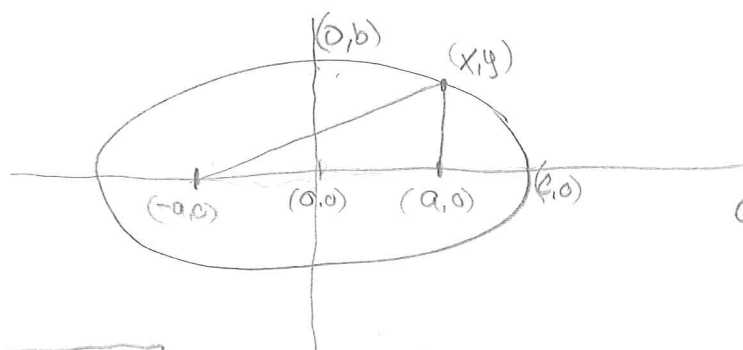




$$c+a+c-a=2c$$

$$2\sqrt{a^2+b^2}=2c$$

$$\Rightarrow a^2+b^2=c^2$$

$$\sqrt{(x+a)^2+y^2} + \sqrt{(x-a)^2+y^2} = 2c$$

$$\Rightarrow \begin{array}{l} (x+a)^2+y^2 = 4c^2 - \frac{1}{4}c\sqrt{(x-a)^2+y^2} + (x-a)^2+y^2 \\ \quad \quad \quad \parallel \quad \quad \quad \parallel \\ x^2+2ax+a^2 \quad \quad \quad x^2+2ax+a^2 \end{array}$$

$$4c\sqrt{(x-a)^2+y^2} = 4c^2 - 4ax \quad \left| \frac{1}{4} + \text{kvadrer} \right.$$

$$\cancel{c^2x^2} - \cancel{2c^2x} + \cancel{a^2} + \cancel{c^2y^2} = c^2(x^2 - 2ax + a^2 + y^2) = c^4 - \cancel{2c^2ax} + \underline{a^2x^2}$$

$$(c^2-a^2)x^2 + c^2y^2 = c^4 - a^2c^2$$

$$b^2x^2 + c^2y^2 = b^2c^2 \quad \left| \frac{1}{b^2c^2} \right.$$

$$\frac{x^2}{c^2} + \frac{y^2}{b^2} = 1$$

standardform, tilsvarende for hyperbol.

$$\left\{ \begin{array}{l} c: \text{skjæring med store halvakse} \\ b: \text{--- " --- lille "} \end{array} \right.$$

Rekke linjer i planet

$$ax + by = c$$

Neste step: Kvadratiske likninger

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

Isometrier:

$$Ax^2 + Bxy + Cy^2 = \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} A & \frac{B}{2} \\ \frac{B}{2} & C \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

Rotasjon

$M =$ Symmetrisk matrise

v_1, v_2 Egenvektorer (λ_1, λ_2)

tillater ortogonal diagonalisering med reelle egenverdier

$$\begin{aligned} v_1^T M v_2 - v_2^T M v_1 &= v_1^T \lambda_2 v_2 - v_2^T \lambda_1 v_1 = \lambda_2 v_1^T v_2 - \lambda_1 v_2^T v_1 \\ &= (\lambda_2 - \lambda_1) v_1^T v_2 \\ \text{Transponer} \\ v_2^T M^T v_1 - v_1^T M^T v_2 &= (\lambda_1 - \lambda_2) v_1^T v_2 \end{aligned}$$

$$\text{Siden } \lambda_1 - \lambda_2 \neq 0, \text{ m\u00e5 } v_1^T \cdot v_2 = 0.$$

$\Rightarrow$ Opp til rotasjon:

$$Ax^2 + Cy^2 + Dx + Ey + F$$

Translasjon

$$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} \frac{D}{2A} \\ \frac{E}{2C} \end{pmatrix}$$

$$A \left(x - \frac{D}{2A} \right)^2 + C \left(y - \frac{E}{2C} \right)^2 + Dx + Ey + F$$

$$= Ax^2 - A \lambda x \frac{D}{2A} + A \frac{D^2}{4A^2} + Cy^2 - C \lambda y \frac{E}{2C} + C \frac{E^2}{4C^2} + Dx + Ey + F$$

$$= Ax^2 + Cy^2 + \left(F + \frac{D^2}{4A} + \frac{E^2}{4C} \right)$$

dvs, opp til isometri.

Kvadratiske ligning $Ax^2 + Cy^2 = K$.

(Merk: små justeringer dersom A eller C = 0.)

Opp til isometrier:

Kjeglesnitt = Kvadratiske ligninger

Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ Hyperbel $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Trigonometrisk parametrisering $\begin{cases} x = a \cos t \\ y = b \sin t \end{cases}$ } Ellipse

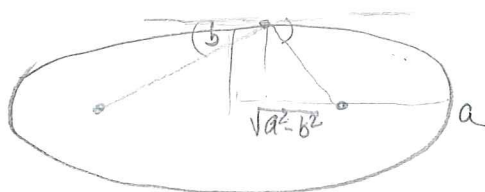
$\begin{cases} x = \frac{a}{\cos t} \\ y = b \tan t \end{cases}$ } Hyperbel.

alt.
 $x = a \cosh u$
 $y = b \sinh u$

Asymptoter til hyperbel:

$$\frac{x}{y} = \frac{a}{\cos t} \cdot \frac{1}{b \tan t} = \frac{a}{b} \frac{1}{\sin t} \rightarrow \pm \frac{a}{b} \text{ når } t \rightarrow \pm \frac{\pi}{2}.$$

eller $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$ (og ikke 1)



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad y^2 = b^2 \left(1 - \frac{x^2}{a^2}\right)$$

↓ deriver m.p. x

$$\frac{2x}{a^2} + \frac{2y}{b^2} \frac{dy}{dx} = 0$$

St. tallet $\frac{dy}{dx} = -\frac{2x}{a^2} \frac{b^2}{2y} = -\frac{b^2}{a^2} \frac{x}{y}$

Tangent Vektor $\therefore \left(1, -\frac{b^2}{a^2} \frac{x}{y}\right)$

Til brennpunktene

$$\frac{1}{\sqrt{(x+\sqrt{a^2-b^2})^2 + y^2}} (x+\sqrt{a^2-b^2}, y)$$

$$\frac{1}{\sqrt{(x-\sqrt{a^2-b^2})^2 + y^2}} (x-\sqrt{a^2-b^2}, y)$$

Prøkket med tangentvektor

$$x \pm \sqrt{a^2-b^2} - \frac{b^2}{a^2} x = \left(1 - \frac{b^2}{a^2}\right) x \pm \sqrt{a^2-b^2}$$

$$\begin{aligned} \sqrt{\left(x \pm \sqrt{a^2-b^2}\right)^2 + y^2} &= \sqrt{x^2 \pm 2x\sqrt{a^2-b^2} + a^2-b^2 + y^2} = \sqrt{x^2 \pm 2x\sqrt{a^2-b^2} + a^2-b^2 + b^2 - \frac{b^2}{a^2}x^2} \\ &= \sqrt{\left(1 - \frac{b^2}{a^2}\right)x^2 \pm 2x\sqrt{a^2-b^2} + a^2} = * \end{aligned}$$

$$\begin{aligned} \left(1 - \frac{b^2}{a^2}\right)x \pm \sqrt{a^2-b^2} &= \left(1 - \frac{b^2}{a^2}\right)x^2 \pm 2x\left(1 - \frac{b^2}{a^2}\right)\sqrt{a^2-b^2} + a^2-b^2 \\ &= \left(1 - \frac{b^2}{a^2}\right) \left(\left(1 - \frac{b^2}{a^2}\right)x^2 \pm 2x\sqrt{a^2-b^2} + a^2 \right) \end{aligned}$$

$$* = \left(1 - \frac{b^2}{a^2}\right)x \pm \sqrt{a^2-b^2} \cdot \frac{1}{\sqrt{1 - \frac{b^2}{a^2}}}$$

$\Rightarrow$ De to produktene er like

$\Rightarrow$ Vinklene er like.