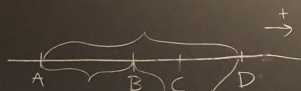


MAT 2500

30.08.2023

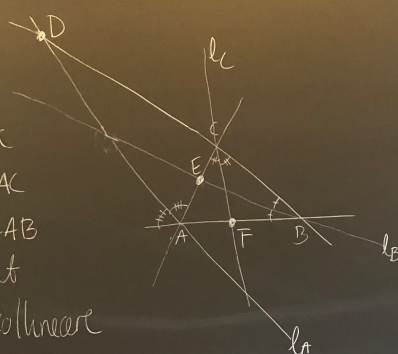
4.4.1

Skal vise $\overline{AD} \cdot \overline{BC} + \overline{BD} \cdot \overline{CA} + \overline{CD} \cdot \overline{AB} = 0$

$$\begin{aligned}
 & \overline{AD} \cdot \overline{BC} + \overline{BD} \cdot \overline{AC} + \overline{CD} \cdot \overline{AB} \\
 &= (\overline{AB} + \overline{BD}) \cdot \overline{BC} + \overline{BD} \cdot \overline{AC} + \overline{CD} \cdot \overline{AB} \\
 &= \overline{AB} \cdot \overline{BC} + \overline{BD} \cdot \overline{BC} + \overline{BD} \cdot \overline{AC} + \overline{CD} \cdot \overline{AB} \\
 &= \overline{AB} (\underbrace{\overline{BC} + \overline{CD}}_{\overline{BD}}) + \overline{BD} (\underbrace{\overline{BC} + \overline{AC}}_{(-1)(\overline{AC} - \overline{BC})}) \\
 &= \overline{AB} \cdot \overline{BD} - \overline{BD} \cdot \overline{AB} \\
 &= 0
 \end{aligned}$$

4.4.3

$D = l_A \cap l_{BC}$
 $E = l_B \cap l_{AC}$
 $F = l_C \cap l_{AB}$
 Skal vise at
 D, E, F er kollineære

OBS: D, E, F er Menelaospkt for $\triangle ABC$

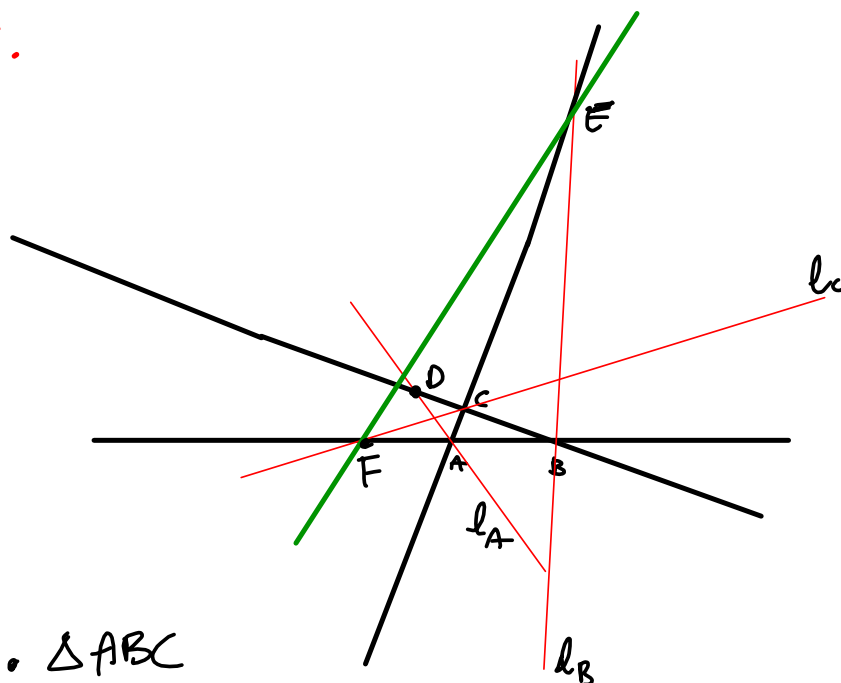
$$\frac{\overline{AF}}{\overline{FB}} \cdot \frac{\overline{BD}}{\overline{DC}} \cdot \frac{\overline{CE}}{\overline{EA}} = \frac{\overline{AC}}{\overline{CB}} \cdot \left(-\frac{\overline{BA}}{\overline{AC}}\right) \cdot \frac{\overline{CB}}{\overline{BA}} = -1$$

Halveringslinje-setning gir

$$\begin{aligned}
 ① \quad \frac{\overline{AF}}{\overline{FB}} &= \frac{\overline{AC}}{\overline{CB}} & ② \quad \frac{\overline{CE}}{\overline{EA}} &= \frac{\overline{CB}}{\overline{BA}} \\
 ③ \quad \frac{\overline{BD}}{\overline{DC}} &= -\frac{\overline{BA}}{\overline{AC}}
 \end{aligned}$$

Ved Menelaos setning
er D, E, F kollineære.

4.4.4.

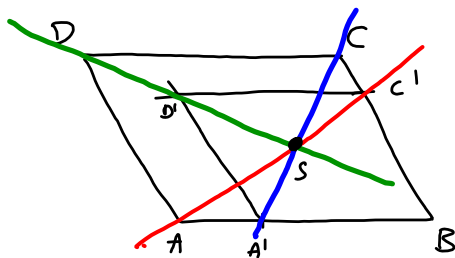


- $\triangle ABC$
- 3 ytre halveringslinjer
- $F = l_{AB} \cap l_C$ $E = l_{AC} \cap l_B$ $D = l_A \cap l_{BC}$
- D, E og F er Menelaos pkt for $\triangle ABC$
- Ved halveringslinje-setningen har vi
 - ① $\frac{\overline{AF}}{\overline{FB}} = -\frac{\overline{AC}}{\overline{CB}}$
 - ② $\frac{\overline{BD}}{\overline{DC}} = -\frac{\overline{BA}}{\overline{AC}}$
 - ③ $\frac{\overline{CE}}{\overline{EA}} = -\frac{\overline{CB}}{\overline{BA}}$
- $\frac{\overline{AF}}{\overline{FB}} \cdot \frac{\overline{BD}}{\overline{DC}} \cdot \frac{\overline{CE}}{\overline{EA}} = \left(-\frac{\overline{AC}}{\overline{CB}}\right) \cdot \left(-\frac{\overline{BA}}{\overline{AC}}\right) \cdot \left(-\frac{\overline{CB}}{\overline{BA}}\right)$

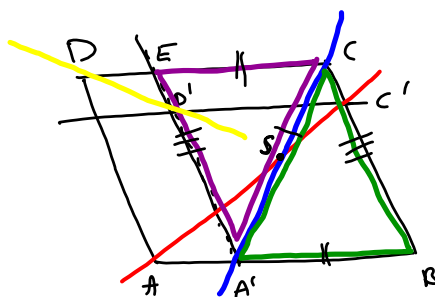
$$= -1,$$

Så ved Menelaos setning er D, E, F kolleare punkter.

4.4.6.



- To parallelogrammer $\square ABCD$ og $\square A'BC'D'$
- Vi skal vise at $l_{DD'} \cap l_{AC'} \cap l_{A'C} = S$.
- Går det i det hele tatt an å angripe med Ceva?



- Se på $\triangle A'BC$
 $S \in l_{A'C}$ $A \in l_{A'B}$ $C' \in l_{BC}$,
 og S, A og $C' \in l_{AC'}$,
 så S, A og C' er kollinear Menelauspunkt for $\triangle A'BC$
 og $\frac{A'A}{AB} \cdot \frac{BC'}{C'C} \cdot \frac{CS}{SA} = -1$.

- Setter av E i forlengelsen av $A'D'$ på l_{CD}
- Se på $\triangle A'CE$ og merk at D, D' og S er Menelauspunkt for denne.

$$\frac{A'S}{SC} \cdot \frac{CD}{DE} \cdot \frac{ED'}{D'A'} = -1$$

- Husk $\triangle A'BC$ er kongruent med $\triangle A'CE$ (da rettlinjet)
 og vi har med forrige parallelogrammer å gjøre!

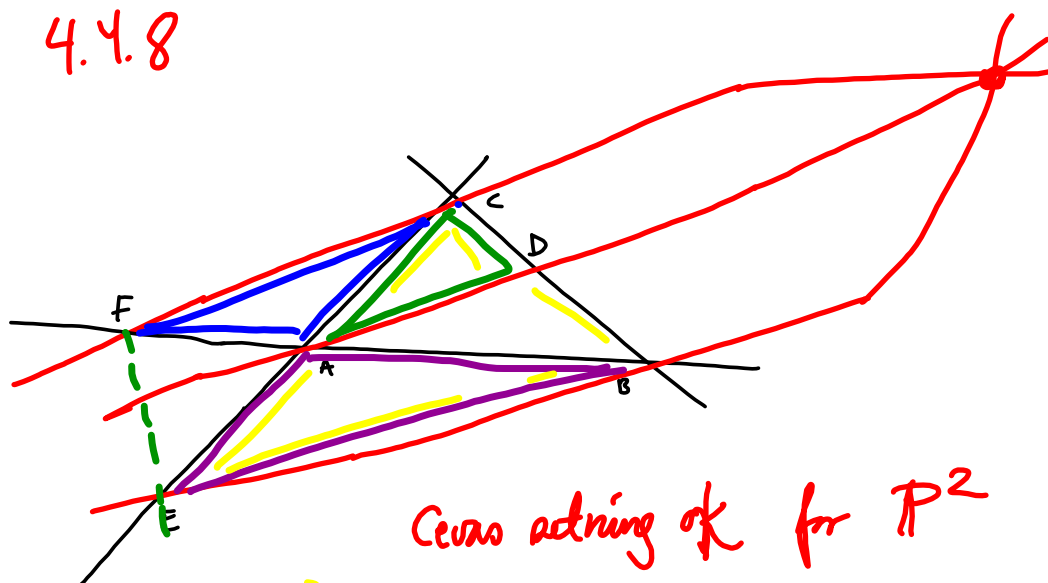
$$\begin{aligned} \overline{ED'} &= \overline{C'C} \\ \overline{D'A'} &= \overline{BC'} \\ \overline{CD} &= \overline{AB} \\ \overline{DE} &= \overline{A'A} \end{aligned}$$

$$= \frac{A'S}{SC} \cdot \frac{AB}{A'A} \cdot \frac{C'C}{BC'} = \frac{1}{-1} = -1$$

↑
ved det vi gjorde for $\triangle A'BC$

- Menelaus setning er D, D' og S kollinear, mao. $S \in l_{DD'}$
- Totalt er $S \in l_{AC'}$, $l_{A'C}$ og $l_{DD'}$, så de tre linjene er konkurrerende!

4.4.8

Ceva's theorem for P^2

$$\frac{\overline{AF}}{\overline{FB}} \cdot \frac{\overline{BD}}{\overline{DC}} \cdot \frac{\overline{CE}}{\overline{EA}}$$

$$\frac{\overline{AF}}{\overline{CA}} = \frac{\overline{AE}}{\overline{BA}}$$

$$\frac{\overline{AF}}{\overline{EA}} = -\frac{\overline{CA}}{\overline{BA}}$$