

Hva antas vi?

- $\exists$ punkter og linjer
- Linjer har længde, længder er additive



$$AB + BC = AC$$

- $\exists$ vinkler: to linjer som skærer hinanden
- Kongruens: Linjer og vinkler forandres ikke når vi flytter dem

- Rett vinkel



Vinkelmål
er additiv

- Areal, rektangel

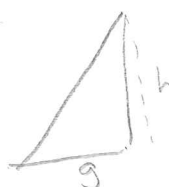


ab

Kongruens, additiv

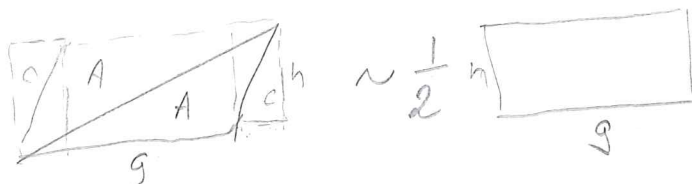
Konsekvenser:

Areal av trekant

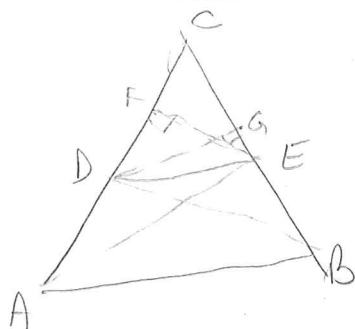


$$\frac{gh}{2}$$

Bevis



Thales setning



$$\text{areal}(\triangle AEB) = \text{areal}(\triangle BDA)$$

$$\Rightarrow \frac{AD \cdot EF}{2} = \frac{EB \cdot DG}{2}$$

$$\text{areal}(\triangle CDE) = \frac{DG \cdot CE}{2} = \frac{EF \cdot CD}{2}$$

$$\left. \begin{array}{l} \frac{AD \cdot EF}{2} = \frac{EB \cdot DG}{2} \\ \frac{DG \cdot CE}{2} = \frac{EF \cdot CD}{2} \end{array} \right\} \frac{AD}{CD} = \frac{EB}{CE}$$

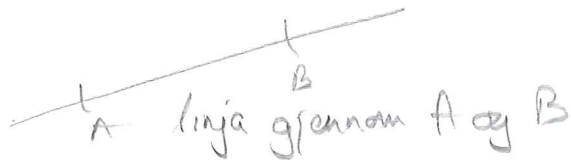
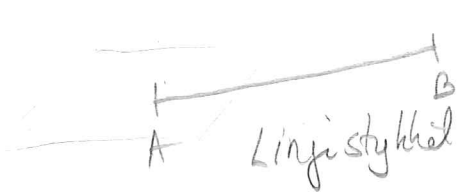
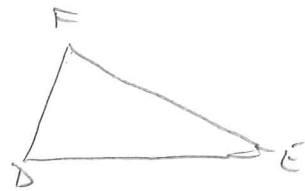
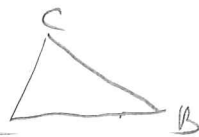
Likeformedede trekanter

$$\triangle ABC \sim \triangle DEF$$

$$\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$$

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$

$$\text{alternativt } \frac{AB}{AC} = \frac{DE}{DF}$$



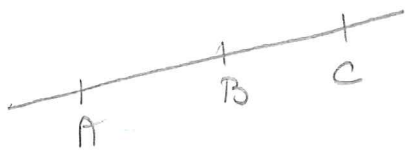
Lengden av AB: $\overline{AB}$

Hvis vi har bestemt oss for en orientering:



$$\begin{aligned} \overline{AB} &= 4 \\ \overline{BA} &= -4 \end{aligned} \quad \left\{ \begin{array}{l} |\overline{AB}| = 4 \\ \underline{\underline{}} \end{array} \right.$$

Regel for fortegn



Kolineære punkter

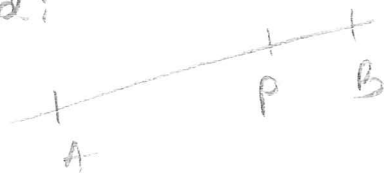
$$\overline{AC} = \overline{AB} + \overline{BC} \Leftrightarrow \text{kolineære}$$

$$\begin{aligned} \overline{AB} &= -\overline{BA} \\ \overline{AA} &= 0 \end{aligned}$$



$$\overline{AB} = \overline{PB} - \overline{PA}$$

Delingsforhold:



P deler AB i forholdet

$$S = \frac{\overline{AP}}{\overline{PB}}$$

P ligger mellom A og B $S > 0$

P ligger utenfor AB $S < 0$

Resultat: Delingsforhold bestemmer delingspunktet
entydig

Bevis

Anta for modsigelse at vi har to forskellige punkter P og Q s.d.

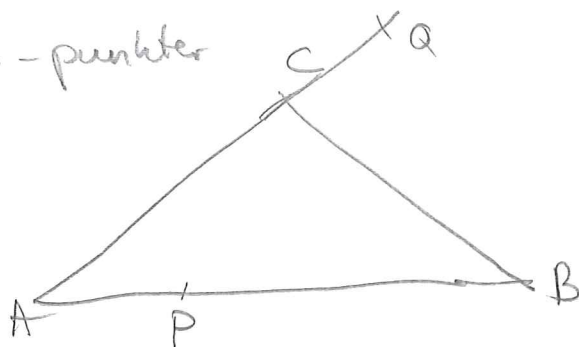
$$\frac{\overline{AP}}{\overline{PB}} = \frac{\overline{AQ}}{\overline{QB}}$$

$$\Rightarrow \frac{\overline{AP}}{\overline{PB}} + \frac{\overline{PB}}{\overline{PB}} = \frac{\overline{AQ}}{\overline{QB}} + \frac{\overline{QB}}{\overline{QB}}$$

$$\Rightarrow \frac{\overline{AP} + \overline{PB}}{\overline{PB}} = \frac{\overline{AQ} + \overline{QB}}{\overline{QB}}$$

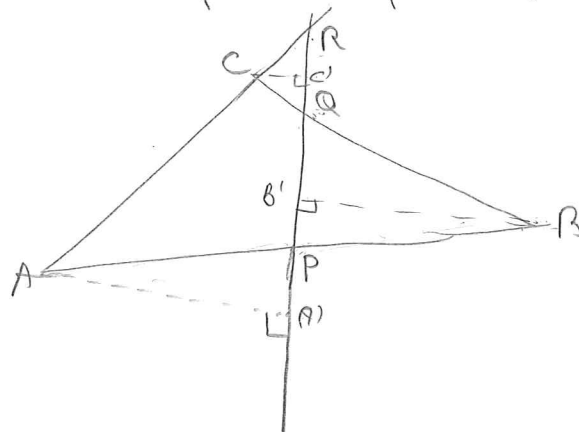
$$\Rightarrow \frac{\overline{AB}}{\overline{PB}} = \frac{\overline{AB}}{\overline{QB}} \Rightarrow \overline{PB} = \overline{QB} \quad \text{QED.}$$

Menelaos-punkter



P: Menelaospunkt for $\triangle ABC$ på AB
Q: ————— på AC.

Når er 3 Menelaos-punkter på linje?



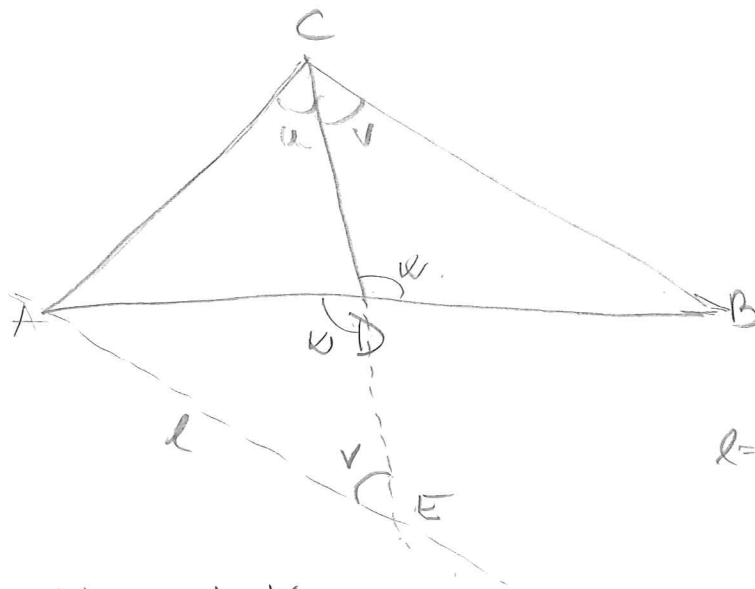
$$\begin{aligned} \triangle AA'P &\sim \triangle BB'P \\ \triangle BB'Q &\sim \triangle CC'Q \\ \triangle CC'R &\sim \triangle AA'R \end{aligned}$$

$$\frac{\overline{AP}}{\overline{PB}} \cdot \frac{\overline{BQ}}{\overline{QC}} \cdot \frac{\overline{CR}}{\overline{RA}} = \frac{\overline{AA'}}{\overline{B'B}} \cdot \frac{\overline{BB'}}{\overline{C'C}} \cdot \frac{\overline{CC'}}{\overline{A'A}} = \frac{\overline{AA'}}{\overline{A'A}} \cdot \frac{\overline{BB'}}{\overline{B'B}} \cdot \frac{\overline{CC'}}{\overline{C'C}} = (-1)^3 = -1.$$

Anta $\frac{\overline{AP}}{\overline{PB}} \cdot \frac{\overline{BQ}}{\overline{QC}} \cdot \frac{\overline{CR}}{\overline{RA}} = -1$. La R' være skjærings mellem PQ og AC.
Vi skal vise at $R = R'$.

$$\text{Vi har vist: } \frac{\overline{AP}}{\overline{PB}} \cdot \frac{\overline{BQ}}{\overline{QC}} \cdot \frac{\overline{CR'}}{\overline{R'A}} = -1$$

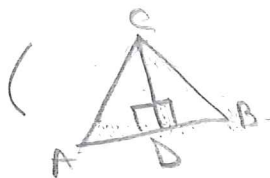
$$\Rightarrow \frac{\overline{CR}}{\overline{RA}} = \frac{\overline{CR'}}{\overline{R'A}} \Rightarrow R = R' \text{ siden delingsforhold bestemmer punkter}$$



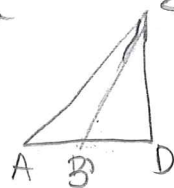
$l = AE \parallel BC$
 E skygger mellom l
 og CD .

$$\frac{AC}{BC} = \frac{AD}{BD} \Leftrightarrow u = v$$

Hvis $u = v$ så er $\triangle EAC$ likebeint og $AC = AE$



$AC = BC \Rightarrow \frac{AC}{CD} = \frac{BC}{CD}$
 Brekker over



og $\triangle EAC \sim \triangle CBD$
 $\Rightarrow \frac{AD}{BD} = \frac{EA}{BC}$

$B'C = BC = AC$
 $\Rightarrow B' = A$

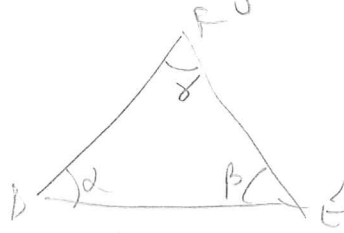
$\Rightarrow \frac{AC}{AD} = \frac{AD}{BD} = AC$

$\frac{AD}{BD} = \frac{AE}{BC} = \frac{AC}{BC}$

Hvis $\frac{AD}{BD} = \frac{AC}{BC} \Rightarrow \frac{AD}{AC} = \frac{BD}{BC} = \frac{AD}{AE} \Rightarrow AC = AE$
 $\Rightarrow u = v$

AAA - teorem

Kongruente vinkler gir konstante forhold



$$\frac{AC}{DF} = \frac{BC}{EF} = \frac{AB}{DE}$$

Bers : Flytt C til F og AC langs DF. Bruk Thalès

Vinkelsummen i en trekant er to rette (π)



Summen av utvendige er 4 rette : 2π

$$2\pi = (\pi - \alpha) + (\pi - \beta) + (\pi - \gamma) = 3\pi - \alpha - \beta - \gamma$$

$$\Rightarrow \alpha + \beta + \gamma = \pi$$