

MAT 2500 27.09.2023

5.10.1 skal finne sentrum og halvakser

$$25x^2 + 9y^2 - 18x + 2y = 0$$

- Skriver om til standard form $\frac{(x-x_0)^2}{a^2} \pm \frac{(y-y_0)^2}{b^2} = 1$

Sentrum (x_0, y_0) Halvakser a og b

Luttsketh

$$(5x)^2 - 2 \cdot 5x \cdot \frac{18}{10} + \left(\frac{9}{5}\right)^2 - \left(\frac{9}{5}\right)^2$$

$$(z \pm w)^2 = z^2 \pm 2zw + w^2$$

$$+ (3y)^2 + 2 \cdot 3y \cdot \frac{1}{3} + \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^2 = 0$$

$$(5x - \frac{9}{5})^2 + (3y + \frac{1}{3})^2 = \left(\frac{9}{5}\right)^2 + \left(\frac{1}{3}\right)^2$$

$$\frac{1}{5^2} = 5^2 \left(x - \frac{9}{25}\right)^2 + 3^2 \left(y + \frac{1}{9}\right)^2 = \frac{754}{225} = \frac{754}{5^2 \cdot 3^2}$$

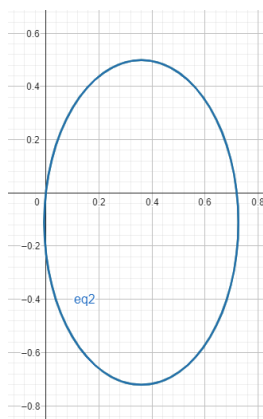
$$\begin{cases} 5x_0 - \frac{9}{5} = 0 \\ x_0 = \frac{9}{25} \\ 3y_0 + \frac{1}{3} = 0 \\ y_0 = -\frac{1}{9} \end{cases}$$

$$\frac{(x - \frac{9}{25})^2}{\frac{754}{5^4 \cdot 3^2}} + \frac{(y + \frac{1}{9})^2}{\frac{754}{5^2 \cdot 3^4}} = 1$$

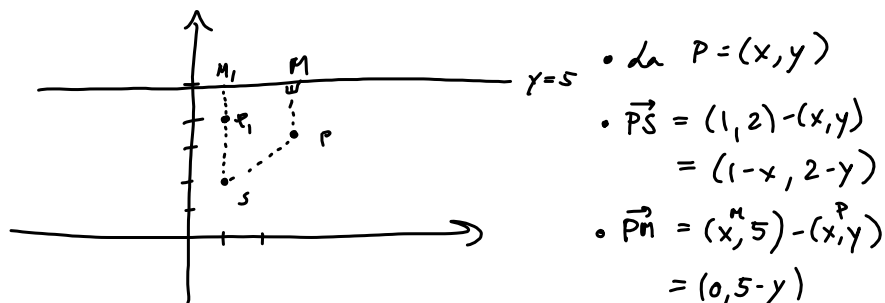
Sentrum $(\frac{9}{25}, -\frac{1}{9})$

$$\text{Halvakser: } a = \frac{\sqrt{754}}{5^2 \cdot 3} < b = \frac{\sqrt{754}}{5 \cdot 3^2}$$

lille halvakse

store halvakse parallell med y-aksen $x = \frac{9}{25}$ 

5.10.3. Skal finne likning og symmetrialer for det geometriske stedet for punkter som har dobbelt så stor avstand til $(1,2)=S$ som til linja $y=5$



BETINGELSE: $\|\vec{PS}\| = 2 \cdot \|\vec{PM}\|$

$$\sqrt{(1-x)^2 + (2-y)^2} = 2 \sqrt{0^2 + (5-y)^2}$$

$$(1-x)^2 + (2-y)^2 = 4(5-y)^2$$

$$(x-1)^2 + 4 - 4y + y^2 = 4(5^2 - 10y + y^2)$$

$$(x-1)^2 + 4 - 4y + y^2 - 100 + 40y - 4y^2 = 0$$

$$(x-1)^2 - 3y^2 + 36y - 96 = 0$$

$$\frac{(x-1)^2}{3} - y^2 + 12y - 32 = 0$$

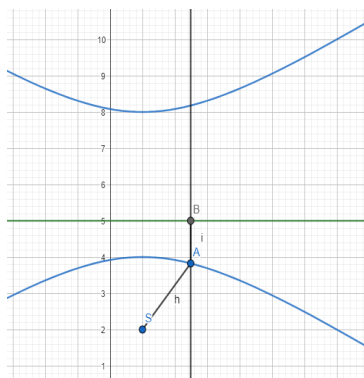
$$\frac{(x-1)^2}{3} - y^2 + 2 \cdot y \cdot 6 - \underbrace{6^2}_{+0} + 6^2 - 32 = 0$$

$$\frac{(x-1)^2}{3} - (y-6)^2 = -4 \quad | :(-4)$$

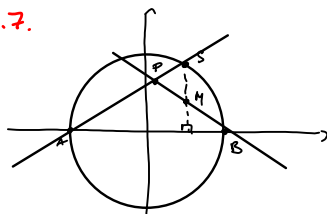
$$-\frac{(x-1)^2}{3 \cdot 4} + \frac{(y-6)^2}{4} = 1$$

Hyperbel med sentrum i $(1,6)$

Symmetrialer $x=1$ og $y=6$



5.10.7.



$$A = (-r, 0)$$

$$B = (r, 0)$$

Skal finne det geometriske stedet for skjæringspunktet mellom AS og BM når S beveger seg langs sirkelen.

$$S = (x_s, y_s) \quad | \quad x_s^2 + y_s^2 = r^2 \quad (1)$$

$$M = (x_s, \frac{1}{2}y_s)$$

Likning for linje gjennom (x_s, y_s) med st. fall k :

$$y - y_s = k(x - x_s)$$

$$AS: \quad y - 0 = \frac{y_s}{x_s + r}(x + r) \quad (2)$$

$$BM: \quad y - 0 = \frac{-\frac{1}{2}y_s}{r - x_s}(x - r) \quad (3)$$

• Kan bruke (2) og (3) til å få x_s og y_s uttrykt ved x, y og r .

• Kan bruke (1) til å finne likningen i x, y og r .

$$(2) \quad (x_s + r)y = y_s(x + r)$$

$$(3) \quad (r - x_s)y = -\frac{1}{2}y_s(x - r)$$

$$(2) + (3) \quad x_s y + r y + r y - x_s y = y_s x + y_s r - \frac{1}{2}y_s x + \frac{1}{2}y_s r$$

$$2ry = y_s \left(\frac{1}{2}x + \frac{3}{2}r \right)$$

$$4ry = y_s(x + 3r)$$

$$y_s = \frac{4ry}{x + 3r}$$

$$(2) \quad x_s y + r y = \frac{4ry}{x + 3r}(x + r)$$

$$x_s = \frac{4r}{x + 3r}(x + r) - r$$

$$= \frac{4rx + 4r^2 - r(x + 3r)}{x + 3r}$$

$$= \frac{3rx + r^2}{x + 3r}$$

$$= \frac{r(3x + r)}{x + 3r}$$

$$(1) \quad x_s^2 + y_s^2 = r^2$$

$$\frac{r^2(3x+r)^2}{(x+3r)^2} + \frac{16r^2 \cdot y^2}{(x+3r)^2} = r^2$$

$$\frac{9x^2 + 6xr + r^2 + 16y^2}{(x+3r)^2} = 1 \quad | \cdot (x+3r)^2$$

$$9x^2 + 6xr + r^2 + 16y^2 = x^2 + 6xr + 9r^2$$

$$8x^2 - 8r^2 + 16y^2 = 0$$

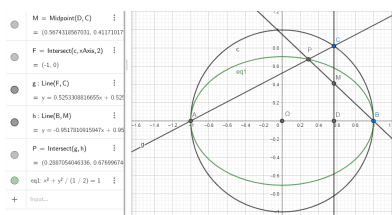
$$x^2 + 2y^2 = r^2$$

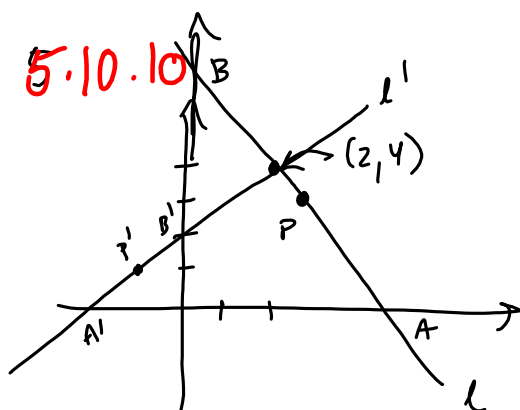
$$\frac{x^2}{r^2} + \frac{y^2}{\left(\frac{r}{\sqrt{2}}\right)^2} = 1$$

Ellipse med sentrum origo og halvaksler r og $\frac{r}{\sqrt{2}}$.

$$2 = \frac{1}{\frac{1}{2}}$$

$$\left(\frac{r}{\sqrt{2}}\right)^2 = \frac{1}{2}$$





P er midtpunkt på AB
skal finne geometrisk sted
når linja l gjennom $(2, 4)$
dreier om $(2, 4)$.

$$l: y - 4 = k(x - 2)$$

$$A = l \cap (y=0) \quad -4 = k(x_A - 2) \quad x_A = 2 - \frac{4}{k}$$

$$A = \left(2 - \frac{4}{k}, 0\right)$$

$$B = l \cap (x=0) \quad y_B - 4 = -2k \quad y_B = 4 - 2k$$

$$B = (0, 4 - 2k)$$

$$P = \left(1 - \frac{2}{k}, 2 - k\right)$$

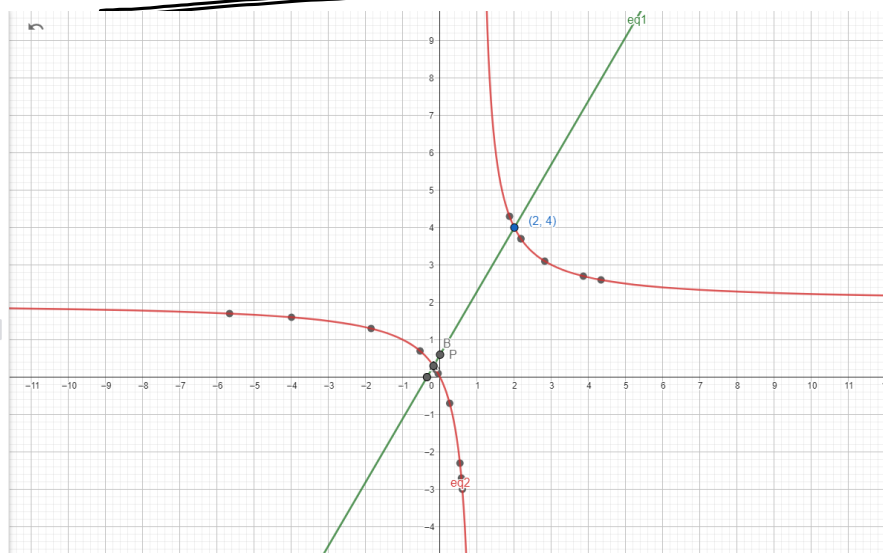
Vi har da $\textcircled{1} x = 1 - \frac{2}{k}$ $\textcircled{2} y = 2 - k$
 $k = 2 - y$

$$x = 1 - \frac{2}{2-y}$$

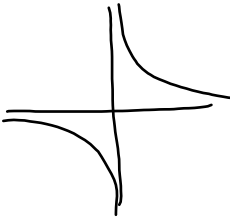
$$(2-y)x = 2-y-2$$

$$xy - y - 2x = 0$$

●	S = (2, 4)	⋮
○	k = 1.7	⋮
●	eq1: $y - 4 = 1.7(x - 2)$	⋮
●	B = Intersect(eq1, xAxis)	⋮
	= (-0.3529411764706, 0)	
●	C = Intersect(eq1, yAxis)	⋮
	= (0, 0.6)	
●	P = Midpoint(B, C)	⋮
	= (-0.1764705882353, 0.3)	
●	eq2: $xy - y - 2x = 0$	⋮
○	eq3: $1 = (x - 3\sqrt{(2)/2})^2 / 2^2 + y -$	⋮
+	Input...	



• $xy = 1$



• $\underbrace{ax^2 + bxy + cy^2}_{\text{quadratic form}} + dx + ey + f = 0$

$$\begin{bmatrix} x & y \end{bmatrix} \cdot M \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

$$M = \begin{bmatrix} a & \frac{1}{2}b \\ \frac{1}{2}b & c \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} x & y \end{bmatrix}}_{\text{row vector}} \underbrace{P D P^T}_{\text{orthogonal matrix}} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\text{column vector}}$$

$$D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$P = \text{Eigenvektormatrisen}$

Isometrien: $\begin{bmatrix} X \\ Y \end{bmatrix} = P^T \begin{bmatrix} x \\ y \end{bmatrix}$

Substitusjon: $\begin{bmatrix} x \\ y \end{bmatrix} = P \begin{bmatrix} X \\ Y \end{bmatrix}$

$xy - y - 2x = 0$

$$M = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$$

Eigenverdi $\pm \frac{1}{2}$

Eigenvektorer:

$$\frac{\sqrt{2}}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \frac{\sqrt{2}}{2} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$P = \frac{\sqrt{2}}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$P^T = \frac{\sqrt{2}}{2} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

er matrisen til $\mathcal{R}(-\frac{\pi}{4})$

$$P \rightarrow \begin{cases} x = \frac{\sqrt{2}}{2}X - \frac{\sqrt{2}}{2}Y \\ y = \frac{\sqrt{2}}{2}X + \frac{\sqrt{2}}{2}Y \end{cases} \quad \Rightarrow \quad 1 = \frac{(X - \frac{3\sqrt{2}}{2})^2}{2^2} - \frac{(Y - \frac{\sqrt{2}}{2})^2}{2^2}$$

