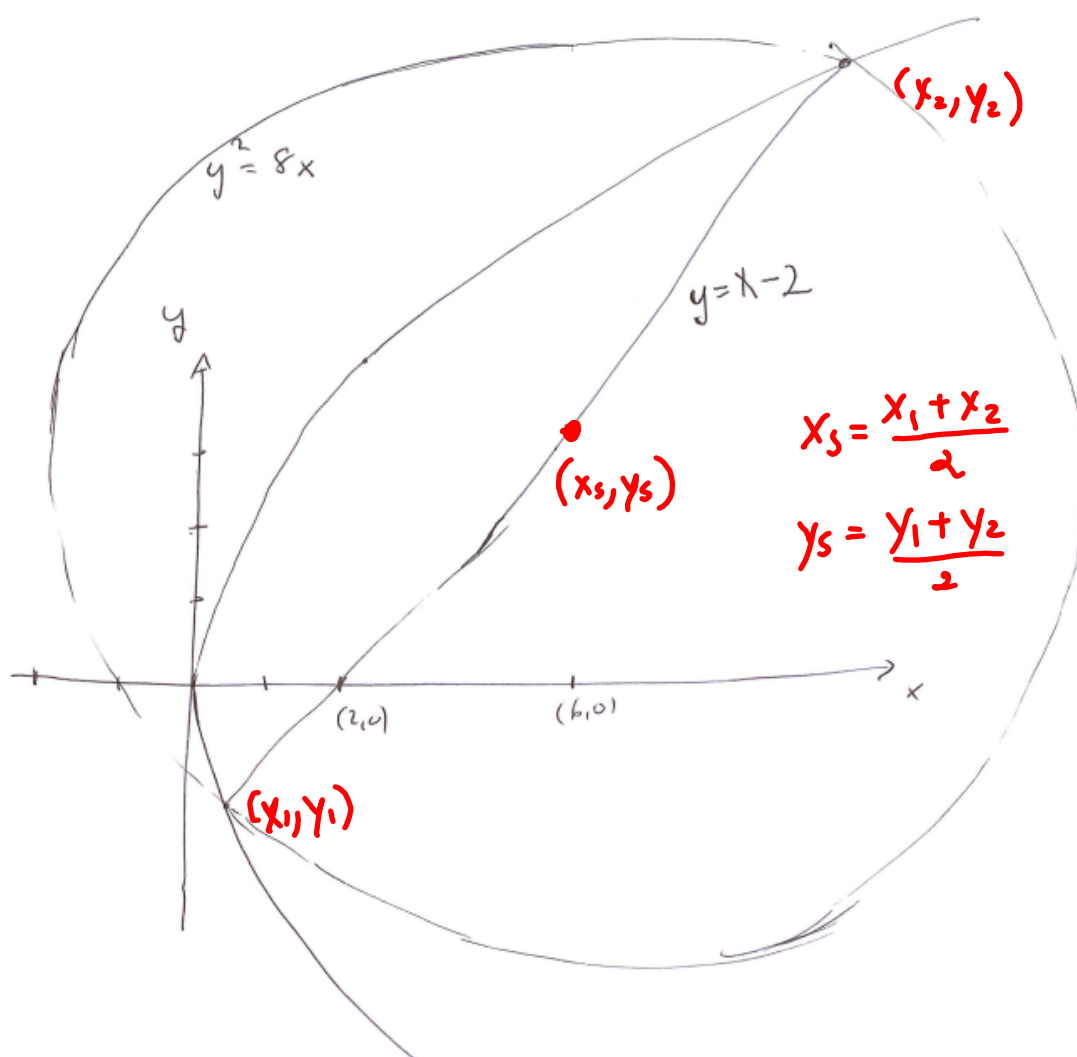


MAT 2500 23.11.2022
SPM FRA EKSAMEN 2015

③

a)



g) $C: y^2 = 8x$ $K: y = k(x-2)$ $x = \frac{y}{k} + 2$

$x = \frac{1}{8}y^2$ $C \cap K: y = \frac{k}{8}y^2 - 2k$

2. gradslinje

$y^2 - \frac{8}{k}y - 16 = 0$

$(y-y_1)(y-y_2) = y^2 - (y_1+y_2)y + y_1y_2 = 0$

$y_1+y_2 = \frac{8}{k}$

$y_s = \frac{4}{k}$

$y_1 = \frac{4}{k} + \sqrt{\frac{16}{k^2} + 16} = \frac{4}{k} + \sqrt{1 + \frac{16}{k^2}}$

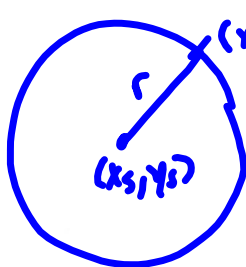
$x_s = \frac{y}{k} + 2 = \frac{4}{k^2} + 2$

endepunkt til korde: $y = k(x-2)$ og $y^2 = 8x$

$y_1 = \frac{4}{k} + \sqrt{\frac{16}{k^2} + 16} = 4\left(\frac{1}{k} + \sqrt{1 + \frac{16}{k^2}}\right)$

$x_1 = \frac{y_1^2}{8} = 2\left(\frac{1}{k^2} + 1 + \frac{16}{k^2} + \frac{2}{k}\sqrt{1 + \frac{16}{k^2}}\right)$
 $= 2\left(\frac{17}{k^2} + 1 + \frac{2}{k}\sqrt{1 + \frac{16}{k^2}}\right)$

radius r:



$r^2 = (x_1 - x_s)^2 + (y_1 - y_s)^2$

$= \left(\frac{4}{k^2} + 2 + \frac{4}{k}\sqrt{1 + \frac{16}{k^2}} - \frac{4}{k^2} - 2\right)^2 + \left(\frac{4}{k} + 4\sqrt{1 + \frac{16}{k^2}} - \frac{4}{k}\right)^2$

$= \frac{16}{k^2}\left(1 + \frac{16}{k^2}\right) + 16\left(1 + \frac{16}{k^2}\right) = 16\left(1 + \frac{16}{k^2} + \frac{16}{k^2}\right)$
 $= 16\left(1 + \frac{32}{k^2}\right)$

Ligning for sirkelen:

$(x - x_s)^2 + (y - y_s)^2 = 16\left(1 + \frac{16}{k^2}\right)$

(*) $\left(x - 2 - \frac{4}{k^2}\right)^2 + \left(y - \frac{4}{k}\right)^2 = 16\left(1 + \frac{16}{k^2}\right)$

g) $x = -2$ tangenter sirkelen da som (*) har dobbel rot når $x = -2$:

(*) med $x = -2$: $(-4 - \frac{4}{k^2})^2 + (y - \frac{4}{k})^2 = 16\left(1 + \frac{16}{k^2}\right)$

$(y - \frac{4}{k})^2 = 16\left(1 + \frac{16}{k^2}\right) - 16\left(1 + \frac{16}{k^2}\right) = 0$

Kan også løses direkte for k .

si vi får dobbel rot og tangens.
 $y = \frac{4}{k}$

REPETISJON

KAP 2 ISOMETRIER OG SYMMETRIER
I PLANET.

4 Isometrier

Orienterings-
bevarende

Orienterings-
reverserende

$\left\{ \begin{array}{l} \text{Translasjoner} \\ \text{Rotasjoner (har fikspunkt)} \end{array} \right.$
 $t_{\vec{a}}(x) = (x_1 + a_1, y + a_2)$
 $S_{O,P} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$

$\left\{ \begin{array}{l} \text{Spæilinger (har fikspunkter)} \\ \text{Glidrefleksjoner} \end{array} \right.$
 $S_{\vec{v}} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
 $R_{\vec{v}} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$

$Q_{\vec{w}_2} = I_2 - \left(\frac{2}{\vec{w}_2^T \vec{w}_2} \right) \vec{w}_2 \vec{w}_2^T \leftarrow O \in l.$

Glidrefleksjoner: $t_{\vec{a}} S_{\vec{v}} = t_{\vec{w}_1} S_{\vec{v}}$

$l = \{ \lambda \vec{v} \mid \lambda \in \mathbb{R} \}$
 $\vec{v} = (\cos \alpha, \sin \alpha)$
 $\vec{v} \parallel l$

$\vec{w}_1 = (\vec{a} \cdot \vec{v}) \vec{v}$
 $\vec{w}_2 = (\vec{a} \cdot \vec{v}^\perp) \vec{v}^\perp$
 $\vec{v}^\perp = (-\sin \alpha, \cos \alpha)$

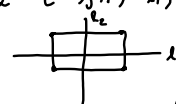
$l' = \{ \frac{1}{2} \vec{w}_2 + \lambda \vec{v} \mid \lambda \in \mathbb{R} \}$

- Regulære polygoner
Hjornemåter

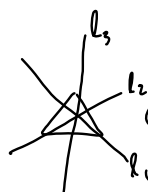


- Trekanter: Innsirkul, Omsirkul
- Endelige undergrupper av $Isom_2$:
Enten C_n eller D_n for passende n .

$C_1 = \{id\}$
 $D_1 = ?$
 $C_2 = \{id, \beta_{\pi}\}$
 $D_2 = \{id, \beta_{\pi}, s_{l_1}, s_{l_2}\}$



- symmetrigruppe til rektangel
- Klein-4-gruppen



$C_3 = \{id, \beta_{\frac{2\pi}{3}}, \beta_{\frac{4\pi}{3}}\}$
 $D_3 = \{C_3, s_{l_1}, s_{l_2}, s_{l_3}\}$

- Fundamentalmåter til L_6 (translasjonsundergrupper)

$\text{Rang } L_6 = 2$
 $M_6 = \{m\vec{a} + n\vec{b} : m, n \in \mathbb{Z}\}$

Standardparallelogram $P = \{0, \vec{a}, \vec{b}, \vec{a} + \vec{b}\}$

Dirichletområdet $D_6 = \{p \in E^2 \mid d(p, 0) \leq d(p, q) \text{ for alle } q \in M_6, q \neq 0\}$

Heksagon dekket inn i trekanter

$\{0, \vec{a}, \vec{b}, -\vec{a}, -\vec{b}, -\vec{a} + \vec{b}, \vec{a} - \vec{b}\}$

Konveksitetshjelpingen av midtpunkt i trekanter!
(Hvor omsentret)

$\bullet$ Primitiviserings $\vec{a}, \vec{b}$ i M_6

KAP 3. SYMMETRI I ROMMET

E^3 Konvekse polyedre

- Euler formel $v - e + f = 2$

- $2e = \sum \deg p$ ($\deg p =$ kamr inn til hjørnet)

- $2e = \sum \deg s$ ($\deg s =$ antall kamr som er kant til flate s)

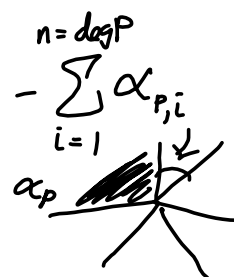
- Vinkeldefekt

- Til et hjørne p : $\alpha_p = 2\pi - \sum_{i=1}^{n=\deg p} \alpha_{p,i}$

- Til polyedret

$$D = \sum_{p \in \mathcal{V}} \alpha_p$$

- Descartes formel $D = 4\pi$



- Schläfli symbol $\{p, q\}$ Regulære p -kamr og q ar deer møtes i hvert hjørne (Til regulære polyedre)

- Platoniske legemer: Tetraeder
Kube
Oktaeder
Ikosaeder
Dodekaeder

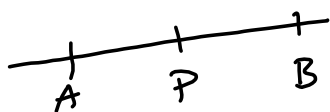
- Arkimediske legemer: 13 som ikke er prismer/antiprismer

- Rotasjonsgrupper: Tetraedergruppen
Oktaedergruppen
Ikosaedergruppen

- Endelige undergrupper av $SO(3)$ ($\Leftrightarrow$ Rotasjoner)
(ortogonale)
($\det A = 1$)

C_n, D_n eller platoniske

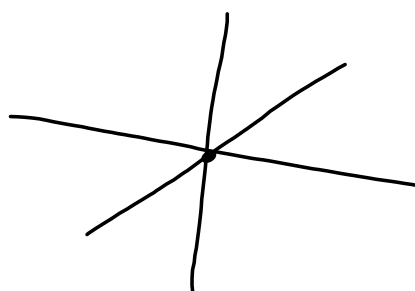
KAP 4 TREKANTER OG SIRKLER



• Kollineare

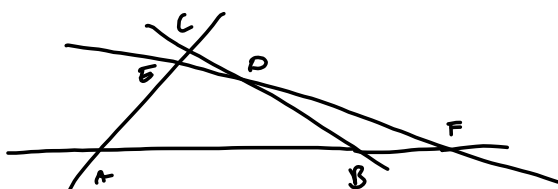
• Delingsforhold

$$\frac{\overline{AP}}{\overline{PB}} \text{ entydig} \quad AP = |\overline{AP}|$$



• Konkurrente

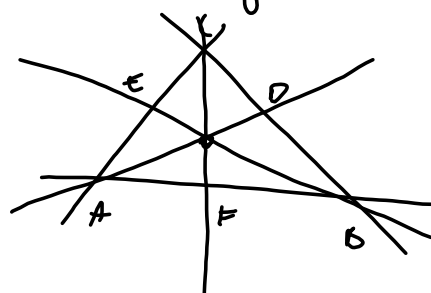
• Menelaos' setning



D, E, F Menelaus' punkt for $\triangle ABC$ er kollineare

$$\Leftrightarrow \frac{\overline{BD}}{\overline{DC}} \cdot \frac{\overline{CE}}{\overline{EA}} \cdot \frac{\overline{AF}}{\overline{FB}} = -1.$$

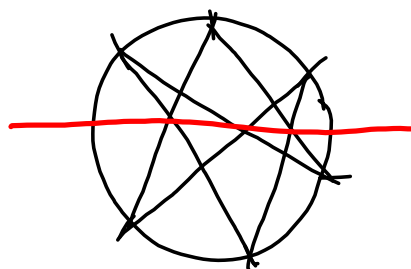
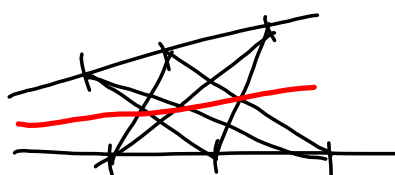
• Ceva's setning



Cevalingjer l_{AD} , l_{BE} , l_{CF} er konkurrente $\Leftrightarrow$

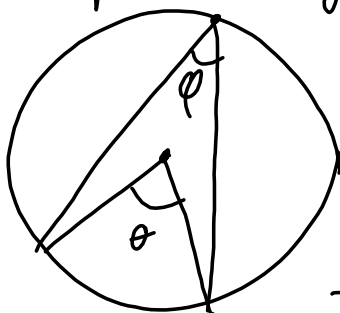
$$\frac{\overline{BD}}{\overline{DC}} \cdot \frac{\overline{CE}}{\overline{EA}} \cdot \frac{\overline{AF}}{\overline{FB}} = 1.$$

• Pappos og Pascal



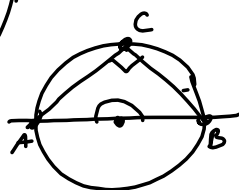
Gjelder også
kjeskneitt

• Periferivinkelsetning

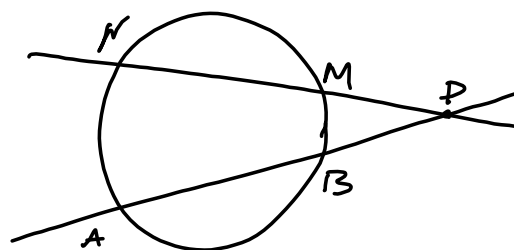


$$\theta = 2\phi$$

Thales



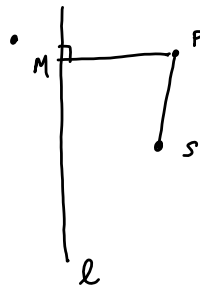
Punkts potens



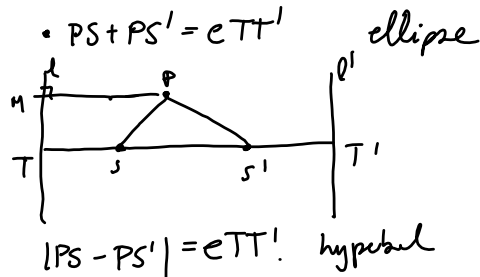
$$\overline{PA} \cdot \overline{PB} = \overline{PN} \cdot \overline{PM}$$

KAP 5 KJEGLESNITT I E^2

• GEOMETRISKE STEDER



- $PS = ePM$
- $0 < e < 1$ ellipse
- $e = 1$ parabel
- $e > 1$ hyperbel



- $PS + PS' = eTT'$ ellipse
- $|PS - PS'| = eTT'$ hyperbel

• $Ax + By = C$

To pkt (x_1, y_1) og (x_2, y_2) gir linje - formel

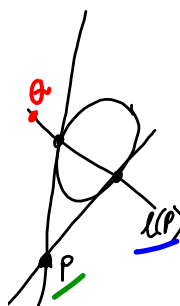
$$\begin{cases} y - y_1 = k(x - x_1) \\ k = \frac{y_2 - y_1}{x_2 - x_1} \end{cases}$$

• $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$
Degenererte og ikke-degenererte

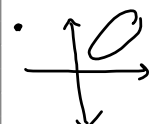
5.6 Kjeglesnitt med akser langs koordinataksene

I Tabell 3 under har vi sammenfattet likninger, brennpunkt, styrelinjer, pol og polare akser for ellipser og hyperbler med sentrum i origo og store/reell akse langs x -aksen, og for parabler med toppunkt i origo og akse langs x -aksen. Nederst har vi lagt til likninger for tangentlinjer gjennom et punkt (p, q) på kjeglesnittene.

	ellipse	hyperbel	parabel
likning	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$y^2 = 4cx$
brennpunkt	$(\pm c, 0)$	$(\pm c, 0)$	$(c, 0)$
Pythagoreisk relasjon	$a^2 = b^2 + c^2$	$a^2 + b^2 = c^2$	
eksentrisitet	$e = \frac{c}{a} (< 1)$	$e = \frac{c}{a} (> 1)$	
styrelinje(r)	$x = \pm \frac{a}{e}$	$x = \pm \frac{a}{e}$	$x = -c$
pol	(x_0, y_0)	(x_0, y_0)	(x_0, y_0)
polare	$\frac{xx_0}{a^2} + \frac{yy_0}{b^2} = 1$	$\frac{xx_0}{a^2} - \frac{yy_0}{b^2} = 1$	$yy_0 = 2c(x + x_0)$
asymptoter		$\frac{x}{a} = \pm \frac{y}{b}$	
trigonometrisk parametrisering	$(a \cos t, b \sin t)$	$(\frac{a}{\cos t}, b \tan t)$	
hyperbolsk parametrisering		$(a \cosh t, b \sinh t)$	
tangentlinjer i punktet (p, q)	$\frac{px}{a^2} + \frac{qy}{b^2} = 1$	$\frac{px}{a^2} - \frac{qy}{b^2} = 1$	$qy = 2c(x + p)$



- Halvaksler parallelle med x og y -akser



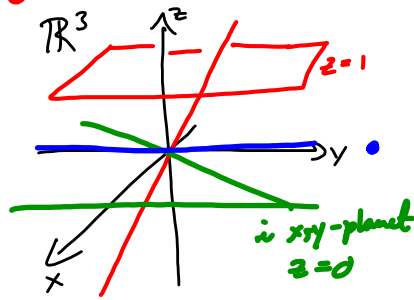
- har med xy -ledd
- Diagonalisere
- Translatere

$Y - q = \frac{y_0}{x_0}(x - p)$

Tabell 3: Kjeglesnitt med akser langs koordinataksene

La h være sekning:
 $Q \in \ell(P) \iff P \in \ell(Q)$

6 PROJEKTIV GEOMETRI



$$\mathbb{P}^2 = (\mathbb{R}^3 \setminus \{0\}) / \sim$$

$$(x_0, x_1, x_2) = (\lambda x_0, \lambda x_1, \lambda x_2) \text{ for all } \lambda \neq 0.$$

$$(x_0 : x_1 : x_2)$$

$$l: ax_0 + bx_1 + cx_2 = 0 \quad l^V: (a : b : c)$$

- Schnitt mellom to linjer:

$$l_1: a_0x_0 + a_1x_1 + a_2x_2 = 0$$

$$l_2: b_0x_0 + b_1x_1 + b_2x_2 = 0$$

$$\vec{u}_P = (a_0, a_1, a_2) \times (b_0, b_1, b_2)$$

$$P = (a_1b_2 - a_2b_1 : a_2b_0 - a_0b_2 : a_0b_1 - b_1a_0)$$

- To plot gir linje

$$P = (p_0 : p_1 : p_2)$$

$$Q = (q_0 : q_1 : q_2)$$

$$l_{PQ}: \begin{vmatrix} x_0 & x_1 & x_2 \\ p_0 & p_1 & p_2 \\ q_0 & q_1 & q_2 \end{vmatrix} = 0.$$

$$P \rightarrow \infty \quad \frac{\overline{AP}}{\overline{PB}} := -1$$

- KRYSSFORHOLD $(A, B), (C, D) \quad A, B, C, D \in l$

$$(A, B, C, D) = \frac{\overline{AC}}{\overline{AD}} \cdot \frac{\overline{BD}}{\overline{BC}}$$

$$= -1 := \text{harmoniske konjugerte}$$

$$\begin{cases} C = pA + qB \\ D = rA + sB \end{cases} \quad \frac{qr}{ps}$$

- Kjedeforhold



$$i \mathbb{P}^2: x_0^2 + x_1^2 - x_2^2 = 0$$

$$ax_0^2 + bx_0x_1 + cx_1^2 + dx_0x_2 + ex_1x_2 + fx_2^2 = 0$$

Hva slags kjedeforhold er dette p\u00e5 $x_2 \neq 0$ (E^2)

$$C \cap U_2$$

$$\text{Se p\u00e5 } D = b^2 - 4ac \quad (x_2 = 0)$$

$$D < 0 \text{ ellipse i } U_2$$

$$D = 0 \text{ parabel i } U_2$$

$$D > 0 \text{ hyperbel i } U_2$$

$$l_{\infty} \cap C: 0 \text{ pkt}$$

$$l_{\infty} \cap C: 1 \text{ pkt}$$

$$\text{tangent}$$

$$l_{\infty} \cap C: 2 \text{ pkt}$$