

MAT 2500

11.10.2023

3.6.1 (I)

Polyeder P

5 hjørner

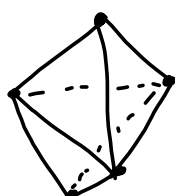
6 sider - trekanter

EULER:

$$v - e + f = 2$$

$$5 - e + 6 = 2$$

$$\underline{\underline{e = 9}}$$



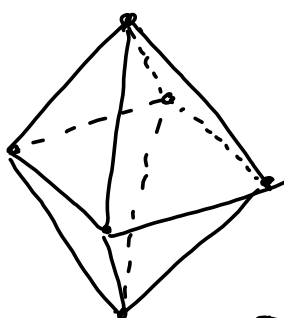
Biprannide pi m trekant

(II)

P

6 hjørner

8 sider - alle trekanter

 P_1 

$$v - e + f = 2$$

$$6 - e + 8 = 2$$

$$\underline{\underline{e = 12}}$$

Bipyramide pi kvadrat
~ oktaeder P_2

?

o

3.6.3. $\{p, q\}$ Schl\"afli-symbol $\{p, q\}$ til P
 $\uparrow$ $\uparrow$
 P -kanter q møter i hvert hjørne $\uparrow$
 regulært polyeder

$e = \# \text{ kanter}$

Skel ut $\frac{1}{e} = \frac{1}{p} + \frac{1}{q} - \frac{1}{2}$

Sehn 3.7.

$$\begin{cases} \bullet v - e + f = 2 \\ \bullet 2e = \sum \deg w = q \cdot v \rightarrow v = \frac{2e}{q} \\ \bullet 2e = \sum \deg s = p \cdot f \rightarrow f = \frac{2e}{p} \end{cases}$$

$$\rightarrow \frac{2e}{q} - e + \frac{2e}{p} = 2 \quad | : 2e$$

$$\frac{1}{q} - \frac{1}{2} + \frac{1}{p} = \frac{1}{e}$$

$\square$

3.6.4.

 P polyeder Γ kantgraphen $U_k = \# \text{ hjørner } i \Gamma \text{ med } \deg w = k \geq 3$ (I) P har trekanter som sideflater

Skal vise

$$\sum_k \left(1 - \frac{k}{6}\right) U_k = 2$$

$$\bullet \quad v - e + f = 2$$

$$\bullet \quad 2e = \sum \deg w = \sum_k k U_k$$

$$\bullet \quad 2e = 3f \quad f = \frac{2e}{3}$$

↑
trekanter

$$\bullet \quad \sum_k U_k - \frac{1}{2} \sum_k k U_k + \frac{1}{3} \sum_k k U_k = 2$$

$$\sum_k U_k \left(1 - \frac{1}{2}k + \frac{1}{3}k\right) = 2$$

$$\sum_k U_k \left(1 - \frac{k}{6}\right) = 2 \quad \square$$

(II)

 n -kanter som flater

$$\bullet \quad 2e = nf \rightarrow f = \frac{2e}{n}$$

$$\sum_k U_k \left(1 - \frac{1}{2}k + \frac{1}{n}k\right) = 2$$

$$\sum_k U_k \left(1 - \frac{(n-2)k}{2n}\right) = 2$$

skal vise at det er umulig med $n \geq 6$.

$$1 - \frac{(n-2)k}{2n} \leq 1 - \frac{(n-2) \cdot 3}{2n} = \frac{2n - 3n + 6}{2n}$$

$$= \frac{6-n}{2n}$$

Hvis $n \geq 6$, så er dette ≤ 0 , som er umulig!