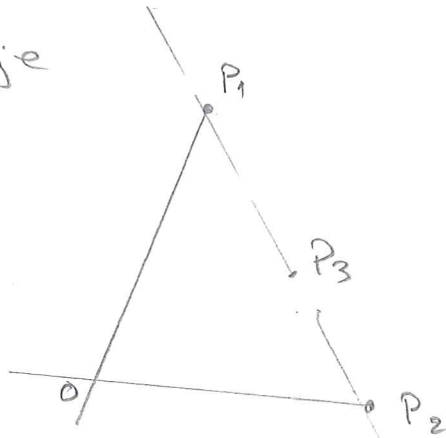


Affine afbildninger tar rette linjer på rette linjer
og kolinearitét på kolinearitét.

Tre punkter på linje



$$P_3 = P_1 + t \cdot (P_2 - P_1) = (1-t) P_1 + t P_2$$

$$\text{dvs } P_3 = \alpha P_1 + \beta P_2, \alpha + \beta = 1$$

$$\Rightarrow \frac{1}{\alpha} P_3 = P_1 + \frac{\beta}{\alpha} P_2 \quad \text{eller} \quad P_1 = -\frac{\beta}{\alpha} P_2 + \frac{1}{\alpha} P_3$$

$$\text{hvor } -\frac{\beta}{\alpha} + \frac{1}{\alpha} = \frac{\alpha}{\alpha} = 1.$$

Affin afbildning

$$T(\bar{x}) = A\bar{x} + \bar{b}$$

$$T(\bar{v}_1) = A\bar{v}_1 + \bar{b}$$

$$T(\bar{v}_2) = A\bar{v}_2 + \bar{b}$$

$$\Rightarrow T(\alpha \bar{v}_1 + \beta \bar{v}_2) = A(\alpha \bar{v}_1 + \beta \bar{v}_2) + \bar{b}$$

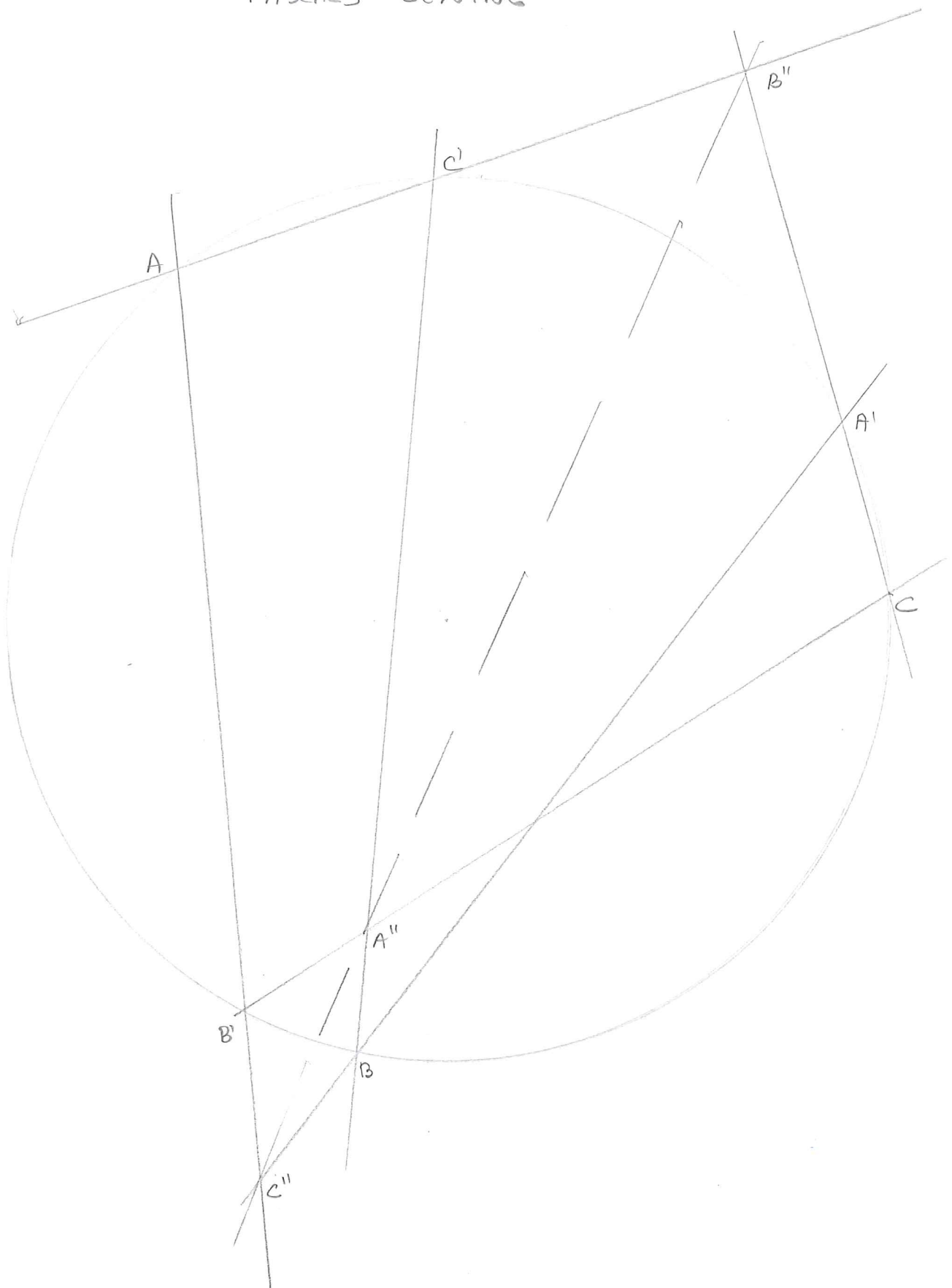
$$= \alpha A\bar{v}_1 + \beta A\bar{v}_2 + \bar{b}$$

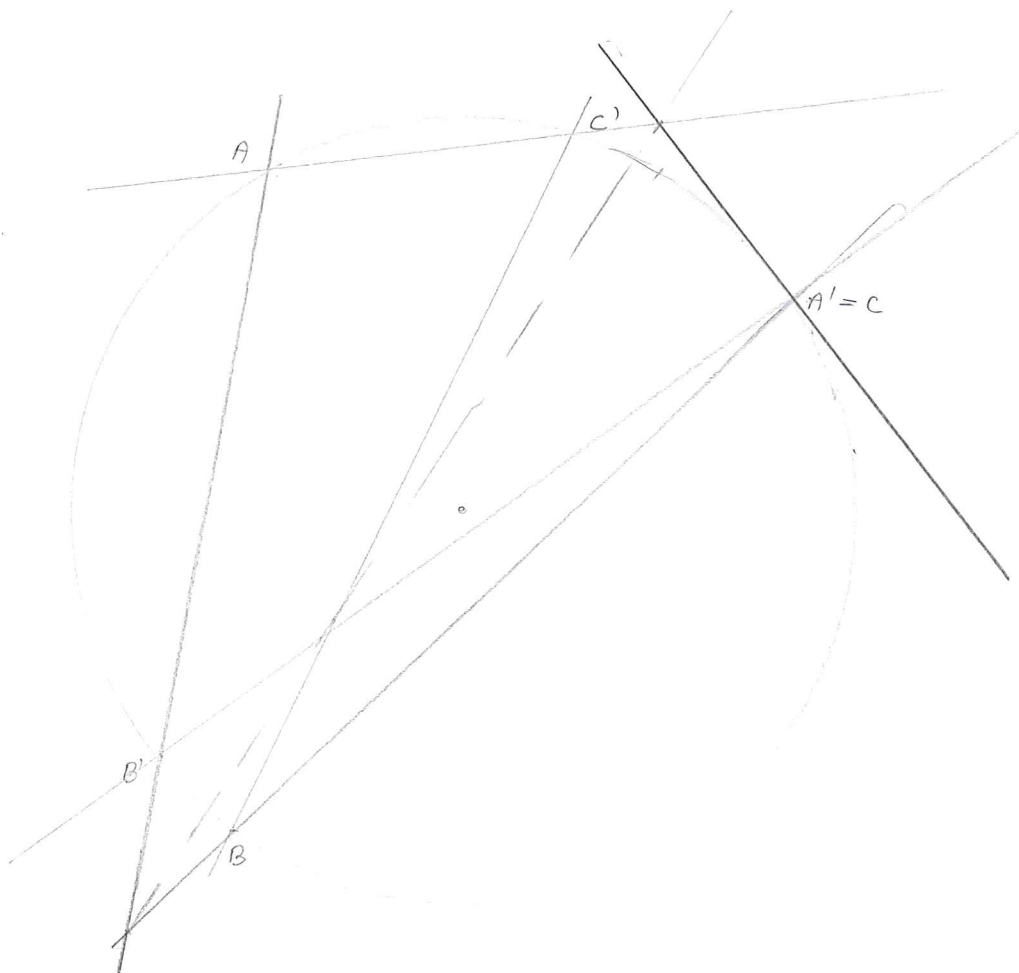
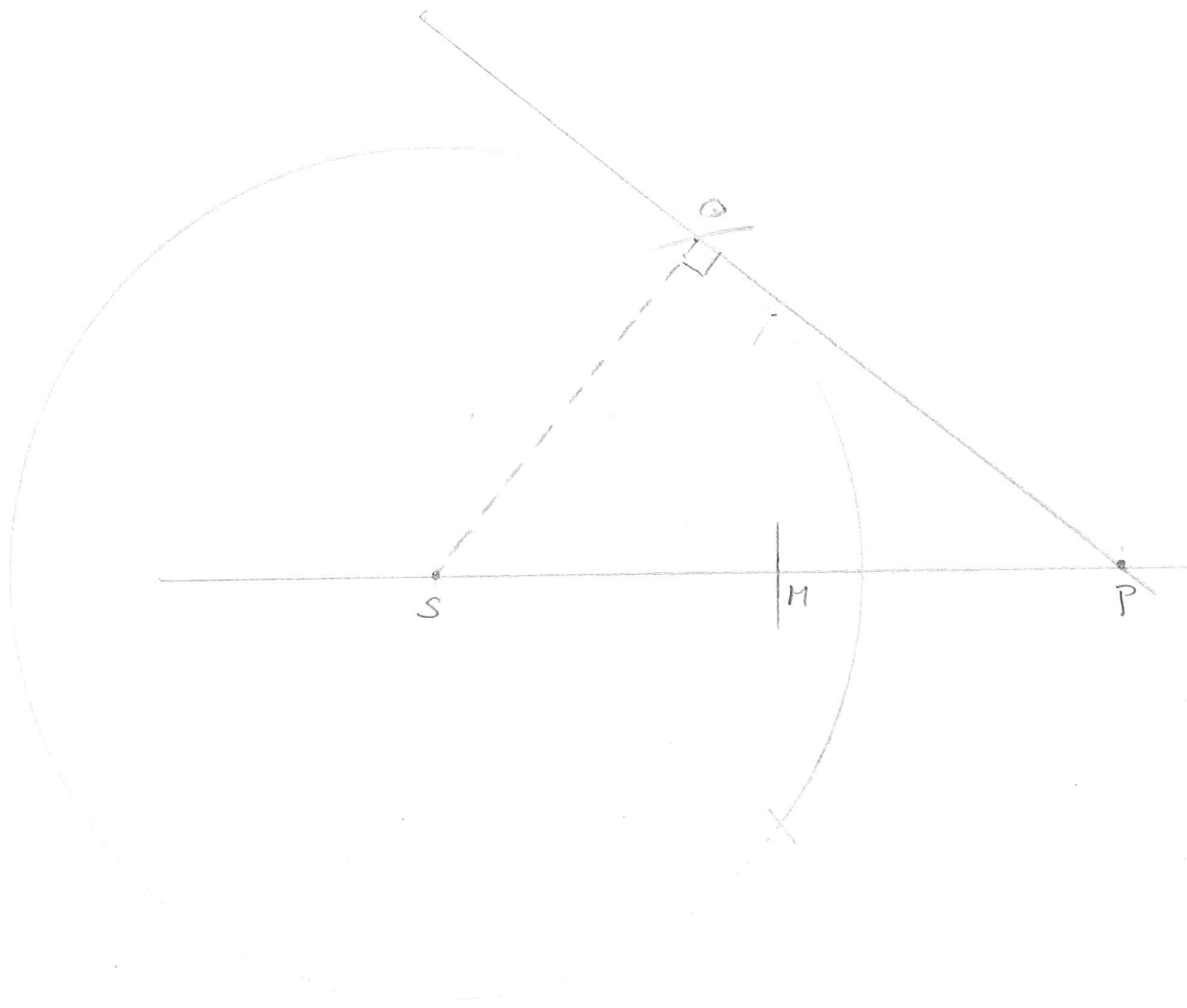
$$= \alpha (T(\bar{v}_1) - \bar{b}) + \beta (T(\bar{v}_2) - \bar{b}) + \bar{b}$$

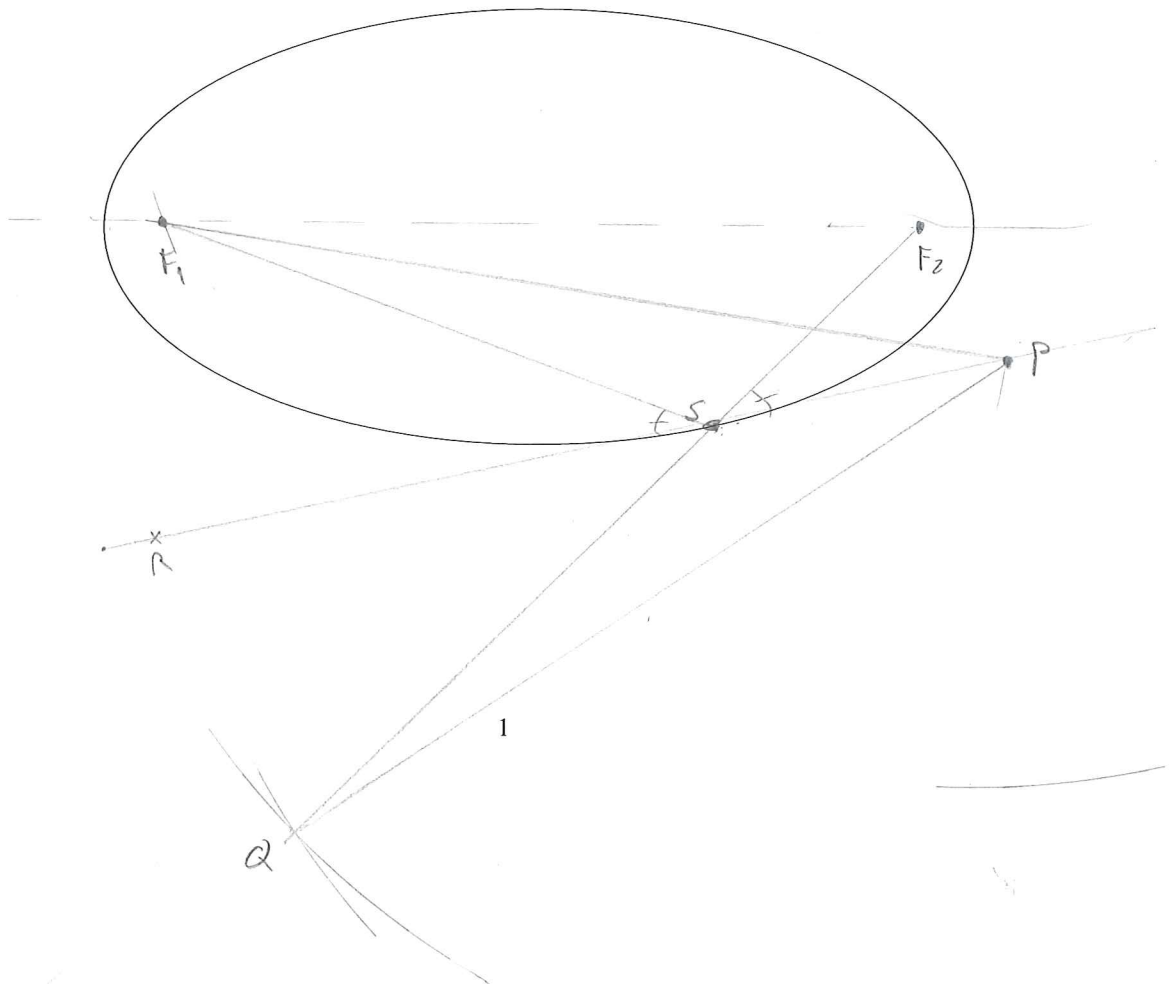
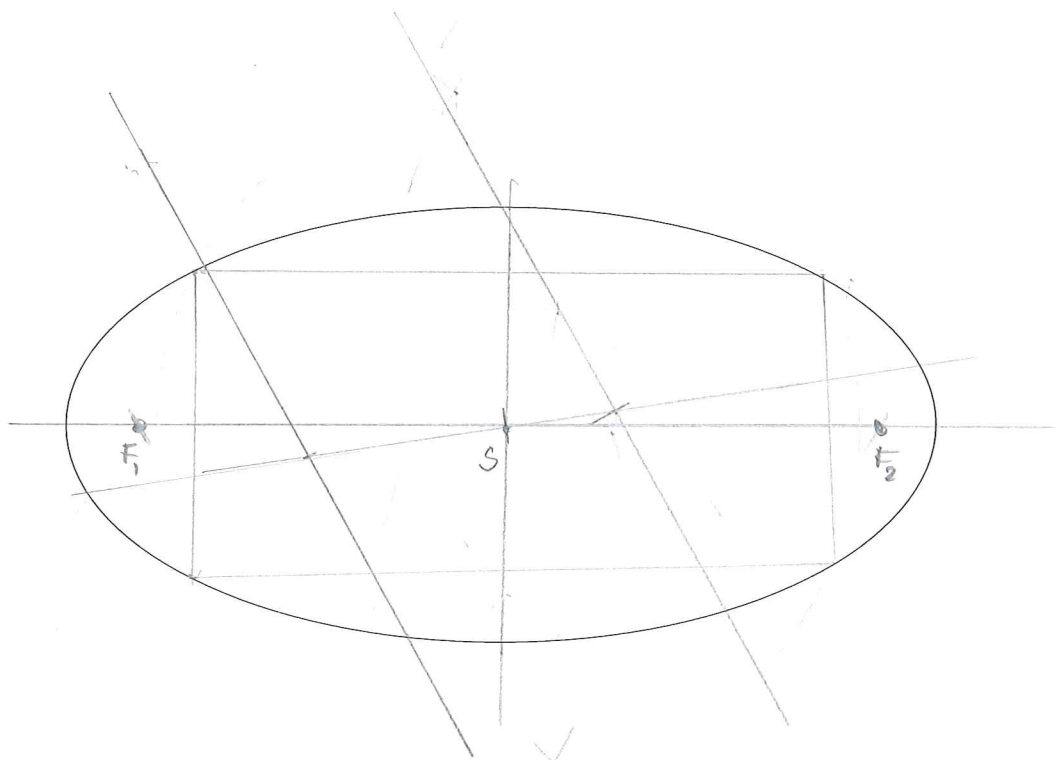
$$= \alpha T(\bar{v}_1) + \beta T(\bar{v}_2) + \underbrace{(\bar{b} - (\alpha + \beta)\bar{b})}_{=0}$$

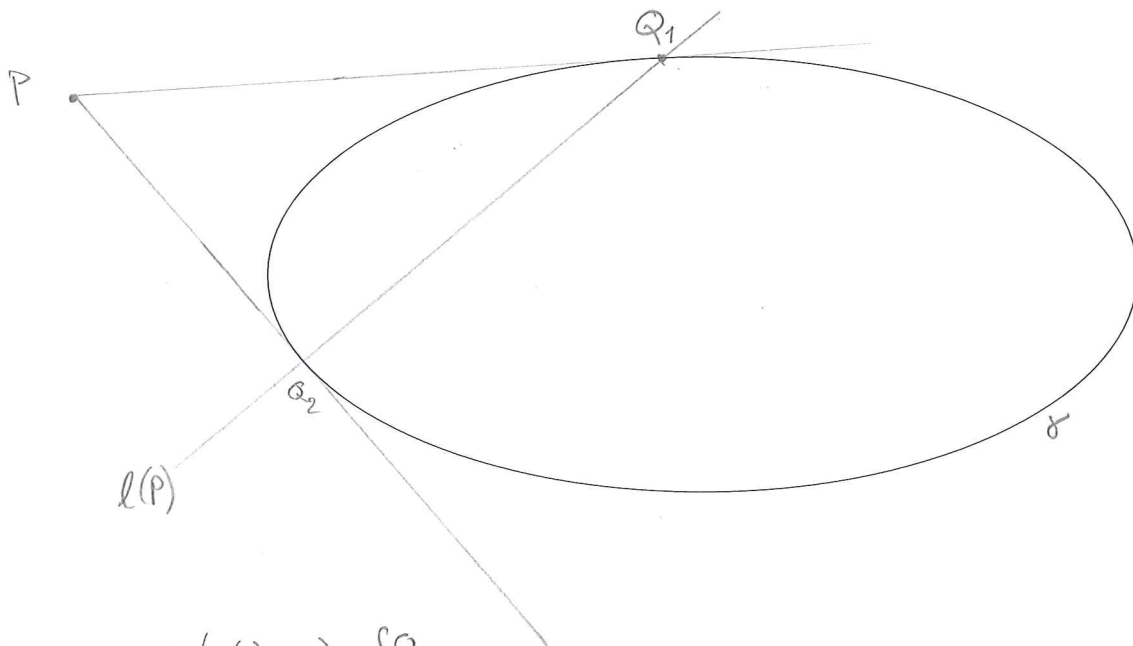
$$\alpha + \beta = 1.$$

PASCALS SENING



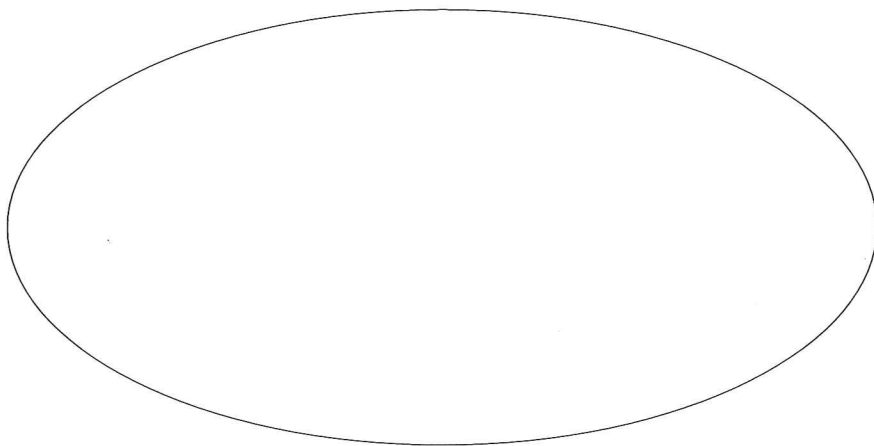






$$P \notin \gamma \Rightarrow \#(l(P) \cap \gamma) = \begin{cases} 0 \\ 2 \end{cases}$$

$Q_i \in l(P) \cap \gamma \Rightarrow \begin{cases} Q_i \in \gamma \text{ betyr } l(Q_i) \text{ er tangent i } Q_i \\ Q_i \in l(P) \text{ betyr } P \in l(Q_i) \end{cases} \quad \} \quad P Q_i \text{ tangent}$



Tangentlinje til $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$:

Deriverer mhp x :

$$\frac{2x}{a^2} + \frac{2y}{b^2} \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{b^2}{a^2} \frac{x}{y}$$

1-punktsformelen i punktet (x_0, y_0) på ellipsen

$$y - y_0 = -\frac{b^2}{a^2} \frac{x_0}{y_0} (x - x_0) \quad | \cdot \frac{y_0}{b^2}$$

$$\frac{yy_0}{b^2} - \frac{y_0^2}{b^2} = -\frac{x_0x}{a^2} + \frac{x_0^2}{a^2}$$

$$\text{gør } \frac{x_0}{a^2}x + \frac{y_0}{b^2}y = 1 \quad \left(\text{eller } \frac{xx_0}{a^2} + \frac{yy_0}{b^2} = 1 \right)$$

Udledt
ellipse-formelen.

Hvis (x_0, y_0) ikke på ellipsen, kan tilsvarende lade $\frac{xx_0}{a^2} + \frac{yy_0}{b^2} = 1$.

Splitter punkt og tangentlinje i punkt !

$P = (x_0, y_0)$ kaldes pol

$P(l)$ pol til polare

$l = \frac{xx_0}{a^2} + \frac{yy_0}{b^2} = 1$ kaldes polare $l(P)$ polare til pol.

La Hiles sætning:

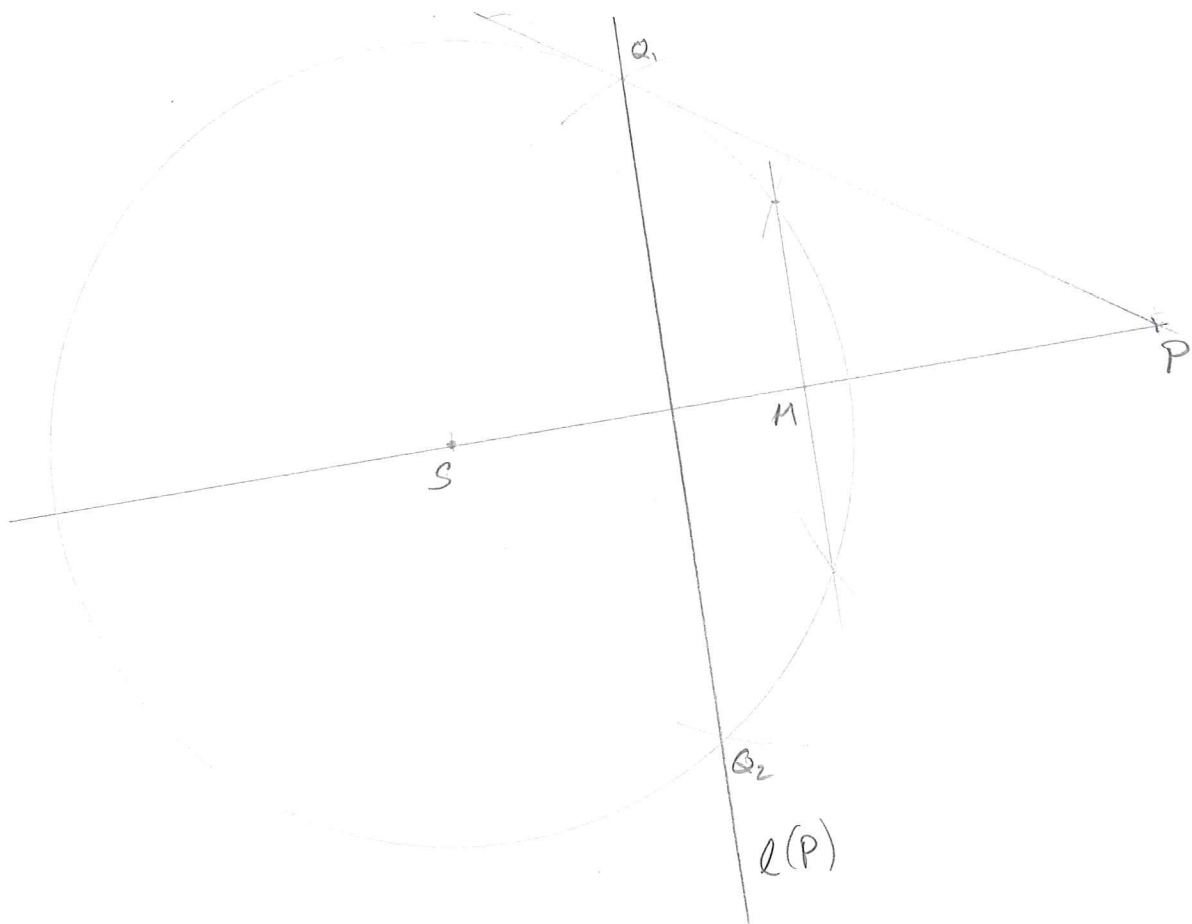
$$Q \in l(P) \Leftrightarrow P \in l(Q)$$

Bevis

$$Q = (x_1, y_1) \quad Q \in l(P) : \frac{x_1 x_0}{a^2} + \frac{y_1 y_0}{b^2} = 1$$

$$P = (x_0, y_0)$$

$$P \in l(Q) : \frac{x_0 x_1}{a^2} + \frac{y_0 y_1}{b^2} = 1.$$



$$S = (0, 0)$$

$$P = (x_0, y_0)$$

$$\Rightarrow M = \frac{1}{2}(x_0, y_0)$$

$$Q_1, Q_2: x^2 + y^2 = 1, \quad (x - \frac{1}{2}x_0)^2 + (y - \frac{1}{2}y_0)^2 = \frac{1}{4}(x_0^2 + y_0^2)$$

$\Downarrow$

$$x^2 - xx_0 + \frac{1}{4}x_0^2 + y^2 - yy_0 + \frac{1}{4}y_0^2 = \frac{1}{4}(x_0^2 + y_0^2)$$

$$\Rightarrow xx_0 + yy_0 = 1. \quad (\text{polar line}).$$

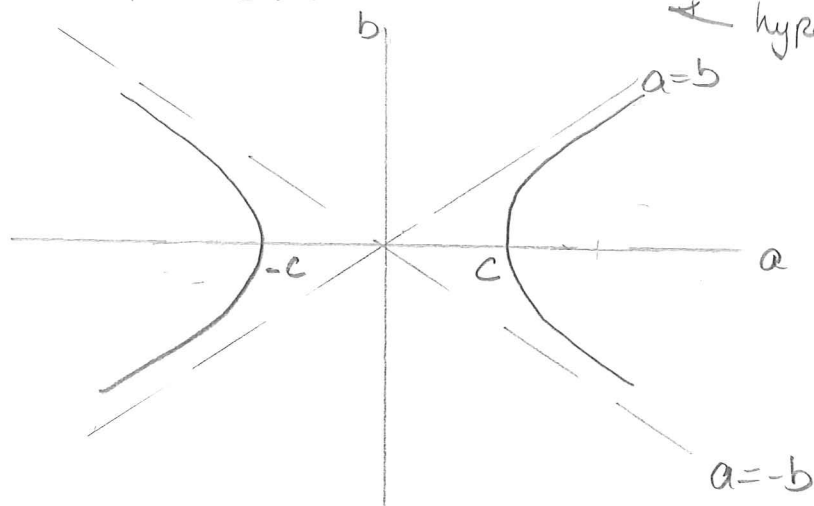
Kjeglesnitt med felles brennpunkter:

$$F_1 = (-c, 0)$$

$$F_2 = (c, 0)$$

Kjeglesnitt $\frac{x^2}{a^2} \pm \frac{y^2}{b^2} = 1$, $c^2 = a^2 - b^2$ ellipse $c^2 = a^2 + b^2$ hyperbel

For hvert par (a, b) med $c^2 = a^2 - b^2$ har vi et kjeglesnitt hyperbel



Skjærings mellom ellipse og hyperbel i (x_0, y_0)

$$\frac{1}{b_1^2} \frac{x^2}{a_1^2} + \frac{y^2}{b_1^2} = 1 \quad \frac{x^2}{a_2^2} - \frac{y^2}{b_2^2} = 1 - \frac{1}{b_1^2} \quad x_0^2 \left(\frac{1}{a_1^2 b_2^2} + \frac{1}{b_1^2 a_2^2} \right) = \frac{1}{b_2^2} + \frac{1}{b_1^2}$$

Tangentlinje $\frac{x x_0}{a_1^2} + \frac{y y_0}{b_1^2} = 1 \quad \frac{x x_0}{a_2^2} - \frac{y y_0}{b_2^2} = 1$

$$x_0^2 = \frac{b_1^2 + b_2^2}{\frac{b_1^2}{a_1^2} + \frac{b_2^2}{a_2^2}}$$

$$y_0^2 = \frac{a_1^2 - a_2^2}{\frac{a_1^2}{b_1^2} + \frac{a_2^2}{b_2^2}}$$

stiftall $y = \frac{b_1^2}{y_0} \left(1 - \frac{x x_0}{a_1^2} \right) \quad y = \frac{b_2^2}{y_0} \left(\frac{x x_0}{a_2^2} - 1 \right)$

$$\left(-\frac{b_1^2}{y_0} \frac{x_0}{a_1^2} \right) \cdot \left(\frac{b_2^2 x_0}{y_0 a_2^2} \right) = -\frac{b_1^2 b_2^2 x_0^2}{a_1^2 a_2^2 y_0^2}$$

$$a_1^2 - b_1^2 = a_2^2 + b_2^2$$

$\Downarrow$

$$a_1^2 - a_2^2 = b_1^2 + b_2^2$$

$$= -\frac{b_1^2 b_2^2 (b_1^2 + b_2^2) \left(\frac{a_1^2}{b_1^2} + \frac{a_2^2}{b_2^2} \right)}{a_1^2 a_2^2 (a_1^2 - a_2^2) \left(\frac{b_1^2}{a_1^2} + \frac{b_2^2}{a_2^2} \right)}$$

$$= -\frac{b_1^2 + b_2^2}{a_1^2 - a_2^2} = \underline{\underline{-1}}$$