

MAT 2500 25.10.2023

6.6.1. $\mathbb{P}_{\mathbb{R}}^2$

$$P = (1:0:0)$$

$$Q = (1:1:0)$$

$$R = (1:0:1)$$

$$S = (1:1:1)$$

$$l_{PQ}: \begin{vmatrix} x_0 & x_1 & x_2 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix} = 0 \quad l_{RS}: \begin{vmatrix} x_0 & x_1 & x_2 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$x_0 \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} - x_1 \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} + x_2 \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 0$$

$$x_0 \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} - x_1 \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + x_2 \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 0$$

$$\underline{\underline{x_2 = 0}} \quad \underline{\underline{-x_0 + x_2 = 0}}$$

$$T = l_{PQ} \cap l_{RS} :$$

$$\vec{v}_T = \hat{l}_{PQ} \times \hat{l}_{RS}$$

$$= (0, 0, 1) \times (-1, 0, 1)$$

$$= (0, -1, 0)$$

$$T = (0:1:0)$$

Alle punkter har

$$P = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$Q = (1, 0)$$

$$R = (0, 1)$$

$$S = (1, 1)$$

$$x = \frac{x_1}{x_0}$$

$$y = \frac{x_2}{x_0}$$

$$l_{pq} : \underline{y} = 0$$

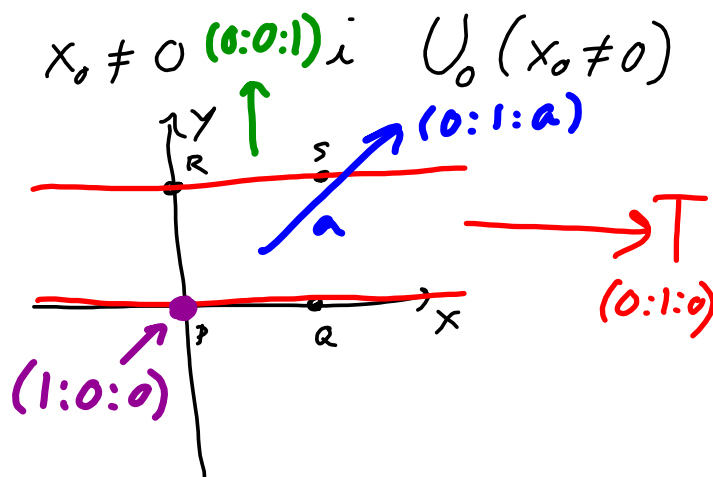
$$\frac{x_2}{x_0} = 0$$

$$\underline{x_2 = 0}$$

$$T = (0 : 1 : 0)$$

$$\downarrow$$

$$x_0 > 0$$



$$l_{rs} : \underline{y} = 1$$

$$\frac{x_2}{x_0} = 1$$

$$x_2 = x_0 \leadsto \underline{-x_0 + x_2 = 0}$$

6.6.7 8 6.6.8.

La $T: \mathbb{P}_{\mathbb{R}}^2 \rightarrow \mathbb{P}_{\mathbb{R}}^2$ ha matrise

$$M = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{pmatrix}$$

Skal finne fikspunkt til T :



$$T(v) = v$$

$$M\vec{v} = \lambda\vec{v}, \lambda \in \mathbb{R}$$

Skal finne egenvektorene til M !

$$\text{Egenverdier: } \begin{vmatrix} 2-\lambda & 1 & 0 \\ 0 & 1-\lambda & -1 \\ 0 & 2 & 4-\lambda \end{vmatrix} = 0$$

$$-\lambda^3 + 7\lambda^2 - 16\lambda + 12 = 0$$

$$-(\lambda-2)^2(\lambda-3) = 0$$

$$\lambda_1 = 2 \text{ (dobbel)} \quad \lambda_2 = 3$$

6.6.8: $T: \mathbb{P}_{\mathbb{R}}^2 \rightarrow \mathbb{P}_{\mathbb{R}}^2$ har en 3×3 -matrise.

↳ Det karakteristiske polynom har grad 3 og har reelle koeffisienter

• Det fins minst en reell egenverdi.

• Til denne fins minst en reell egenvektor, som er et fikspunkt for T i $\mathbb{P}_{\mathbb{R}}^2$.

6.6.7. Egenvektorer: $M\vec{v}_i = \lambda_i \vec{v}_i$

$$\lambda_1 = 2: \vec{v}_1: \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 2 & 2 & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} y=0 \\ z=0 \\ x=y \\ x=y \\ x=y \end{matrix}$$

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} t, t \in \mathbb{R}$$

$$\lambda_2 = 3: \vec{v}_2: \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -2 & -1 & 0 \\ 0 & 2 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} x=y \\ y=-\frac{1}{2}z \\ z=t, t \in \mathbb{R} \end{matrix}$$

$$\vec{v}_2 = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} \hat{t}, \hat{t} \in \mathbb{R}$$

$$\begin{matrix} x=y \\ y=-\frac{1}{2}z \\ z=t, t \in \mathbb{R} \end{matrix}$$

Fikspunkt for T er $(1:0:0)$ og $(-1:-1:2)$
 $(1:1:-2)$

SPM: Finn filosofier ^{i $\mathbb{R}^3$} for T !
 $\downarrow$
 plan gjennom origo i $\mathbb{R}^3$

$$M(a_1 \vec{v}_1 + a_2 \vec{v}_2) = a_1 \lambda_1 \vec{v}_1 + a_2 \lambda_2 \vec{v}_2 \in \text{span}\{\vec{v}_1, \vec{v}_2\}$$

En filosofi er linja gjennom v_1 og v_2

$$L: \begin{vmatrix} x_0 & x_1 & x_2 \\ 1 & 0 & 0 \\ -1 & -1 & 2 \end{vmatrix} = 0$$

$$-2x_1 - x_2 = 0 \sim \underline{2x_1 + x_2 = 0}$$

MEN!

$\lambda_1 = 2$ har multiplisitet 2
 men bare en egenvektor!!

M er DEFECTIV

Det fins en GENERALISERT EGENVEKTOR

$$M\vec{v}_3 = \vec{v}_1 + \lambda_1 \vec{v}_3$$

$$(M - 2I)\vec{v}_3 = \vec{v}_1$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 2 & 2 & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{matrix} y=1 \\ z=-1 \\ x \text{ fri} \end{matrix} \quad \vec{v}_3 = \begin{pmatrix} t \\ 1 \\ -1 \end{pmatrix}, \quad t \in \mathbb{R}$$

Velger $t=0$. $\underline{\vec{v}_3 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}}$

Obs: $M\vec{v}_3 = \vec{v}_1 + \lambda_1 \vec{v}_3$

$$M(a_1 \vec{v}_1 + a_3 \vec{v}_3) = a_1 \lambda_1 \vec{v}_1 + a_3 \lambda_1 \vec{v}_3 + \vec{v}_1 \in \text{span}\{\vec{v}_1, \vec{v}_3\}$$

$$\hat{L}: \begin{vmatrix} x_0 & x_1 & x_2 \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix} = 0$$

$$\underline{\underline{x_1 + x_2 = 0}}$$

$$M(a_2 \vec{v}_2 + a_3 \vec{v}_3) = a_2 \lambda_2 \vec{v}_2 + a_3 \lambda_1 \vec{v}_3 + \vec{v}_1 \notin \text{span}\{\vec{v}_2, \vec{v}_3\}$$

En firkant $l: ax_0 + bx_1 + cx_2 = 0$

da $(x_0 : x_1 : x_2) \in l$

Se på $T(l) = l$

$$\begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2x_0 + x_1 \\ x_1 - x_2 \\ 2x_1 + 4x_2 \end{pmatrix} \quad \begin{matrix} \swarrow l \\ \searrow l \end{matrix}$$

$$a(2x_0 + x_1) + b(x_1 - x_2) + c(2x_1 + 4x_2) = 0$$

$$\begin{cases} \underline{2a} x_0 + \underline{(a+b+2c)} x_1 + \underline{(-b+4c)} x_2 = 0 \\ \underline{\lambda a} x_0 + \underline{\lambda b} x_1 + \underline{\lambda c} x_2 = 0 \end{cases}$$

$$\text{I } \lambda a = 2a \quad \rightarrow \lambda = 2 \text{ eller } a = 0$$

$$\text{II } \lambda b = a + b + 2c$$

$$\text{III } \lambda c = -b + 4c$$

$$a=0: \text{II } \lambda b = b + 2c \rightarrow \underline{b}(\lambda - 1) = 2c$$

$$\text{III } \underline{b = c(4 - \lambda)}$$

$$\underline{c(4 - \lambda)(\lambda - 1) = 2c}$$

$$c=0 \\ \downarrow \\ \text{umulig}$$

$$(4 - \lambda)(\lambda - 1) = 2$$

$$\lambda = 2 \text{ eller } \lambda = 3$$

$$\lambda = 2: 2b = b + 2c$$

$$\underline{b = 2c}$$

$$2cx_1 + cx_2 = 0$$

$$l: \underline{2x_1 + x_2 = 0}$$

$$\lambda = 3: 3b = b + 2c$$

$$2b = 2c$$

$$\underline{b = c}$$

$$cx_1 + cx_2 = 0$$

$$\hat{l}: \underline{X_1 + X_2 = 0}$$

6.6.9. $\mathbb{P}_{\mathbb{R}}^2$ og se på $U_2 = \{(x_0 : x_1 : x_2) \in \mathbb{P}_{\mathbb{R}}^2 \mid \text{s.a. } x_2 \neq 0\}$

ψ
 C $(C \cap U_2)$

(i) $\mathbb{P}_{\mathbb{R}}^2: 2x_0^2 + 2x_1^2 + 4x_0x_1 - 10x_0x_2 = 0$ $x_2 \neq 0$
si deler
på x_2^2

$2\left(\frac{x_0}{x_2}\right)^2 + 2\left(\frac{x_1}{x_2}\right)^2 + 4\left(\frac{x_0x_1}{x_2^2}\right) - 10\left(\frac{x_0x_2}{x_2^2}\right) = 0$ $1:2$

$\left(\frac{x_0}{x_2}\right)^2 + \left(\frac{x_1}{x_2}\right)^2 + 2\left(\frac{x_0x_1}{x_2^2}\right) - 5\left(\frac{x_0}{x_2}\right) = 0$

$U_2 \quad x = \frac{x_0}{x_2} \quad y = \frac{x_1}{x_2}$

$C \cap U_2: x^2 + y^2 + 2xy - 5x = 0$

$\bullet \quad b^2 - 4ac = 2^2 - 4 \cdot 1 \cdot 1 = 0 \rightarrow \text{PARABEL}$

$X = \frac{2\sqrt{2}}{5} Y^2 + Y$

(ii) $C: 2x_0^2 - 3x_0x_1 + x_1^2 - 5x_0x_2 - 2x_1x_2 + 6x_2^2 = 0$

$C \cap U_2: 2x^2 - 3xy + y^2 - 5x - 2y + 6 = 0$

$x_2 = 1$
 $x_0 = x$
 $x_1 = y$

$x_2 = 0 \quad 2x_0^2 - 3x_0x_1 + x_1^2 = 0 \quad C \cap l_{\infty}$

$b^2 - 4ac = (-3)^2 - 4 \cdot 2 \cdot 1 = 1 > 0$

To realle løsninger $\Rightarrow C \cap U_2$ er hyperbel

