

5.10

17. $(x-a)^2 + (y-b)^2 = R^2$

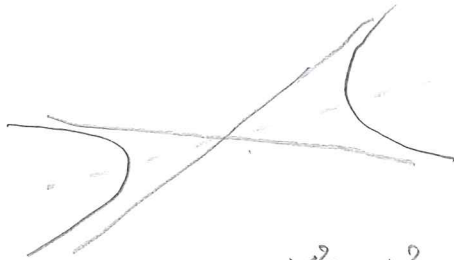
Tre punkter bestemmer en sirkel

generell
posisjon

sentrum : 2 koordinater

radius : 1 størrelse

18



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{standard form}$$

Rotasjon : $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Flytte sentrum (x_0, y_0)

Til sammen 5 parametre

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

6 parametre, -1 : normalisering

Asymptoter : $y = \pm \frac{b}{a}x$

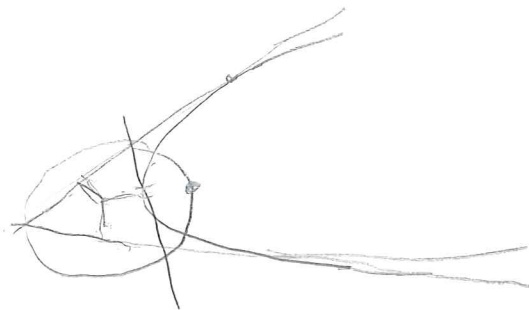
Rotasjon $\frac{(x\cos\theta + y\sin\theta)^2}{a^2} - \frac{(x\sin\theta - y\cos\theta)^2}{b^2}$

$$= \left(\frac{\cos^2\theta}{a^2} - \frac{\sin^2\theta}{b^2} \right) x^2 + \left(2\frac{\cos\theta\sin\theta}{a^2} + 2\frac{\sin\theta\cos\theta}{b^2} \right) xy + \left(\frac{\sin^2\theta}{a^2} - \frac{\cos^2\theta}{b^2} \right) y^2 = 1$$

$$x \mapsto x + x_0$$

$$y \mapsto y + y_0$$

19.



2x

Kan auch normalform : $y = 4\frac{x^2}{4}$, Brennpunkt $(0,1)$

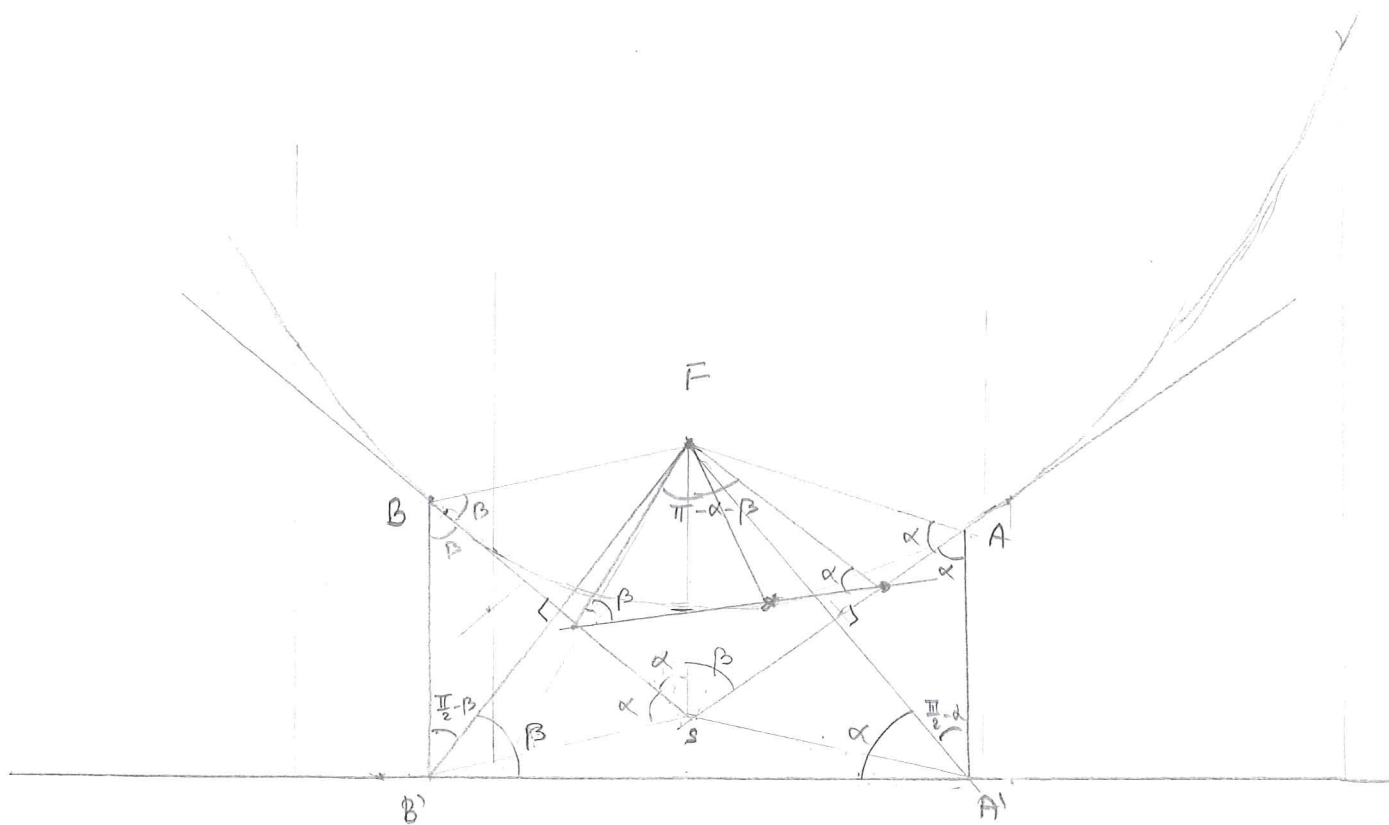
Tangent-gerade $y - \frac{x_0^2}{4} = \frac{x_0}{2}(x - x_0)$

$$\Rightarrow y = \frac{x_0}{2}x - \frac{x_0^2}{2} + \frac{x_0^2}{4} = \frac{x_0}{2}x - \frac{x_0^2}{4}$$

$$\left. \begin{array}{l} y - \frac{x_0}{2}x = \frac{x_0^2}{4} \\ y - \frac{x_1}{2}x = \frac{x_1^2}{4} \end{array} \right\} \left(\frac{x_1}{2} - \frac{x_0}{2} \right) x = \frac{x_0^2}{4} - \frac{x_1^2}{4} = \frac{x_0 - x_1}{2} \cdot \frac{x_0 + x_1}{2}$$

$$\Rightarrow x = -\frac{x_0 + x_1}{2}$$

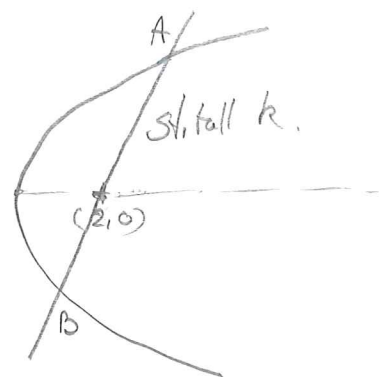
$$\begin{aligned} y &= \frac{x_0}{2} \cdot \frac{x_0 + x_1}{2} - \frac{x_0^2}{4} = -\frac{x_1^2}{4} - \frac{x_0 x_1}{4} - \frac{x_0^2}{4} \\ &= -\frac{x_0}{4}(x_0 + x_1) \end{aligned}$$



$$FS = B'S = SA'$$

#15 oppgave 3.

Parabel $y^2 = 8x$



Linje gjennom (2,0) med st. tall k

$$y = k(x-2) = kx - 2k$$

Snitt med $y^2 = 8x \Leftrightarrow x = \frac{y^2}{8}$

$$\Rightarrow y = \frac{ky^2}{8} - 2k \Leftrightarrow y^2 - \frac{8}{k}y - 16 = 0$$

$$\Rightarrow y = \frac{\frac{8}{k} \pm \sqrt{\frac{64}{k^2} + 64}}{2} = \frac{4}{k} \pm 4\sqrt{\frac{1}{k^2} + 1}$$

$$\Rightarrow \frac{y_1 + y_2}{2} = \frac{4}{k} \quad x = \frac{y + 2k}{k} = \frac{y}{k} + 2$$

$$= \frac{4}{k^2} + 2$$

Mid
Sentrum $(\frac{4}{k^2} + 2, \frac{4}{k})$

radius

$$\frac{1}{2} \sqrt{\left(\frac{8}{k} \sqrt{\frac{1}{k^2} + 1}\right)^2 + \left(8 \sqrt{\frac{1}{k^2} + 1}\right)^2}$$

$$= \frac{1}{2} \sqrt{\frac{64}{k^2} \left(\frac{1}{k^2} + 1\right) + 64 \left(\frac{1}{k^2} + 1\right)}$$

$$= \frac{1}{2} \sqrt{\frac{64}{k^4} + \frac{128}{k^2} + 64}$$

$$= \frac{1}{2} \left(\frac{8}{k^2} + 8 \right) = 4 \left(\frac{1}{k^2} + 1 \right)$$

A: $x = \frac{4}{k^2} + \frac{4}{k} \sqrt{\frac{1}{k^2} + 1} + 2$

$$y = \frac{4}{k} + 4 \sqrt{\frac{1}{k^2} + 1}$$

B: $x = \frac{4}{k^2} - \frac{4}{k} \sqrt{\frac{1}{k^2} + 1} + 2$

$$y = \frac{4}{k} - 4 \sqrt{\frac{1}{k^2} + 1}$$

Likning $\left(x - \left(\frac{4}{k^2} + 2\right)\right)^2 + \left(y - \frac{4}{k}\right)^2 = 16 \left(\frac{1}{k^2} + 1\right)^2$

setter inn $x = -2$:

$$\left(-2 - \frac{4}{k^2} - 2\right)^2 + \left(y - \frac{4}{k}\right)^2 = 16 \left(\frac{1}{k^2} + 1\right)^2$$

$$16 + \frac{32}{k^2} + \frac{16}{k^4} + \left(y - \frac{4}{k}\right)^2 = \frac{16}{k^4} + \frac{32}{k^2} + 16$$

dvs $\left(y - \frac{4}{k}\right)^2 = 0$ Tangent

Midtpunkt til korden $(\frac{4}{k^2} + 2, \frac{4}{k})$

dvs $x = \frac{4}{k^2} + 2 \quad y = \frac{4}{k}$

$\Rightarrow x = \frac{1}{4}y^2 + 2$ Parabel.

Når $k \rightarrow 0$ har vi $(x, y) \rightarrow (\infty, \infty)$

$k \rightarrow \infty \quad \text{---} \quad (x, y) \rightarrow (2, 0)$

