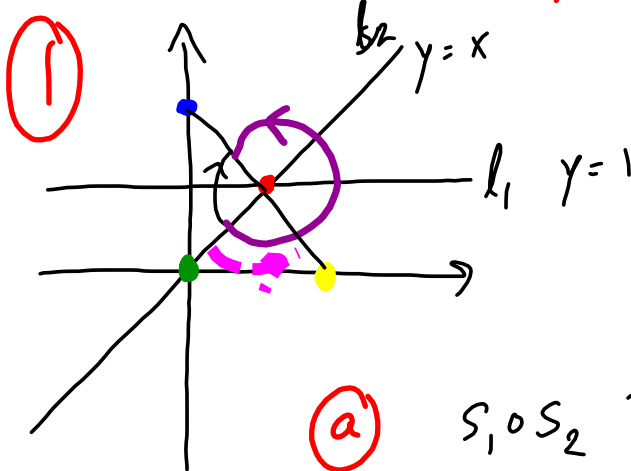


EKSAMEN 2019

MAT 2500 25.11.2022

 s_1 speiling om l_1 s_2 — l_2

② $\left. \begin{matrix} s_1 \circ s_2 \\ s_2 \circ s_1 \end{matrix} \right\}$ har fikspunkt $O = (1,1) = l_1 \cap l_2$, som er likt.

③ Rotasjonssenter er lik fikspunkt: $O = (1,1)$.

$$s_1 \circ s_2 \left(\underset{\substack{\parallel \\ P}}{0,0} \right) = s_1(0,0) = \underline{(0,2)} = Q$$

$$\vec{OP} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \quad \vec{OQ} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\cos \theta = \frac{(-1)(-1) + (-1)(1)}{\sqrt{2} \sqrt{2}} = 0$$

$$\underline{\underline{\theta = \frac{3\pi}{2}}}$$

$$s_2 \circ s_1(0,0) = s_2(0,2) = \underline{(2,0)} = Q'$$

$$\vec{OP} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \quad \vec{OQ'} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\cos \theta' = 0$$

$$\underline{\underline{\theta' = \frac{\pi}{2}}}$$

②

 $\mathbb{RP}^2$

$$l: x_0 + 2x_1 + x_2 = 0 \quad m: x_0 - x_1 + x_2 = 0$$

$$\begin{aligned} \vec{v}_p &= (1, 2, 1) \times (1, -1, 1) \\ &= (2 \cdot 1 - 1 \cdot (-1), 1 \cdot 1 - 1 \cdot 1, 1 \cdot (-1) - 1 \cdot 2) \\ &= (3, 0, -3) \end{aligned}$$

$$P = \underline{(1:0:-1)} = \underline{(-1:0:1)}$$

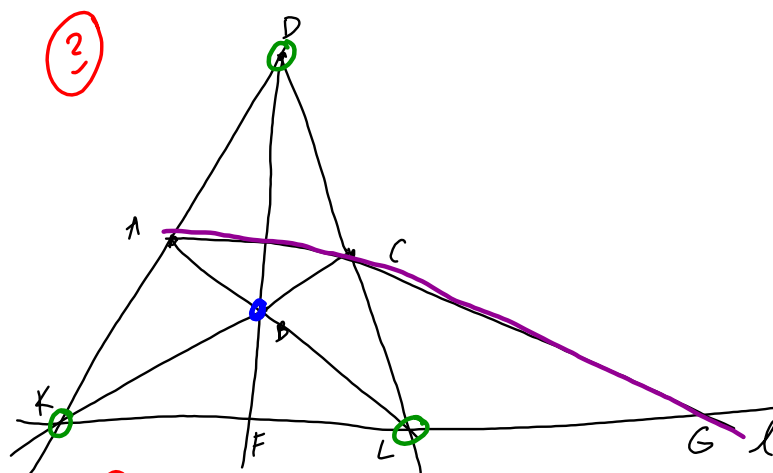
⑥

$$Q = (0:-1:1)$$

$$n = l_{QP}: \begin{vmatrix} x_0 & x_1 & x_2 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{vmatrix} = 0$$

$$-x_0 - x_1 - x_2 = 0$$

$$\underline{\underline{x_0 + x_1 + x_2 = 0}}$$



- (a) Vi har 3 Cevalinjer DKC , DLF , DLA for $\triangle DKL$. De møtes i B , så ved Cevas setning er:

$$\frac{\overline{DA}}{\overline{AK}} \cdot \frac{\overline{KF}}{\overline{FL}} \cdot \frac{\overline{LC}}{\overline{CD}} = 1,$$

som var det vi skulle vise.

- (b) A, C, G er tre Menelaospått for $\triangle DKL$ som er kollineære. Ved Menelaos' setning er

$$\frac{\overline{DA}}{\overline{AK}} \cdot \frac{\overline{KG}}{\overline{GL}} \cdot \frac{\overline{LC}}{\overline{CD}} = -1, \text{ som var det vi skulle vise.}$$

- (c) Oppgaven sier:
 (A, B) og (C, D) er harmoniske konjugerte
 dersom $(AB, CD) = \frac{\overline{AC}}{\overline{AD}} \cdot \frac{\overline{BD}}{\overline{BC}} = -1$.

(K, L) og (F, G)

$$(KL, FG) = \frac{\overline{KF}}{\overline{KG}} \cdot \frac{\overline{LG}}{\overline{LF}}$$

$$\frac{(a)}{(b)} = \frac{\cancel{\overline{DA}} \cdot \cancel{\overline{KF}} \cdot \cancel{\overline{LC}}}{\cancel{\overline{AK}} \cdot \cancel{\overline{FL}} \cdot \cancel{\overline{CD}}} = \frac{1}{-1}$$

$$\frac{\overline{DA}}{\overline{AK}} \cdot \frac{\overline{KG}}{\overline{GL}} \cdot \frac{\overline{LC}}{\overline{CD}} = -1$$

$$\frac{\overline{KF}}{\overline{FL}} \cdot \frac{\overline{GL}}{\overline{KG}} = -1$$

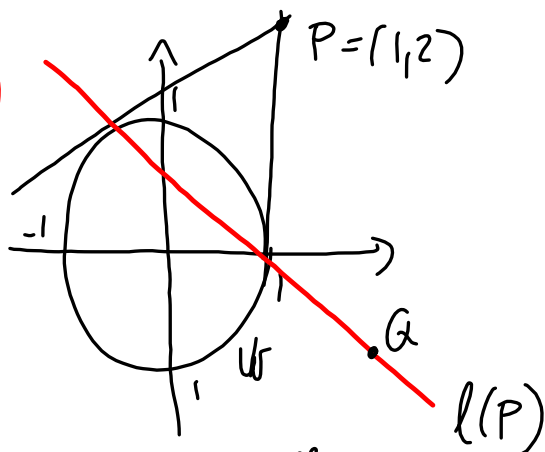
$$\frac{\overline{KF}}{\overline{KG}} \cdot \frac{\overline{LG}}{\overline{LF}} = -1$$

Så begge disse

$$(-1)(-1) = 1$$

(4)

(a)



$$W: x^2 + y^2 = 1$$

$l(P)$: Gitt i tabell:

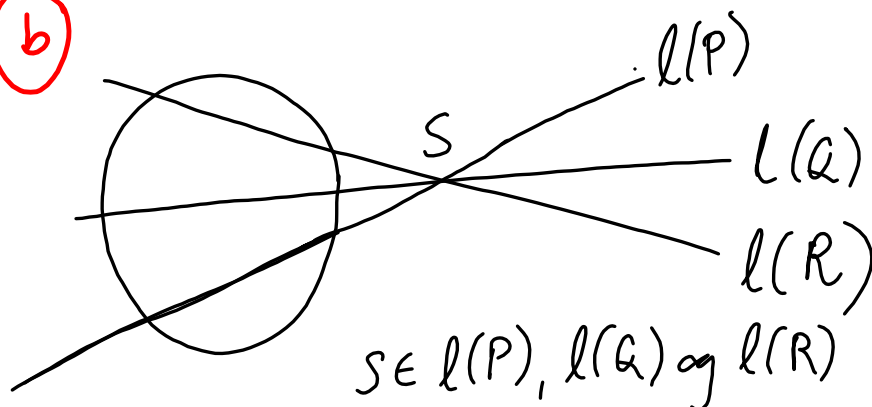
$$x_P \cdot x + y_P \cdot y = 1$$

$$1 \cdot x + 2 \cdot y = 1$$

$Q \in l(P)$ betyr $1 \cdot x_Q + 2 \cdot y_Q = 1$,
 men da er $x_P \cdot x_Q + y_P \cdot y_Q = 1$
 men $l(Q): x_Q \cdot x + y_Q \cdot y = 1$,
 så $P \in l(Q)$.

MAO: La Hires setning sier at $Q \in l(P) \Leftrightarrow P \in l(Q)$.

(b)



$$S \in l(P), l(Q) \text{ og } l(R)$$

$\forall a$ eller La Hires setning:

$$\left. \begin{array}{l} P \in l(S) \\ Q \in l(S) \\ R \in l(S) \end{array} \right\} \text{ dvs. } P, Q, R \text{ er p\aa} \underline{\text{sammene linje.}}$$

5 a) 7 konveks polygoner E^3

$$v = 12$$

$$e = 24$$

V/ Eulers formel er $v - e + f = 2$,

$$\begin{aligned} \text{sa } f &= 2 + e - v \\ &= 2 + 24 - 12 \\ &= \underline{\underline{14}} \end{aligned}$$

b) 7 beting av 7 trekanter
0 firkanter

$$\tau + \phi = f = 14$$

$$3\tau + 4\phi = 2e = 48$$

$$\begin{pmatrix} 1 & 1 & 14 \\ 3 & 4 & 48 \end{pmatrix} \xrightarrow{(-1)} \begin{pmatrix} 1 & 1 & 14 \\ 0 & 1 & 6 \end{pmatrix} \xrightarrow{(-1)}$$

$$\sim \underline{\underline{\begin{pmatrix} 1 & 0 & 8 \\ 0 & 1 & 6 \end{pmatrix}}}$$

$$\tau = 8$$

$$\underline{\underline{\phi = 6}}$$

E-2021

(4) (a) P polyeder i $\mathbb{E}^3$

$$V = 24$$

$$C = 60$$

$$\text{Euler: } \phi = 2 + C - V = 2 + 60 - 24 = \underline{\underline{38}}$$

 τ trekanter ϕ firkanter

$$\tau + \phi = 38$$

$$3\tau + 4\phi = 2 \cdot 60$$

$$\begin{pmatrix} 1 & 1 & 38 \\ 3 & 4 & 120 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 38 \\ 0 & 1 & 6 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 32 \\ 0 & 1 & 6 \end{pmatrix}$$

$$\underline{\underline{\tau = 32 \text{ og } \phi = 6}}$$

(b) I hvert hjørne møtes fast konfigurasjon av trekanter og firkanter; hvor mange av hver

$$\underline{32} \cdot \underline{3} = \underline{24} \cdot \underline{C}$$

$$\underline{\underline{C = 4}}$$

$$6 \cdot 4 = 24 \cdot d$$

$$\underline{\underline{d = 1}}$$

1 firkant og 4 trekanter møtes i hvert hjørne

ALTERNATIV:

$$2e = \sum \deg p = V \cdot \deg p$$

$$2 \cdot 60 = 24 \cdot \deg p$$

$$\deg p = 5 \text{ da } 5 \text{ kanter/flater in i hvert hjørne.}$$

$$\begin{matrix} 5 \\ 1+4 \end{matrix} \quad 1 \cdot \frac{\pi}{3} + 4 \cdot \frac{\pi}{2} > 2\pi \quad \text{Umulig}$$

$$2+3 \quad 2 \cdot \frac{\pi}{3} + 3 \cdot \frac{\pi}{2} > 2\pi \quad \text{---}$$

$$3+2 \quad 3 \cdot \frac{\pi}{3} + 2 \cdot \frac{\pi}{2} = 2\pi \quad \text{--- " ---}$$

$$4+1 \quad 4 \cdot \frac{\pi}{2} + 1 \cdot \frac{\pi}{3} < 2\pi \quad \text{OK.}$$

$$\textcircled{3} \textcircled{a} \quad \ell: 2x_0 + x_1 + 2x_2 = 0$$

$$m: x_0 - x_2 = 0$$

$$P = m \cap \ell$$

$$\vec{v}_P = (2, 1, 2) \times (1, 0, -1)$$

$$= (-1, 4, -1)$$

$$P = (-1:4:-1) = (1:-4:1)$$

$$\textcircled{b} \quad n: 3x_0 + x_1 + x_2 = 0$$

$$P \in n: 3 \cdot 1 + (-4) + 1 = 0 \quad \text{ok.}$$

$$\ell \quad \ell^\vee = (2:1:2)$$

$$m^\vee = (1:0:-1)$$

$$n^\vee = (3:1:1)$$

Skal finne k gjennom P s.a.

$$(m^\vee, n^\vee; k^\vee, \ell^\vee) = -1.$$

$$\frac{q \cdot r}{p \cdot s}$$

$$k^\vee = p m^\vee + q n^\vee$$

$$\ell^\vee = r m^\vee + s n^\vee$$

$$(k_0:k_1:k_2) = p(1:0:-1) + q(3:1:1)$$

$$(2:1:2) = p(1:0:-1) + q(3:1:1)$$

$$s=1, r=-1$$

$$\begin{cases} 2 = r + 3s \\ 1 = 0 \cdot r + s \\ 2 = -r + s \end{cases} \rightarrow \begin{matrix} s=1 \\ r=-1 \end{matrix}$$

$$\text{Husk: } \frac{q \cdot r}{p \cdot s} = -1$$

$$\frac{q}{p} = 1 \quad q = p$$

$$(k_0:k_1:k_2) = p[(1:0:-1) + (3:1:1)]$$

$$= p(4:1:0)$$

$$k^\vee = (k_0:k_1:k_2) = (4:1:0)$$

Linjen k er gitt ved $4x_0 + x_1 = 0$
Sjekk $P \in k: 4 \cdot 1 + (-4) = 0$ ok!

$$\textcircled{c) \quad C \in \mathbb{P}^2 \quad 2x_0^2 + x_0x_1 - x_1x_2 - 3x_2^2 = 0$$

$$\underline{C \cap l}: \quad l: x_1 = -2(x_0 + x_2)$$

$$C \cap l: 2x_0^2 + x_0(-2)(x_0 + x_2) - (-2)(x_0 + x_2)x_2 - 3x_2^2$$

$$\underline{2x_0^2 - 2x_0^2 - 2x_0x_2 + 2x_0x_2 + 2x_2^2 - 3x_2^2} = 0$$

$$-x_2^2 = 0$$

$$x_2 = 0 \quad x_1 = -2x_0 \quad \underline{(1: -2: 0)}_{\text{dobbelt}}$$

$$\underline{C \cap m}: \quad$$

$$m: x_0 = x_2$$

$$C \cap m: 2x_2^2 + x_2x_1 - x_1x_2 - 3x_2^2 = 0$$

$$\underline{-x_2^2 = 0}$$

$$x_2 = 0 \quad x_0 = 0 \quad x_1 \neq 0$$

$$\underline{(0: 1: 0)}_{\text{dobbelt}}$$

$C \cap l$ og $C \cap m$ ligger på $x_2 = 0$

$C \cap (x_2 = 0) = 2$ punkter,

der er $C \cap (x_2 \neq 0)$ en hyperbel.