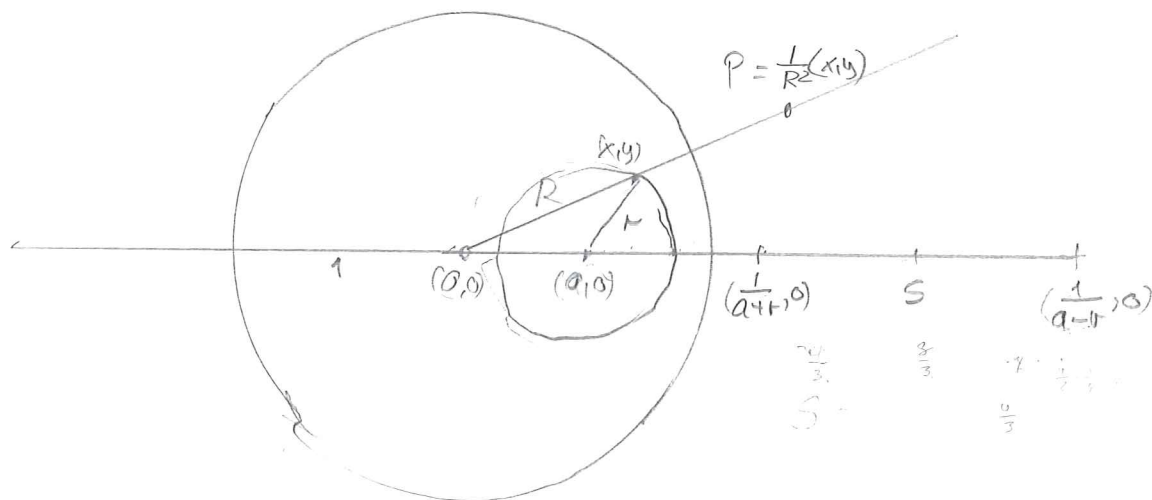


Inversjon over en sirkel

$$x^2 + y^2 = R^2$$

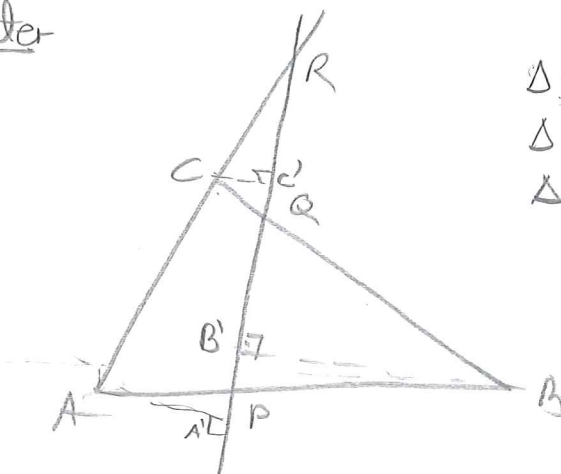
$$(x-a)^2 + y^2 = r^2 \Rightarrow x^2 + y^2 = r^2 - a^2 + 2ax$$



S: x-koordinat: $\frac{1}{2} \left(\frac{1}{a+r} + \frac{1}{a-r} \right) = \frac{a}{a^2 - r^2}$

$$\begin{aligned} \|P-S\|^2 &= \left\| \frac{1}{R^2}(x,y) - \left(\frac{a}{a^2 - r^2}, 0 \right) \right\|^2 \\ &= \left(\frac{x}{R^2} - \frac{a}{a^2 - r^2} \right)^2 + \left(\frac{y}{R^2} \right)^2 \\ &= \frac{x^2}{R^4} - \frac{2ax}{R^2(a^2 - r^2)} + \frac{a^2}{(a^2 - r^2)^2} + \frac{y^2}{R^4} \\ &= \frac{x^2 + y^2}{R^4} - 2 \frac{x}{R^2} \frac{a}{a^2 - r^2} + \left(\frac{a}{a^2 - r^2} \right)^2 \\ &= \frac{1}{R^2} - 2 \frac{x}{R^2} \frac{a}{a^2 - r^2} + \left(\frac{a}{a^2 - r^2} \right)^2 \\ &= \frac{a^2 - r^2 - 2ax}{R^2(a^2 - r^2)} + \left(\frac{a}{a^2 - r^2} \right)^2 = -\frac{1}{a^2 - r^2} + \left(\frac{a}{a^2 - r^2} \right)^2 \\ &= \frac{-a^2 + r^2 + a^2}{(a^2 - r^2)^2} = \left(\frac{r}{a^2 - r^2} \right)^2 \end{aligned}$$

Menelaos-punkter



$$\begin{aligned}\triangle AA'P &\sim \triangle BB'P \\ \triangle BB'Q &\sim \triangle CC'Q \\ \triangle CC'R &\sim \triangle AA'R\end{aligned}$$

Menelaos setning

$$\gamma = \frac{\overline{AP}}{\overline{PB}} \cdot \frac{\overline{BQ}}{\overline{QC}} \cdot \frac{\overline{CR}}{\overline{RA}} = -1 \Leftrightarrow P, Q, R \text{ kollineære}$$

Bevis

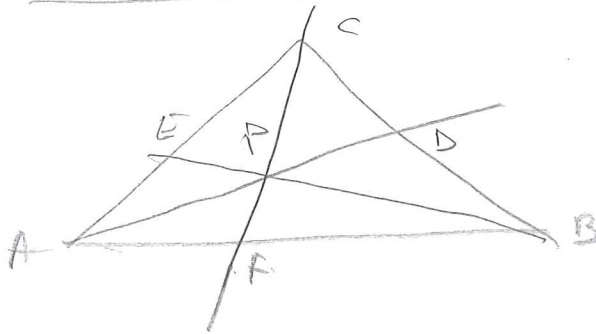
$$\gamma = \frac{\overline{AA'}}{\overline{BB'}} \cdot \frac{\overline{BB'}}{\overline{CC'}} \cdot \frac{\overline{CC'}}{\overline{A'A}} = \frac{\overline{AA'}}{\overline{A'A}} \cdot \frac{\overline{BB'}}{\overline{BB'}} \cdot \frac{\overline{CC'}}{\overline{CC'}} = (-1)^3 = -1.$$

Motsetning: R' skjærsnitt mellom PQ og AC.

$$\frac{\overline{AP}}{\overline{PB}} \cdot \frac{\overline{BQ}}{\overline{QC}} \cdot \frac{\overline{CR'}}{\overline{RA}} = -1 \quad \text{sammen med } \gamma \text{ gir}$$

$$\frac{\overline{CR'}}{\overline{RA}} = \frac{\overline{CR}}{\overline{RA}} \Rightarrow R = R'$$

Ceva - linje



Cevas setning

$$\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = 1 \Leftrightarrow \text{linjene skjærer hverandre i et punkt (konkurrente)}$$

Bevis

Menelaos på $\triangle ABD$ og $\triangle ADC$ gir

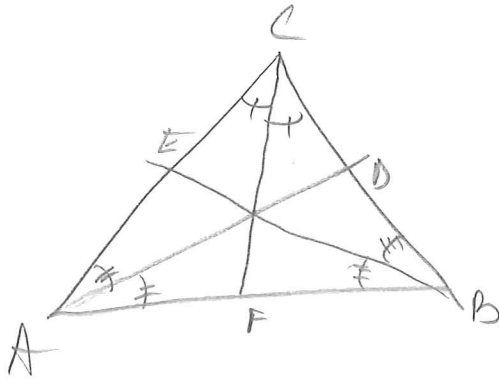
$$\frac{BE}{ED} \cdot \frac{DP}{PA} \cdot \frac{AF}{FB} = \frac{DB}{BC} \cdot \frac{CE}{EA} \cdot \frac{AP}{PD} = -1$$

Produkt og forkortning gir Ceva

Motsatt: Anta ikke konkurrent, La F' være skjæring med linje gjennom C , P og AB

Samme sann over $\Rightarrow F = F'$

Anvendelser av Ceva

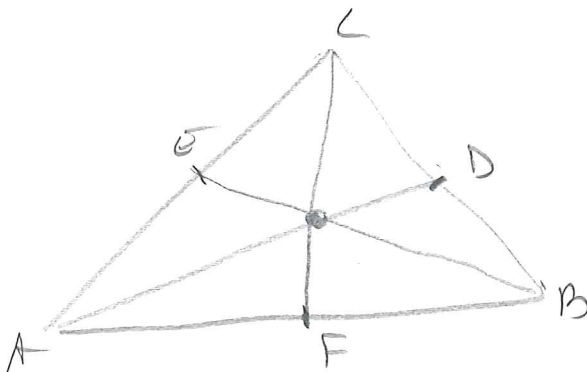


Vinkelhalvering
Incenter

$$\frac{AC}{AB} = \frac{CD}{DB} \quad \frac{AC}{CB} = \frac{AF}{FB} \quad \frac{BC}{AB} = \frac{EC}{AE}$$

$\Rightarrow$

$$\frac{AE}{EC} \cdot \frac{CD}{DB} \cdot \frac{BF}{FA} = \frac{AB}{BC} \cdot \frac{AC}{AB} \cdot \frac{CB}{AC} = 1.$$



Median punkt

$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1 \cdot 1 \cdot 1 = \underline{\underline{1}}$$