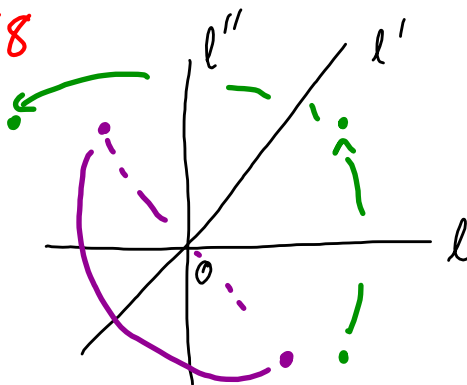


15.11.2022

EKSAMEN 2018

 $E^2 \quad (x, y)$ $l: y=0$ $l': y=x$ $l'': x=0$ Spiegelinger $S_l, S_{l'}, S_{l''}$ Rotasjon ρ_π

(a) I Vis $S_{l''} \circ S_l$ er rotasjon og finn filopunkt & rotasjonsvinkel

- orienteringsbevarende som sammensetning av to orienteringsreverserende isometrier.

$$l \cap l'' = \emptyset$$

S_l og $S_{l''}$ fikserer begge $\emptyset$, så

$\emptyset$ er filopunkt for sammensetningen.

- V/ sætn 2.25 er $S_{l''} \circ S_l$ en rotasjon

$$S_{l''} \circ S_l \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = S_{l''} \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Vinkel mellom $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ og $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ er π

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|}$$

$$\underline{S_{l''} \circ S_l = \rho_{\pi, \emptyset}}$$

II

$$S_{l'} \circ S_{l''} \circ S_l = S_{l'} \circ \rho_{\pi, \emptyset}$$

- orienteringsreverserende som sammensetning av tre orienteringsreverserende isometrier

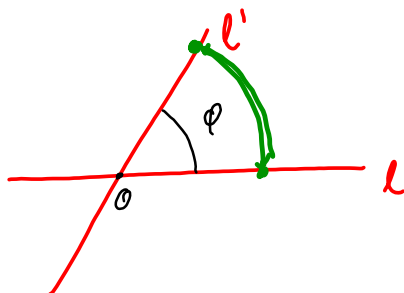
$$l' \cap l'' \cap l = \emptyset, \text{ så } \emptyset \text{ er et filopunkt}$$

Se at $(-a, a)$ er filopunkter,

så $S_{l'} \circ S_{l''} \circ S_l$ fikserer linja $y = -x$

og det er en speiling om $y = -x$.

16.11.2022
DISKUSJON (2006)



$$l \cap l' = \emptyset$$

$$l: y=0$$

$$l': y=ax$$

$$a = \tan \varphi$$

$$\varphi = \arctan a$$

$S_{l'} \circ S_l$: $\left. \begin{array}{l} \text{• orienteringsbevarende} \\ \text{• fikspunkt } \emptyset \end{array} \right\} \text{rotasjon } \beta_{\emptyset, \emptyset} = \beta_{\emptyset}$

Hva er θ ?

$$\bullet S_l \quad \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\bullet S_{l'} = \beta_{\varphi} S_l \beta_{-\varphi}$$

$$\begin{bmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \cos(-\varphi) & -\sin(-\varphi) \\ \sin(-\varphi) & \cos(-\varphi) \end{bmatrix}$$

$$= \begin{bmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{bmatrix} \begin{bmatrix} \cos(-\varphi) & -\sin(-\varphi) \\ -\sin(-\varphi) & -\cos(-\varphi) \end{bmatrix}$$

$$= \begin{bmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{bmatrix} \begin{bmatrix} \cos \varphi & \sin \varphi \\ \sin \varphi & -\cos \varphi \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 \varphi - \sin^2 \varphi & 2 \cos \varphi \sin \varphi \\ 2 \cos \varphi \sin \varphi & \sin^2 \varphi - \cos^2 \varphi \end{bmatrix}$$

$$\begin{array}{l} \cos(\theta_1 + \theta_2) \\ \sin(\theta_1 + \theta_2) \end{array}$$

$$= \begin{bmatrix} \cos 2\varphi & \sin 2\varphi \\ \sin 2\varphi & -\cos 2\varphi \end{bmatrix}$$

$$S_{l'} \circ S_l : \begin{bmatrix} \cos 2\varphi & \sin 2\varphi \\ \sin 2\varphi & -\cos 2\varphi \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\varphi & -\sin 2\varphi \\ \sin 2\varphi & \cos 2\varphi \end{bmatrix}$$

detta er st. matrisen til

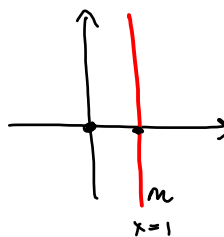
$\beta_{2\varphi}$

TOTALT: $S_{l'} \circ S_l$ er en rotasjon om $l \cap l'$ med vinkel $\theta = 2\varphi$, der φ er vinkel mellom l og l' ,
 $\theta = 2 \arctan a$.

E2018 - ①

⑥ m: $x=1$ β_{π}

- Skal vise at $g = \delta_m \circ \beta_{\pi}$ er en glidespeiling.



- Måke at $\delta_m = t_{\vec{a}} \circ \delta_{l''} \circ t_{-\vec{a}}$ med l'' y -aksen ($x=0$)
 $\vec{a} = (1, 0)$

- g er orienteringsreverserende
- Nødt å vise at g ikke har fikspunkt.

$$g = t_{\vec{a}} \circ \delta_{l''} \circ t_{-\vec{a}} \circ \beta_{\pi}$$

Antar $g\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ y \end{bmatrix}$ fins og får motsigelse:

$$t_{\vec{a}} \circ \delta_{l''} \circ t_{-\vec{a}} \left(\begin{bmatrix} \cos \pi & -\sin \pi \\ \sin \pi & \cos \pi \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} \right)$$

$$= t_{\vec{a}} \circ \delta_{l''} \circ t_{-\vec{a}} \left(\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} \right)$$

$$\vec{a} = (1, 0)$$

$$= t_{\vec{a}} \circ \delta_{l''} \circ t_{-\vec{a}} \left(\begin{bmatrix} -x \\ -y \end{bmatrix} \right)$$

$$= t_{\vec{a}} \circ \delta_{l''} \left(\begin{bmatrix} -x-1 \\ -y \end{bmatrix} \right)$$

$$= t_{\vec{a}} \left(\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -x-1 \\ -y \end{bmatrix} \right)$$

$$= t_{\vec{a}} \left(\begin{bmatrix} x+1 \\ -y \end{bmatrix} \right)$$

$$= \begin{bmatrix} x+2 \\ -y \end{bmatrix} \neq \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{aligned} y &= -y \\ y &= 0 \\ x+2 &\neq x \end{aligned}$$

så g er en glidespeiling.

$$\begin{aligned} g\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) &= \begin{bmatrix} x+2 \\ -y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix} \\ &= t_{\vec{b}} \circ S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) \end{aligned}$$

$$\text{der } \vec{b} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

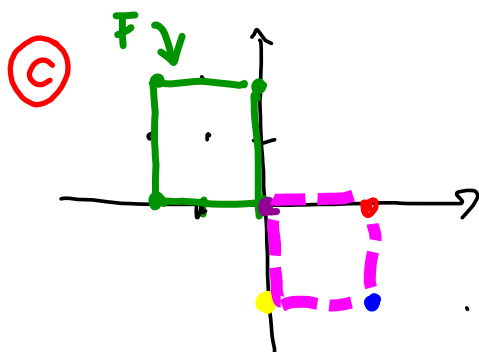
S er speiling om x -aksen.

Så $\vec{b}$ er translasjonsvektoren

og x -aksen er speilingshøyde vi ønsker.

Husk omskriving i boks for 2.29:

$$t_{\vec{a}} \circ S_{l'} = t_{\vec{a}'} \circ S_{l'} \quad \text{der } \vec{a}' \parallel l'$$



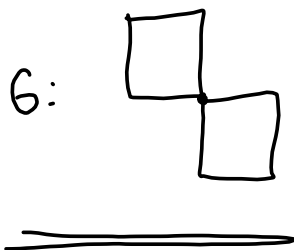
La $G = F \vee g(F)$

$$g(F): g(0,0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$g(0,2) = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$g(-2,0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$g(-2,2) = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

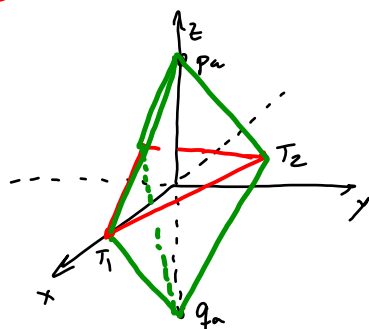


Symmetriene til G
 id
 $\beta\pi, \emptyset$

$$s_1 \quad l: y = x$$

$$s_2 \quad l': y = -x$$

② E^3 (x, y, z)



$$T_1(1,0,0)$$

$$T_2(-\frac{1}{2}, \frac{\sqrt{3}}{2}, 0)$$

$$(-\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0)$$

For $a > 0$

$$P_a = (0, 0, a)$$

$$q_a = (0, 0, -a)$$

P_a konvekshylining.

① Skal bestemme a s.a. P_a er begrenset av kongruente, like-sidede trekanten.

$$\text{Kraav: } \|\vec{T_1 T_2}\| = \|\vec{T_1 P_a}\|$$

$$\|(-\frac{1}{2}, \frac{\sqrt{3}}{2}, 0) - (1, 0, 0)\| = \|(0, 0, a) - (1, 0, 0)\|$$

$$\|(-\frac{3}{2}, \frac{\sqrt{3}}{2}, 0)\|^2 = \|(-1, 0, a)\|^2$$

$$(-\frac{3}{2})^2 + (\frac{\sqrt{3}}{2})^2 = (-1)^2 + a^2$$

$$\frac{9}{4} + \frac{3}{4} = 1 + a^2$$

$$2 = a^2$$

$$\underline{a = \pm \sqrt{2}}, \quad \underline{a = \sqrt{2}}$$

Når $a = \sqrt{2}$ vil P_a være begrenset av regulære trekanter.

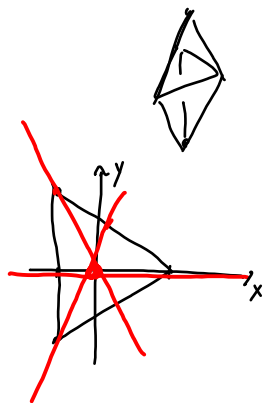
thorfor er $P = P_{\sqrt{2}}$ ikke et reguleret polyeder?

1 $P_{\sqrt{2}}$ vil det være 3 flater som møtes.

1 T_1 vil det være 4 $\dots$

Så $P_{\sqrt{2}}$ er ikke reguleret.

② Rotasjonsgruppen til P .



• id

• Rotasjoner om z -aksen

med vinkler $\frac{2\pi}{3}$ og $\frac{4\pi}{3}$.

• Rotasjoner om x -aksen

med vinkel π

• Rotasjoner om linjer i planet $z=0$
(x, y)-planet

$$\left. \begin{array}{l} y = \sqrt{3}x \\ y = -\sqrt{3}x \end{array} \right\} \text{ med vinkel } \pi$$

• D_3 i planet.

③ $\mathbb{RP}^2$ $(X_0 : X_1 : X_2)$
 $\mathbb{R}^2 \hookrightarrow \mathbb{RP}^2 \quad X_2 \neq 0.$

$$Q_a : X_0^2 + aX_1^2 - X_2^2 = 0$$

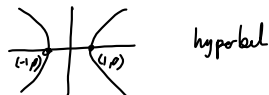
④ Tegn $Q_a \cap \mathbb{RP}^2$ for $a = -1, a = 0$ og $a = 1$.

$$\mathbb{R}^2 : X_2 \neq 0 :$$

$$Q_a : \frac{X_0^2}{X_2^2} + a \frac{X_1^2}{X_2^2} - \frac{X_2^2}{X_2^2} = 0$$

$$(x, y) = \left(\frac{X_0}{X_2}, \frac{X_1}{X_2} \right) : x^2 + ay^2 = 1$$

$Q_a \cap \mathbb{RP}^2 : a = -1 \quad x^2 - y^2 = 1$

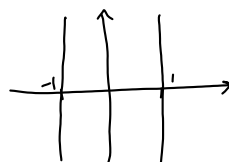


hyperbol

$a = 0$

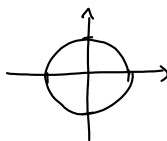
$$x^2 = 1 \quad (x^2 - 1) = 0$$

$$x = \pm 1 \quad (x-1)(x+1) = 0$$



$a = 1$

$$x^2 + y^2 = 1$$



sirkel

⑤ $Q_a : X_0^2 + aX_1^2 - X_2^2 = 0$

$$\mathbb{R}^2 : X_2 \neq 0 \quad l_{\infty} : X_2 = 0$$

Regner $D = 0^2 - 4 \cdot 1 \cdot a = -4a$

$D > 0$ of $Q_a \cap \mathbb{RP}^2$ hyperbol for $a < 0$.
 $D < 0$ of $Q_a \cap \mathbb{RP}^2$ ellipse for $a > 0$. } $\times$ pi forknet
 pi $x^2 + ay^2 = 1$.

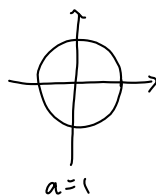
⑥ • Her sett at Q_0 er to linjer.

$$Q_0 : X_0^2 - X_2^2 = (X_0 - X_2)(X_0 + X_2) = 0$$

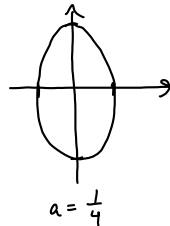
$$L_1 : X_0 - X_2 = 0$$

$$L_2 : X_0 + X_2 = 0$$

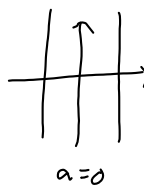
• $\lim_{a \rightarrow 0^+} Q_a \quad x^2 + ay^2 = 1$



$$a = 1$$



$$a = \frac{1}{4}$$



$$a = 0$$

$$x^2 + \frac{y^2}{a^2} = 1$$

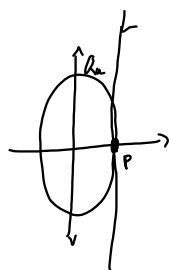
Ellipsen blir stadig mer sirkelaktig inn til den degenererer til to linjer.

d) $P = (1:0:1)$
 $Q = (1:1:1)$

$$L: \begin{vmatrix} x_0 & x_1 & x_2 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$-x_0 + x_2 = 0$$

$$\underline{x_0 = x_2}$$



$L \cap Q: x_0^2 + ax_1^2 - x_0^2 = 0$
 $ax_1^2 = 0$
 Q er en dobbel rot for $x_1 = 0$
 så L vil tangere Q i punktet $P = (1:0:1)$.

$\downarrow$
(1,0)

e) $R = L \cap L_\infty = (0:1:0)$
 $(x_0 = x_2) \quad (x_2 = 0)$

$\vec{U}_R = (1, 0, -1) \times (0, 0, 1)$
 $= (0, -1, 0)$

$L': \begin{vmatrix} x_0 & x_1 & x_2 \\ 0 & 1 & 0 \\ c_0 & c_1 & c_2 \end{vmatrix} = 0$

• Linje L' gjennom R har formen
 $-c_0 x_2 + c_2 x_0 = 0, \quad x_2 = \frac{c_2 x_0}{c_0}$

• L' skal tangere Q

$L' \cap Q: x_0^2 + ax_1^2 - \left(\frac{c_2}{c_0}\right)^2 x_0^2 = 0$

$\left(1 - \frac{c_2^2}{c_0^2}\right) x_0^2 + ax_1^2 = 0$

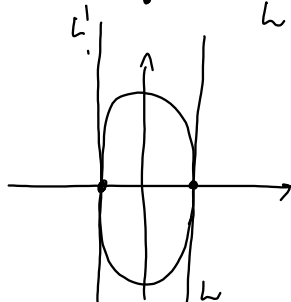
Har dobbel rot $\Leftrightarrow$ diskriminant lik 0:

$0^2 - 4 \cdot \left(1 - \frac{c_2^2}{c_0^2}\right) \cdot a = 0 \quad a \neq 0$

så $1 = \frac{c_2^2}{c_0^2}$ så $c_0^2 = c_2^2$

Enten $\underline{c_0 = c_2}$ eller $\underline{c_0 = -c_2}$

$L': \begin{array}{ll} c_2 x_0 - c_0 x_2 = 0 & c_2 x_0 + c_2 x_2 = 0 \\ c_2 x_0 - c_2 x_2 = 0 & x_0 + x_2 = 0 \\ (0:1:0) \quad x_0 - x_2 = 0 & L' \\ \quad \quad \quad L & (x = -1) \end{array}$



Så $L': x_0 + x_2 = 0$
 er tangent til Q .