

Hvorfor studere polyedre?

- Klassisk (Platon / Arkimedes)
- Av alle 3-dim legemer er disse de "enkleste"
- Stykkevis ^{flat} versjon av mer generelle legemer.

F.eks: Gauss-Bonet-teorem

$$\int_M K dA + \sum_i \nu_i^* = 2\pi \chi(M)$$

$M \uparrow$

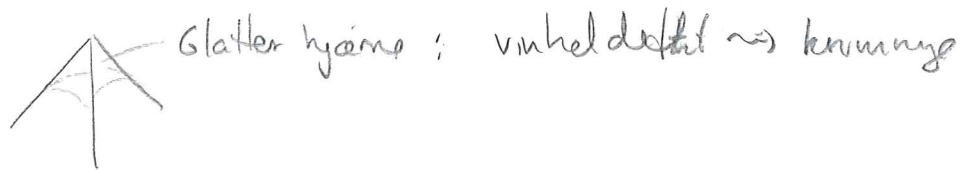
Gauss-krumning

$$K = \kappa_1 \cdot \kappa_2$$

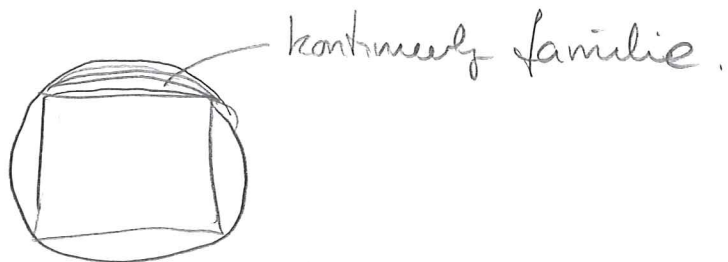
principal-
krumning
(max/min)

$\uparrow$

Vinkeldefekt



Topologisk ekvivalens $\longleftrightarrow$ Homotopi



Udfordring: Klassifisere polyedre

Konvexs mengde: K

Konvex kombinasjon
 $\forall x, y \in K \Rightarrow \lambda x + (1-\lambda)y \in K \quad 0 \leq \lambda \leq 1$

Konvex innhullings av $\{x_1, \dots, x_n\}$

$$\left\{ \sum_{i=1}^n \lambda_i x_i \mid \sum \lambda_i = 1, \lambda_i \geq 0 \right\}$$

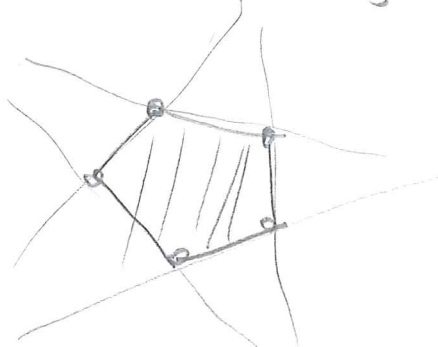
Eksempler

Konvex polygon: Konvexe innhullings av end. mange pnt i $\mathbb{R}^2$

n - polyeder: ——— i ———

$\mathbb{R}^3$

$\approx$ Schnitt av endelig mange halvrom

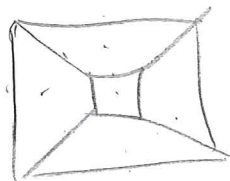


Hjørner, Kanter, Flater

Schlägel diagram:

projeksjon fra generell posisjon

Eks: Kube



Tetraeder



Eulers formel

$$\chi = V - e + f = 2$$

for et konvekst polyeder

Bevis resultatet for sammenhengende grafer.

$$e=0.$$

$$V=1 \quad e=0 \quad f=1 \quad \chi=2$$

$$e=1$$

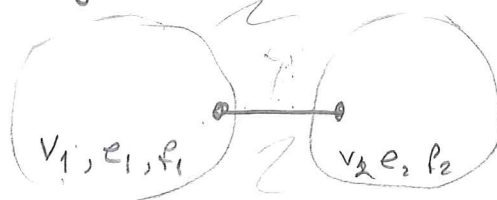


$$V=2 \quad e=1 \quad f=1 \quad \chi=2$$

Anta sant for $e \leq n$, og la $e(T) = n+1$

Fjern en kant. To muligheter:

1. Sammenhengende:



$$V_1 - e_1 + f_1 = 2 \quad V_2 - e_2 + f_2 = 2$$

$$(V_1 + V_2) - (e_1 + e_2 + 1) + (f_1 + f_2 - 1) =$$

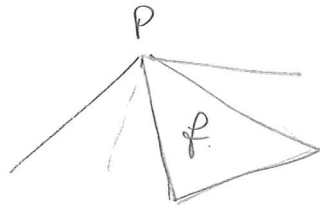
$$= (V_1 - e_1 + f_1) + (V_2 - e_2 + f_2) - 1 - 1 = 2 + 2 - 2 = 2.$$

Sammenhengende



$$V_1 - e_1 + f_1 = 2$$

$$V - (e-1) + (f-1) = V - e + f = 2.$$



$\deg p$: antall kanter
som møtes

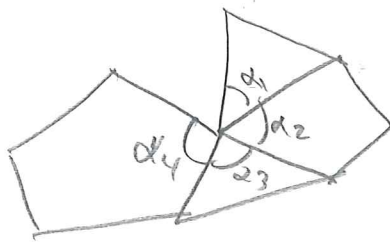
$\deg f$: antall kanter

For et polyeder:

$$2e = \sum \deg p.$$

$$2e = \sum \deg f.$$

Vinkel-defekt



$$v_i^* = 2\pi - \sum \alpha_i$$

Samlet vinkeldefekt

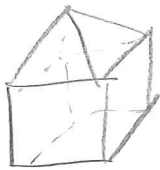
$$\sum v_i^*$$

Descartes' teorem (Gauss-Bonnet)

$$\sum v_i^* = 4\pi \quad (\text{for et konvekst polyeder})$$

Ikke-konvekst:

Må operere med positiv og negativ defekt
(og krumning)



4 hjørner 3 firkanter

$$4 \cdot (2\pi - 3 \cdot \frac{\pi}{2}) = 2\pi$$

$$\triangle: \frac{\pi}{3}$$

$$\square: \frac{\pi}{2}$$

$$\triangle: \frac{3\pi}{5}$$

4 hjørner 2 firkanter + 2 trekanter

$$4 \cdot (2\pi - 2 \cdot \frac{\pi}{2} - 2 \cdot \frac{\pi}{3}) = \frac{4\pi}{3}$$

1 hjørne 4 trekanter

$$2\pi - 4 \cdot \frac{\pi}{3} = \frac{2\pi}{3}$$

Tilsammen $2\pi + \frac{4\pi}{3} + \frac{2\pi}{3} = 4\pi$

#			
4	3		
4		2	2
4	2		2
4	4		1
4	5		
4	4	1	

$$4 \cdot (2\pi - 3 \cdot \frac{\pi}{2}) = 2\pi$$

$$4 \cdot (2\pi - 2 \cdot \frac{\pi}{2} - 2 \cdot \frac{3\pi}{5}) = -\frac{4}{5}\pi$$

$$4 \cdot (2\pi - 2 \cdot \frac{\pi}{3} - 2 \cdot \frac{3\pi}{5}) = \frac{8}{15}\pi$$

$$4 \cdot (2\pi - 4 \cdot \frac{\pi}{3} - \frac{3\pi}{5}) = \frac{4}{15}\pi$$

$$4 \cdot (2\pi - 5 \cdot \frac{\pi}{3}) = \frac{4}{3}\pi$$

$$4 \cdot (2\pi - 4 \cdot \frac{\pi}{3} - \frac{\pi}{2}) = \frac{2}{3}\pi$$

Til sammen $2\pi - \frac{4}{5}\pi + \frac{8}{15}\pi + \frac{4}{15}\pi + \frac{4}{3}\pi + \frac{2}{3}\pi$

$$= (2 - \frac{4}{5} + \frac{8}{15} + \frac{4}{15} + \frac{4}{3} + \frac{2}{3})\pi = 4\pi$$