

MAT 2500 15.11.2022

SETN 6.34. (ikke tre på linje)
 5 pkt i $\mathbb{P}_R^2$ ($\mathbb{RP}^2$) i generell posisjon
 ligger på nøyaktig én koadmisk kurve

FORMEL:

5 pkt $P_i = (\alpha_i : \beta_i : \gamma_i)$

$$\bullet \alpha \alpha_i^2 + \beta \alpha_i \beta_i + \dots + \gamma \gamma_i^2 = 0$$

$$\bullet \alpha x_0^2 + \beta x_0 x_1 + \dots + \gamma x_2^2 = 0$$

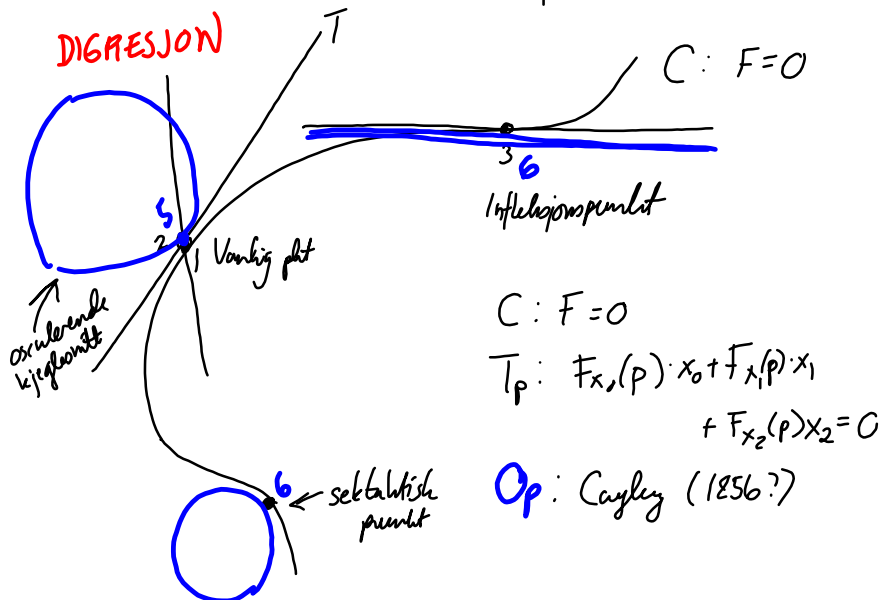
6 lineær linjer i 6 variabler α -f.

FORMEL FOR LINJE

To pkt gir linje:

$$\begin{vmatrix} x_0 & x_1 & x_2 \\ a_0 & a_1 & a_2 \\ b_0 & b_1 & b_2 \end{vmatrix} = 0$$

$$0 = Q = \begin{vmatrix} x_0^2 & x_0 x_1 & x_1^2 & x_0 x_2 & x_1 x_2 & x_2^2 \\ \alpha_1^2 & \alpha_1 \beta_1 & \dots & \dots & \dots & \gamma_1^2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \alpha_5^2 & \alpha_5 \beta_5 & \dots & \dots & \dots & \gamma_5^2 \end{vmatrix}$$



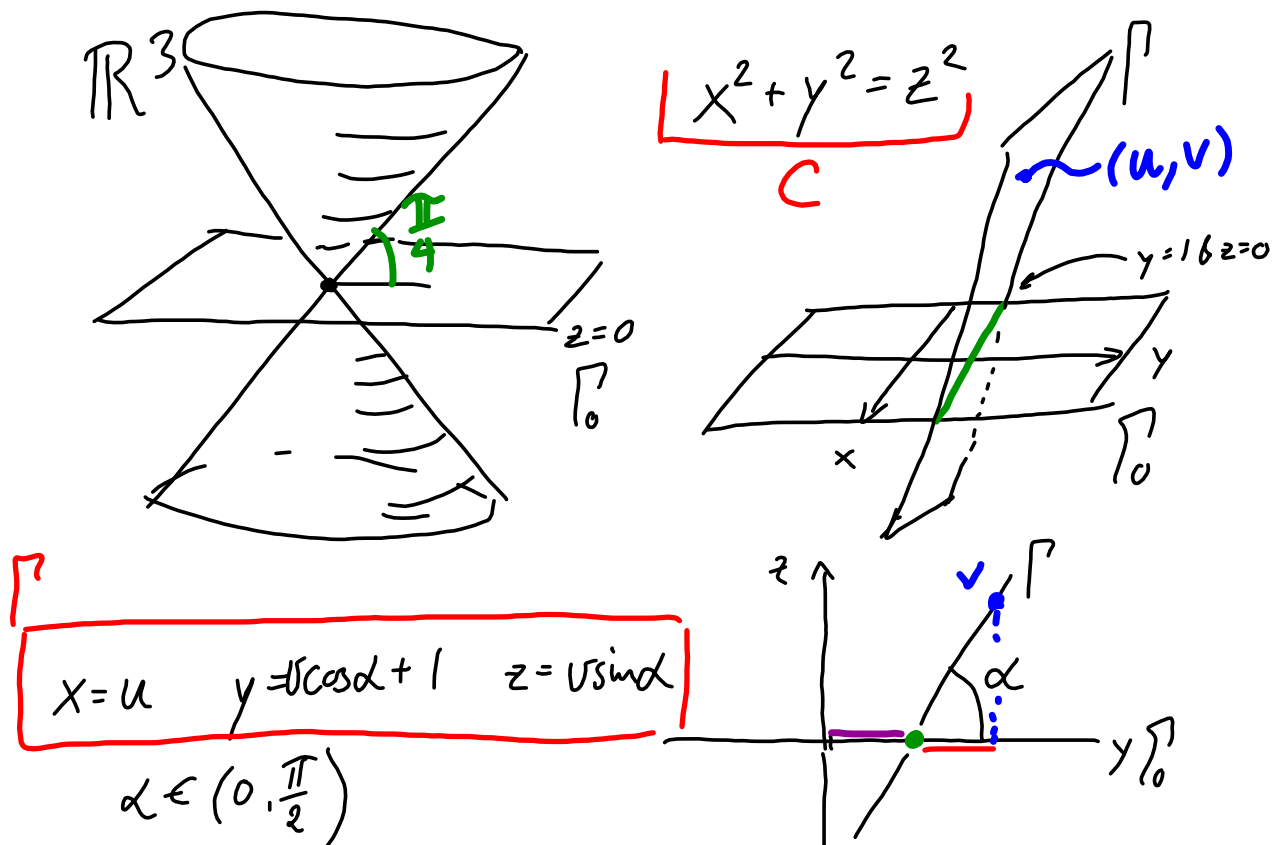
Kan finne kurver som identifiserer de "rare" punktene;

• infleksjonspunkt
 Hessekurve
 $\leq d(3(d-2))$

$$H = \begin{vmatrix} F_{x_0 x_0} & F_{x_0 x_1} & F_{x_0 x_2} \\ F_{x_0 x_1} & F_{x_1 x_1} & F_{x_1 x_2} \\ F_{x_0 x_2} & F_{x_1 x_2} & F_{x_2 x_2} \end{vmatrix} = 0$$

• sekantlinje pkt
 2-Hessekurve
 $\leq d(12d-27)$

6.5. SNITT AV KJEBLER



$C \cap \Gamma$

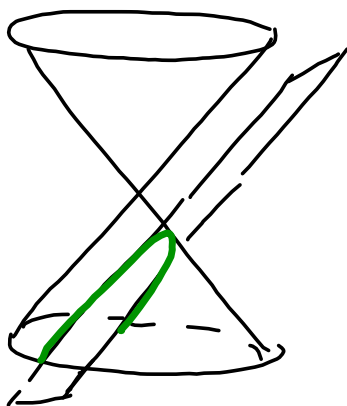
$$u^2 + (u \cos \alpha + 1)^2 = u^2 \sin^2 \alpha$$

$$u^2 + u^2 \cos^2 \alpha + 2u \cos \alpha + 1 = u^2 \sin^2 \alpha$$

$$u^2 + u^2 (\underbrace{\cos^2 \alpha - \sin^2 \alpha}_{\cos 2\alpha}) + 2u \cos \alpha + 1 = 0$$

$\alpha = \frac{\pi}{4}$

$$u^2 + u^2 \cdot \underbrace{\cos^2 \frac{\pi}{4}}_0 + 2u \underbrace{\cos \frac{\pi}{4}}_{\frac{\sqrt{2}}{2}} + 1 = 0$$



$$u^2 + \sqrt{2}u + 1 = 0$$

$$u = \frac{-1 - u^2}{\sqrt{2}}$$

$$u = -\frac{u^2}{\sqrt{2}} - \frac{1}{\sqrt{2}}$$

som er en parabel

$$\alpha \neq \frac{\pi}{4} \quad \cos 2\alpha \neq 0$$

sluttes om:

$$\frac{u^2}{\cos 2\alpha} + \underbrace{v^2 + 2v \frac{\cos \alpha}{\cos 2\alpha} + \frac{1}{\cos 2\alpha}}_{\text{fullfører kvadratet}} = 0$$

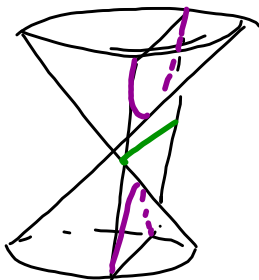
$$\frac{u^2}{\cos 2\alpha} + \left(v + \frac{\cos \alpha}{\cos 2\alpha}\right)^2 - \frac{\cos^2 \alpha}{\cos^2 2\alpha} + \frac{1}{\cos 2\alpha} = 0$$

$$\frac{u^2}{\cos 2\alpha} + \left(v + \frac{\cos \alpha}{\cos 2\alpha}\right)^2 = \frac{\cos^2 \alpha - \cos 2\alpha}{\cos^2 2\alpha} \quad (\cos^2 \alpha - \sin^2 \alpha)$$

$$= \frac{\sin^2 \alpha}{\cos^2 2\alpha} \quad \alpha \neq 0$$

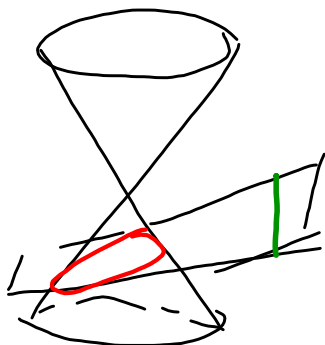
$$\boxed{\frac{u^2}{\frac{\sin^2 \alpha}{\cos 2\alpha}} + \frac{\left(v + \frac{\cos \alpha}{\cos 2\alpha}\right)^2}{\frac{\sin^2 \alpha}{\cos^2 2\alpha}} = 1}$$

$$\alpha > \frac{\pi}{4}$$



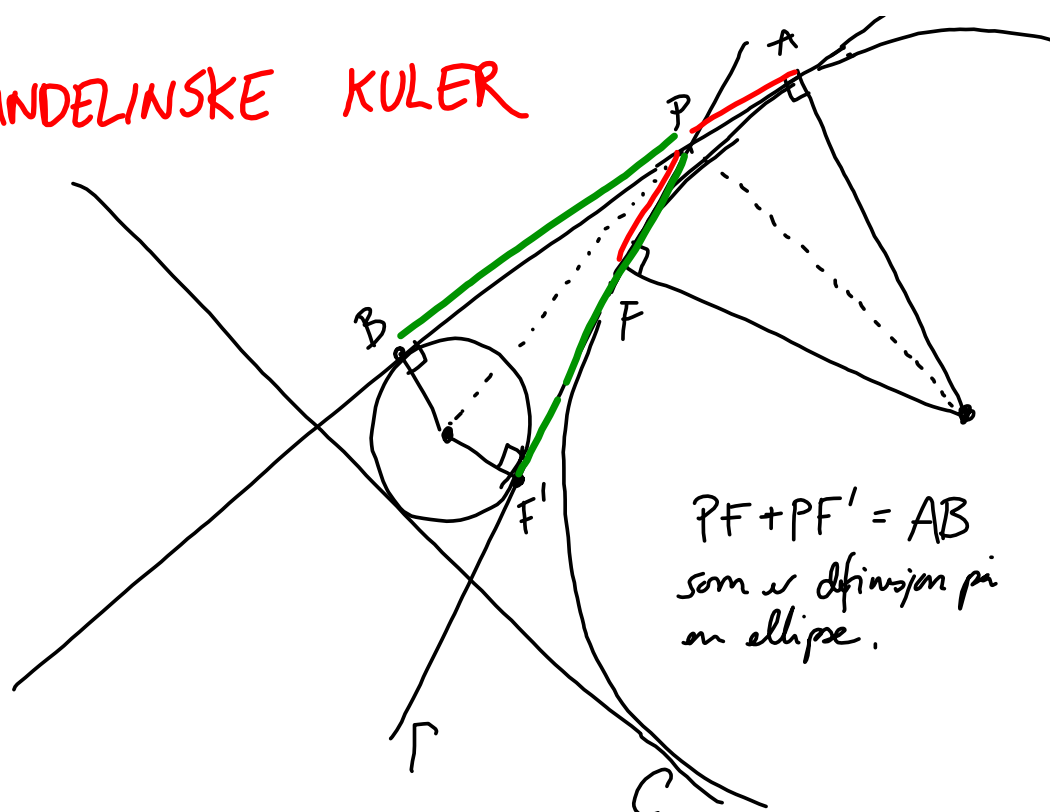
$\cos 2\alpha < 0$
 u^2 får negativt fortegn,
 så vi har en hyperbel

$$\alpha < \frac{\pi}{4}$$



$\cos 2\alpha > 0$
 u^2 og v^2 har begge positivt fortegn,
 så vi har en ellipse.

DANDELINSKE KULER



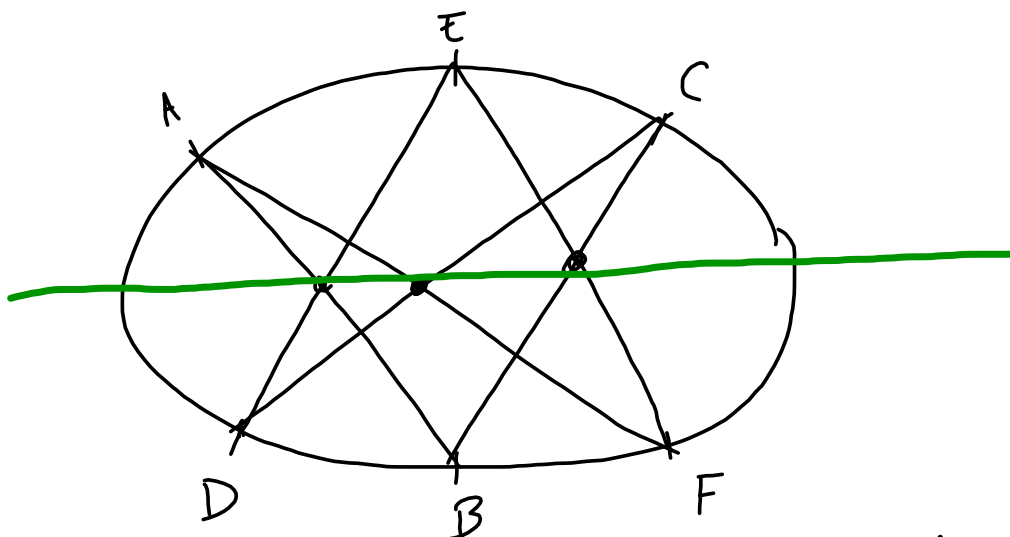
$PF + PF' = AB$
 som er definisjon på
 en ellipse.

To par av kongruente trekanter

SETN 6.35 . PASCALS SETNING FOR KJEGLESNITT

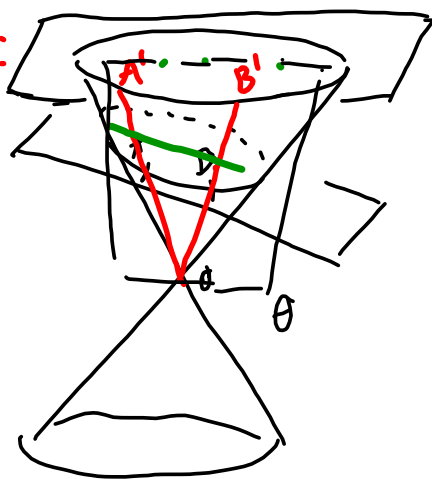
(4.17 - for sirkel
Menelaos' setn)

Hvis A, B, C, D, E, F er på et kjeglesnitt



så er $l_{AB} \cap l_{DE}$, $l_{BC} \cap l_{EF}$ og $l_{FA} \cap l_{CD}$
kollineære.

BEVIS:

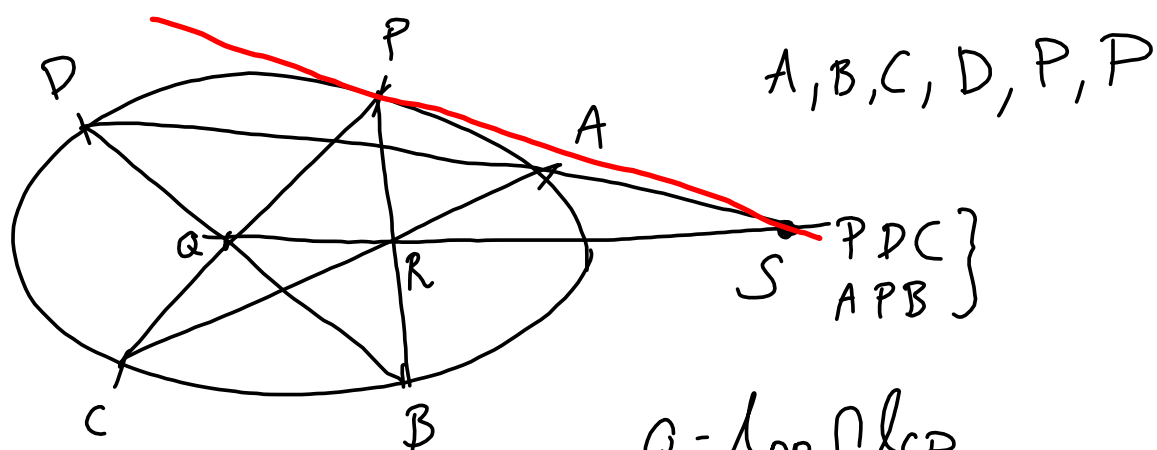


Ta et plan Γ parallelt med
 $z=0$, der er $\Gamma \cap C$
en sirkel, og her holder
Pascals setning.

De tre kollineære skjæringspunkt
i sirkelen gir sammen med O
et plan θ .

Γ gir $\Gamma \cap \theta$ en linje
gjennom skjæringspunkt

ANVENDELSE (KONSTRUKSJON AV TANGENTLINJE)



$$Q = l_{PB} \cap l_{CP}$$

$$R = l_{PB} \cap l_{CA} \quad \text{vid Pascal}$$

$$S := l_{QR} \cap l_{DA} = l_{PP} \cap l_{DA}$$

$$l_{PS} = l_{PP}$$