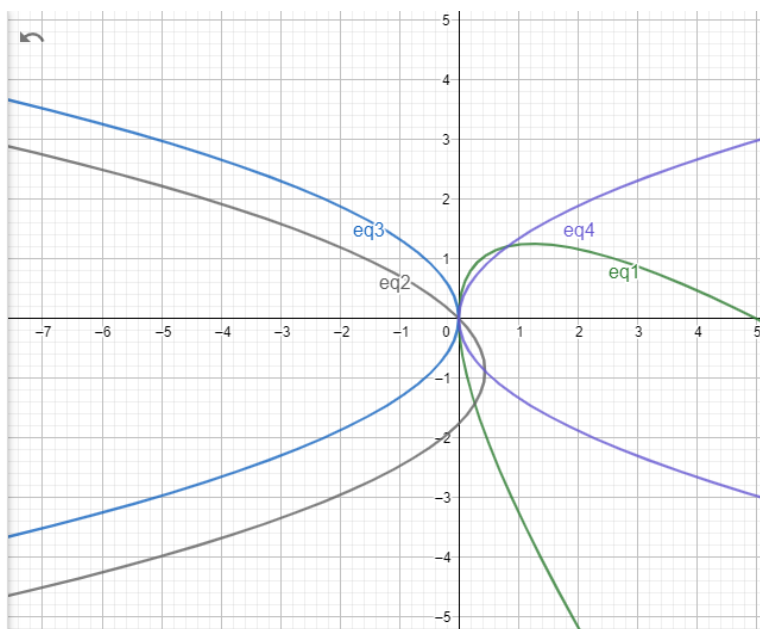
$$\theta = \frac{\pi}{4}$$

$$(x \ y) \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} - 1 = 0$$

$$\boxed{\frac{x^2}{2} - \frac{y^2}{2} = 1}$$



●	eq1: $x^2 + 2x y + y^2 - 5x = 0$	⋮
●	eq3: $y^2 = -5 \sqrt{2} / 4 x$	⋮
●	eq2: $(y + 5\sqrt{2} / 8)^2 = -5 \sqrt{2} / 4 (x - 5\sqrt{2} / 16)$	⋮
●	eq4: $y^2 = 5\sqrt{2} / 4 x$	⋮
○	$u = \text{Vector}\left(\left(5 \cdot \frac{\sqrt{2}}{16}, -5 \cdot \frac{\sqrt{2}}{8}\right), (0, 0)\right)$ $\rightarrow \begin{pmatrix} -0.4419417382416 \\ 0.8838834764832 \end{pmatrix}$	⋮
+	Input...	



FRÅ GENERELL LIGNING TIL STANDARDFORM:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

- ① Diagonaliser $Ax^2 + Bxy + Cy^2$
(for $B' = 0$)
- ② Speilte om $x = y$
- ③ Translasjon for å fjerne $Dx + Ey$,
gjøre etter å ha fullført kvadrat.
- ④ For parabel: Speiling om y -aksen.

5.3. PARAMETRISERINGER

ELLIPSEN

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\cos^2 t + \sin^2 t = 1$$

$$x = a \cdot \cos t \quad 0 \leq t < 2\pi$$

$$y = b \cdot \sin t$$

er en parametrisering av
ellipsen.

HYPERBELLEN

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$1 + \tan^2 t = \frac{1}{\cos^2 t}$$

$$\frac{1}{\cos^2 t} - \tan^2 t = 1$$

$$x = \frac{a}{\cos t} \quad -\pi \leq t < \pi$$

$$y = b \tan t \quad t \neq \pm \frac{\pi}{2}$$

er en parametrisering av
hyperbelen.

OBS: $\lim_{t \rightarrow \pm \frac{\pi}{2}} \frac{x}{y} = \lim_{t \rightarrow \pm \frac{\pi}{2}} \frac{\frac{a}{\cos t}}{b \tan t}$

$$= \lim_{t \rightarrow \pm \frac{\pi}{2}} \frac{a}{b \sin t} = \pm \frac{a}{b}$$

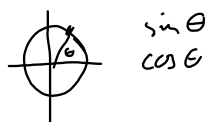
$$bx = \pm ay$$

$ay = bx$ $-ay = bx$
er asymptotene til hyperbelen!

$$\left(\frac{x^2}{a^2} - \frac{y^2}{b^2} \right) = \underbrace{\left(\frac{x}{a} - \frac{y}{b} \right)}_0 \underbrace{\left(\frac{x}{a} + \frac{y}{b} \right)}_0 = 0$$

Begynn med disse ved tegning!

HYPERBOLSK PARAMETRISERING



$$\cosh u = \frac{e^u + e^{-u}}{2}$$

$$\sinh u = \frac{e^u - e^{-u}}{2}$$

$$\cosh^2 A - \sinh^2 A = 1$$

"sinh-hyperbel"
 $x^2 - y^2 = 1$

