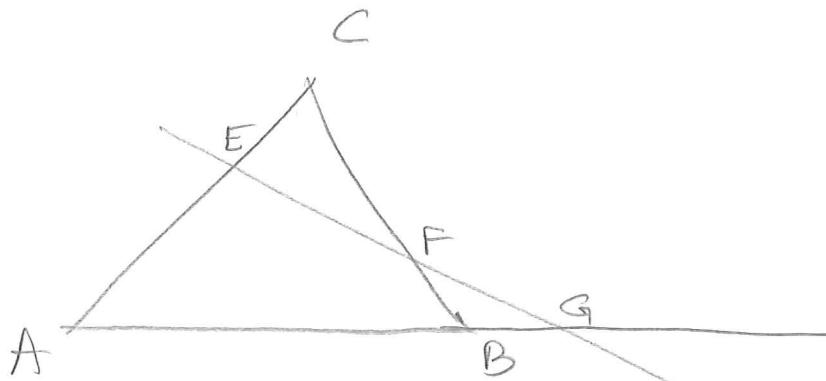
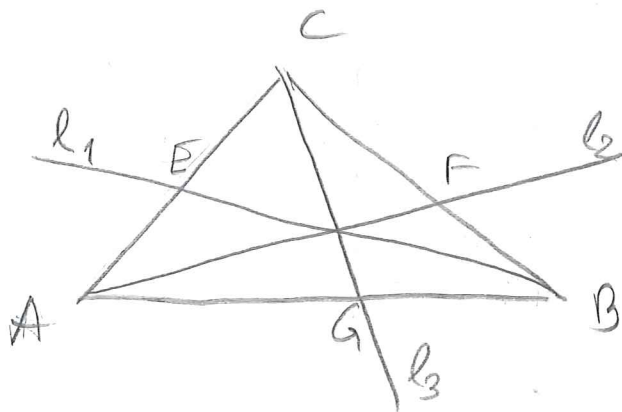


Repetition



Menelauspunkter

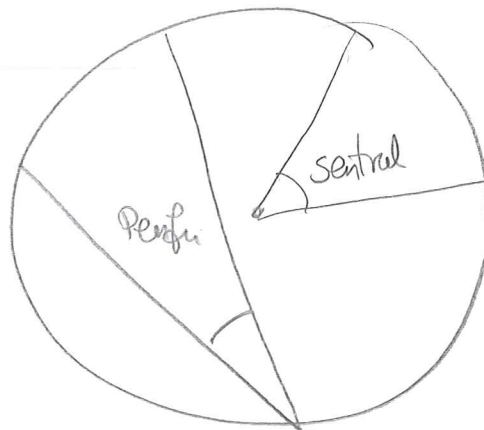
Setzung: E, F, G $\Leftrightarrow \frac{\overline{AE}}{\overline{EC}} \cdot \frac{\overline{CF}}{\overline{FB}} \cdot \frac{\overline{BG}}{\overline{GA}} = -1$
kollinear



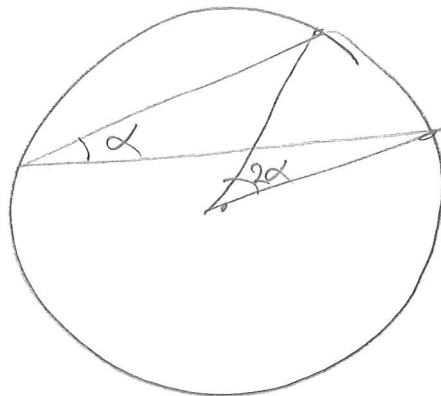
Ceva-Linien l_1, l_2, l_3 er konvergente:

$$\frac{\overline{AE}}{\overline{EC}} \cdot \frac{\overline{CF}}{\overline{FB}} \cdot \frac{\overline{BG}}{\overline{GA}} = 1.$$

Sentral- og perifer vinkel

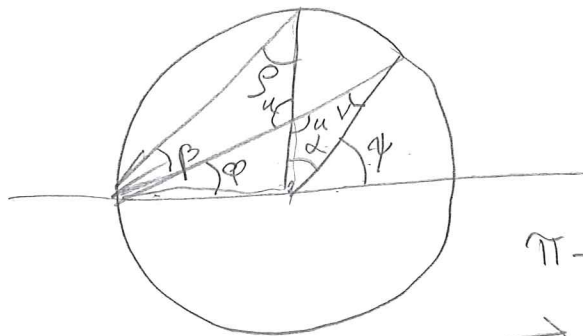


Setning



Sentralvinkelen er dobbelt så stor som perifer vinkel

Bevis



$$\pi - (\alpha + \psi) = \pi - (\beta + \varphi)$$

$$\Rightarrow \alpha + \psi = \beta + \varphi$$

(trækker alle)

$$\beta + \varphi = \psi$$

$$\varphi = \psi$$

$$\beta + \psi + \alpha = \pi$$

$$\alpha + \psi + \alpha = \pi$$

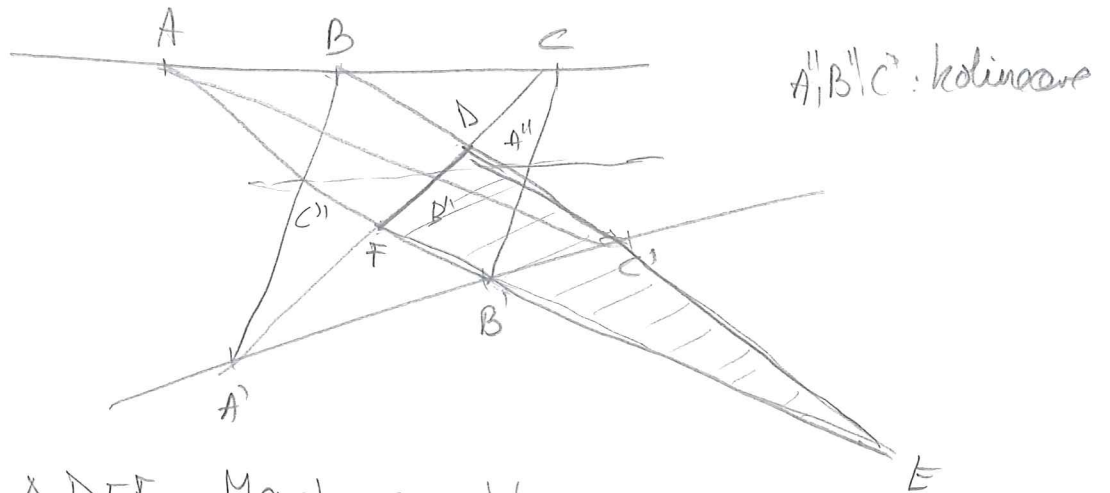
$$\beta + \psi = \alpha + \psi$$

$$\parallel \quad \parallel$$

$$\beta + \beta + \varphi$$

$$\Rightarrow \alpha = 2\beta$$

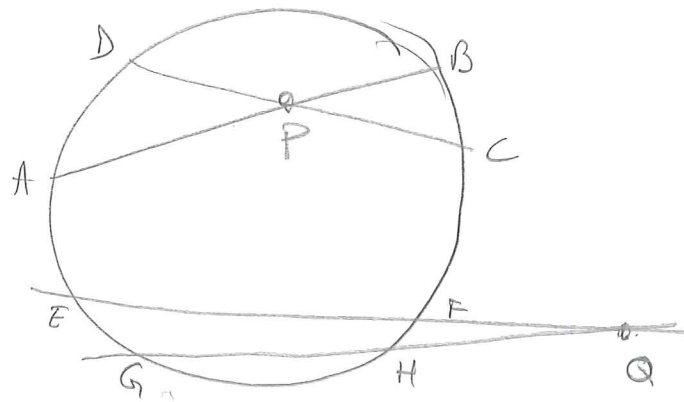
Pappos setzung



Dreieck $\triangle DEF$. Menelaospunkte:

$$\begin{array}{l}
 ABC \\
 A'B'C' \\
 A, B'', C' \\
 A', B, C'' \\
 A'', B', C
 \end{array}
 \left\{
 \begin{array}{l}
 \frac{EB}{BD} \cdot \frac{DC}{CF} \cdot \frac{FA}{AE} = 1 \\
 \frac{EC'}{C'D} \cdot \frac{DA}{AF} \cdot \frac{FB}{BE} = 1 \\
 \frac{EC'}{C'D} \cdot \frac{DB''}{B'F} \cdot \frac{FA}{AE} = 1 \\
 \frac{EB}{BD} \cdot \frac{DA}{AF} \cdot \frac{FC''}{C'E} = 1 \\
 \frac{EA''}{A'D} \cdot \frac{DC}{CF} \cdot \frac{FB}{B'E} = 1
 \end{array}
 \right.$$

$$\frac{DB''}{B'F} \cdot \frac{FC''}{C'E} \cdot \frac{EA''}{A'D} = 1$$

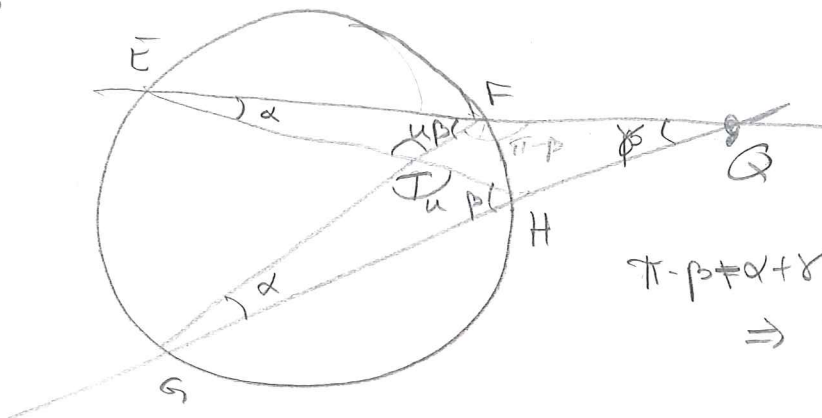


$$\overline{PA} \cdot \overline{PB} = \overline{PC} \cdot \overline{PD}$$

$$\overline{QF} \cdot \overline{QE} = \overline{QH} \cdot \overline{QG}$$

Punkts potens

Bew3



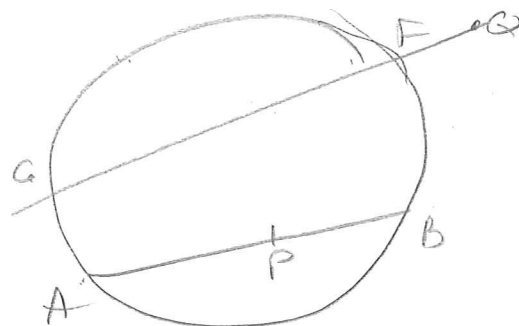
$$\pi - \beta = \alpha + \beta = \pi$$

$$\Rightarrow \beta = \alpha + \beta$$

$$\triangle FQG \sim \triangle HQE$$

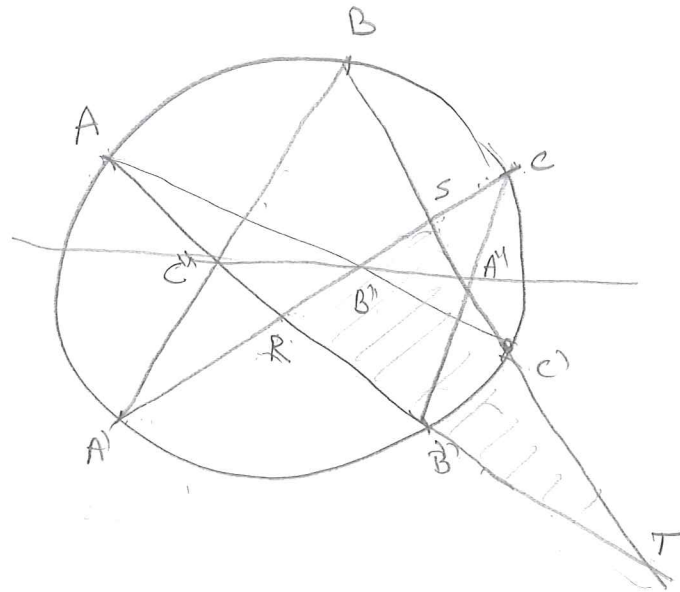
$$\Rightarrow \frac{FQ}{HQ} = \frac{QG}{QE} \Rightarrow$$

$$\underline{\underline{FQ \cdot EQ = HQ \cdot QG}}$$



$\overline{PA} \cdot \overline{PB} < 0$ inni
 $\overline{QE} \cdot \overline{QG} > 0$ vetente
 $= 0$ p[er] s[er]kela

Pascals setning



A'', B'', C'' : kollinear

$\triangle RST$

Menelaospunkter $\{B, C'', A'\}$, $\{C', A, B''\}$, $\{C, A'', B'\}$

$$\left(\frac{RA}{AS} \cdot \frac{SB}{BT} \cdot \frac{TC''}{C'R} \right) \cdot \left(\frac{RB''}{B'S} \cdot \frac{SC}{CT} \cdot \frac{TA}{AR} \right) \cdot \left(\frac{RC}{CS} \cdot \frac{SA''}{A'T} \cdot \frac{TB'}{B'R} \right) = 1$$

$$\frac{SA''}{A'T} \cdot \frac{TC''}{C'R} \cdot \frac{RB''}{B'S} = \frac{A'S}{RA} \cdot \frac{BT}{SB} \cdot \frac{C'T}{SC'} \cdot \frac{AR}{TA} \cdot \frac{CS}{RC} \cdot \frac{B'R}{TB'}$$

Ved punktens potens.

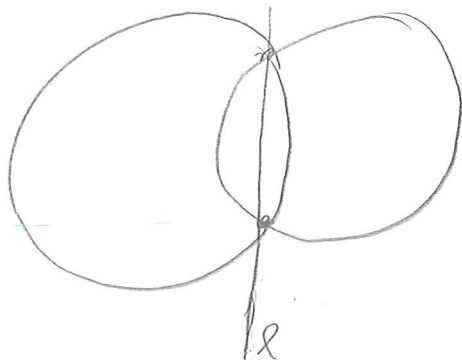
$$RA \cdot RB' = RA' \cdot RC$$

$$SC \cdot SA' = SB \cdot SC'$$

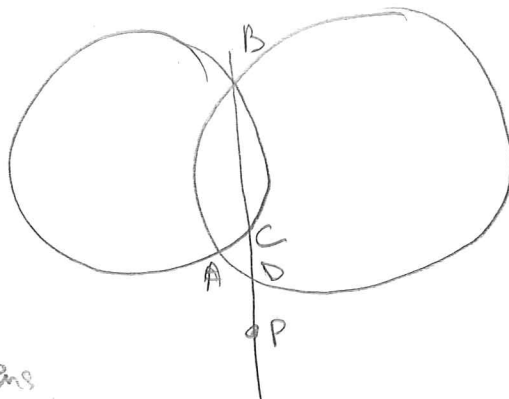
$$TA \cdot TB' = TB \cdot TC'$$

$$= \frac{SB}{RA} \cdot \frac{TA}{SB} \cdot \frac{TB'}{SC'} \cdot \frac{RA'}{TA} \cdot \frac{SC}{RC} \cdot \frac{RC}{TB'} = 1$$

Radikalakse: samme potens til to sider

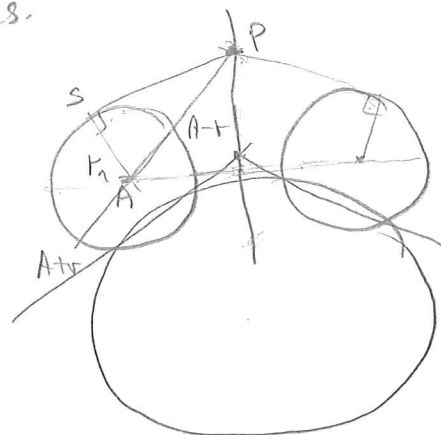


Alle punkter på l har samme potens.

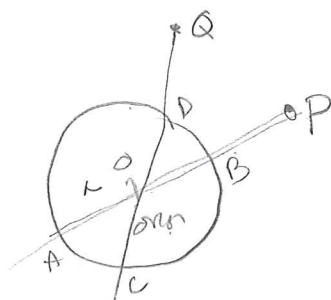


Samme potens

$$\Rightarrow PB \cdot PC = PB \cdot PD \Rightarrow PC = PD \Rightarrow C = D = A$$



Alle punkter med samme potens



$$PB \cdot PA = (PO - r)(PO + r) = PO^2 - r^2$$

$$QD \cdot QC = PO^2 - r^2$$

$$\text{Likhed} \Rightarrow PO = QO$$

$\Rightarrow P, Q$ på samme stråle om O .

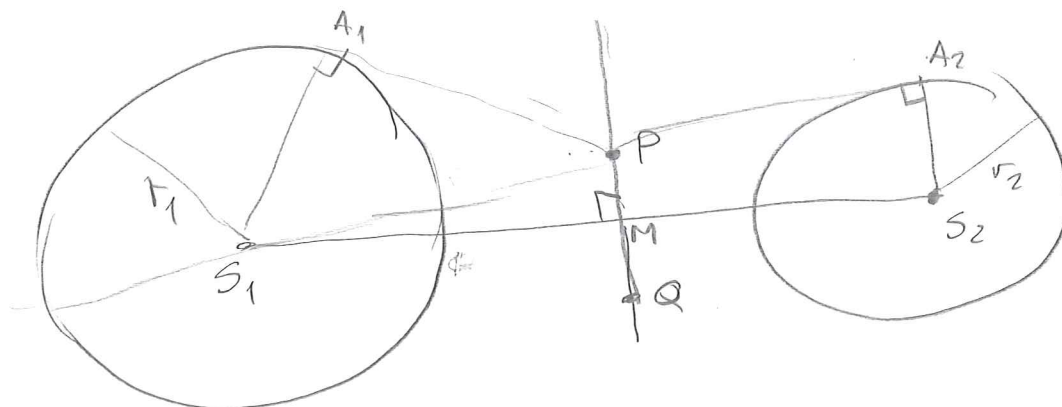
Pythagoras

$$AP^2 = AS^2 + SP^2$$

$$\Rightarrow SP^2 = (AP - AS)(AP + AS)$$

$$= (AP - r)(AP + r)$$

$$= P's \text{ potens}$$



Hvis P har samme potens på de to sødler, så gælder $(PS_1 - r_1)(PS_1 + r_1) = (PS_2 - r_2)(PS_2 + r_2)$
 og

$$\begin{aligned}
 (MS_1 - r_1)(MS_1 + r_1) &= MS_1^2 - r_1^2 \\
 &= PS_1^2 - PM^2 - r_1^2 \\
 &= PS_1^2 - r_1^2 - PM^2 \\
 &= (PS_1 - r_1)(PS_2 + r_2) - PM^2 \\
 &= (PS_2 - r_2)(PS_2 + r_2) - PM^2 = (MS_2 - r_2)(MS_2 + r_2)
 \end{aligned}$$

La Q ligge på linja gennem M og P;

$$\begin{aligned}
 (QS_1 - r_1)(QS_1 + r_1) &= QS_1^2 - r_1^2 - QM^2 + QM^2 \\
 &= QS_1^2 - QM^2 - r_1^2 + QM^2 \\
 &= MS_1^2 - r_1^2 + QM^2 \\
 &= (MS_1 - r_1)(MS_2 + r_2) + QM^2 \\
 &= (MS_2 - r_2)(MS_2 + r_2) + QM^2 \\
 &= MS_2^2 - r_2^2 + QM^2 \\
 &= MS_2^2 + QM^2 - r_2^2 \\
 &= QS_2^2 - r_2^2 = (QS_2 - r_2)(QS_2 + r_2)
 \end{aligned}$$