

MAT 2500 01.11.2023

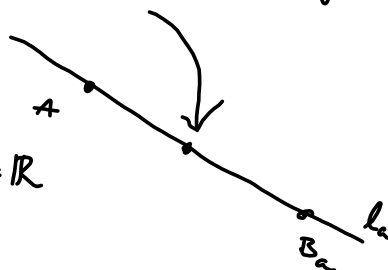
H 2016-4

$(x_0 : x_1 : x_2)$ homogene koordinater i $\mathbb{P}^2$

(a)

$$A = (1 : 0 : 1)$$

$$B_a = (0 : a : 1), a \in \mathbb{R}$$



$$(0 : 0 : 0)$$

$$(0 : 0 : 1)$$

$$(0 : 1 : 0)$$

$$(1 : 0 : 0)$$

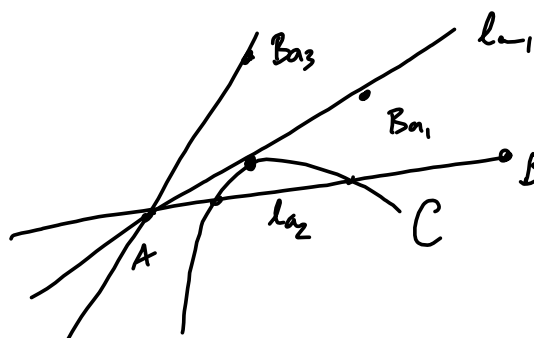
$$l_a: \begin{vmatrix} x_0 & x_1 & x_2 \\ 1 & 0 & 1 \\ 0 & a & 1 \end{vmatrix} = 0$$

$$x_0 \begin{vmatrix} 0 & 1 \\ a & 1 \end{vmatrix} - x_1 \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} + x_2 \begin{vmatrix} 1 & 0 \\ 0 & a \end{vmatrix} = 0$$

$$-ax_0 - x_1 + ax_2 = 0$$

$$\underline{\underline{ax_0 + x_1 - ax_2 = 0}}$$

(b)



$$C : x_2^2 - x_0 x_1 = 0$$

Skal bestemme a s.a.
 l_a tangenter C

Tangenter $\Leftrightarrow$ ett skjæringspunkt $C \cap l_a$

$\Leftrightarrow$ en andregradsligning med dobbelrot

$\Leftrightarrow$ diskriminanten $= 0$

$$\rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$l_a : ax_0 + x_1 - ax_2 = 0 \quad x_1 = \underline{a(x_2 - x_0)}$$

$C \cap l_a$: setter inn for x_1 i likningen for C

$$x_2^2 - x_0 \cdot \underline{a(x_2 - x_0)} = 0$$

$$x_2^2 - ax_0 x_2 + ax_0^2 = 0$$

homogen ligning
i 2. variable

$$x_0 = 1 : x_2^2 - ax_2 + a = 0$$

$$x_2 = 1 : ax_0^2 - ax_0 + 1 = 0$$

$$\Delta b^2 - 4ac = 0$$

$$(-a)^2 - 4 \cdot 1 \cdot a = 0$$

$$a(a-4) = 0$$

$$\underline{\underline{a=0 \text{ eller } a=4}}$$

l_a tangenter C når $a=0$ eller $a=4$

$$\mathbb{R}^2 \cong U_2 \quad (x_2 \neq 0)$$

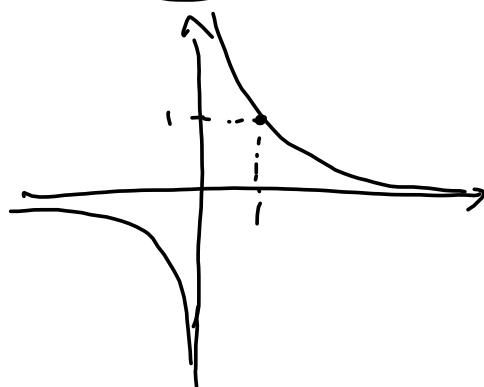
$$A = (1, 0)$$

$$B_a = (0, a)$$

C: $1^2 - xy = 0$ $x = \frac{x_1}{x_2}$

$$y = \frac{x_1}{x_2}$$

$$y = \frac{1}{x}$$

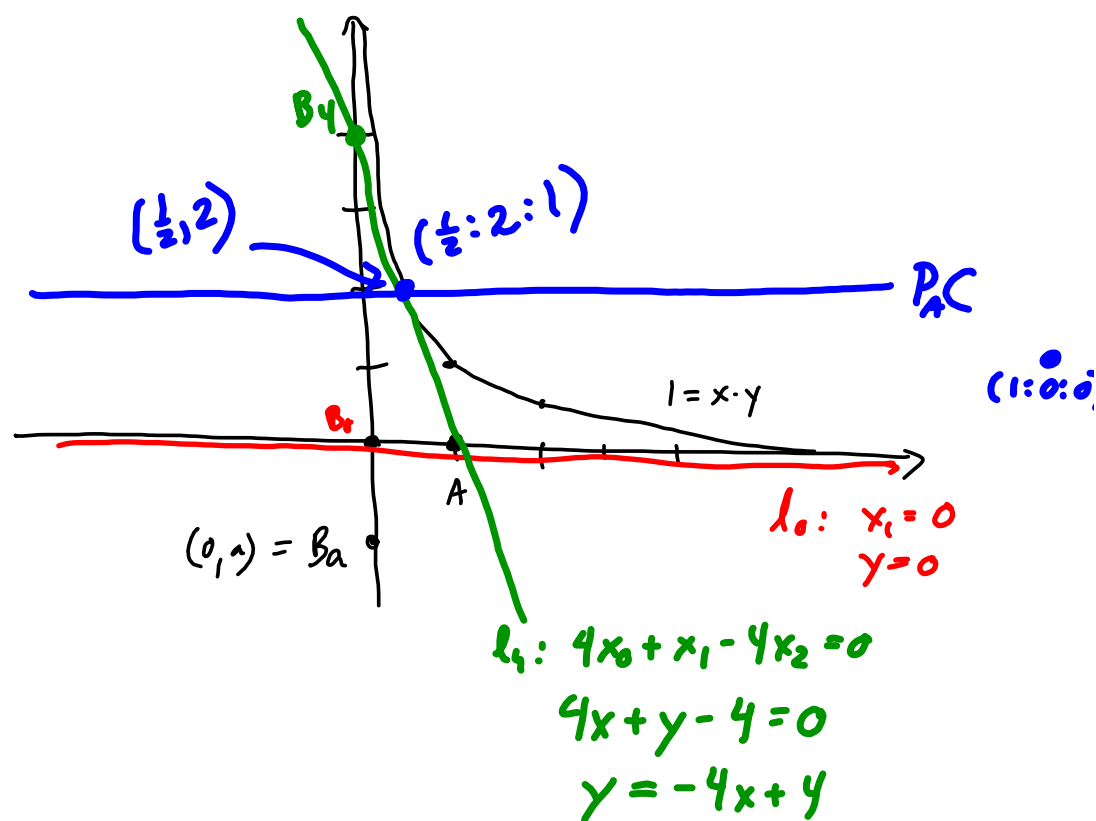


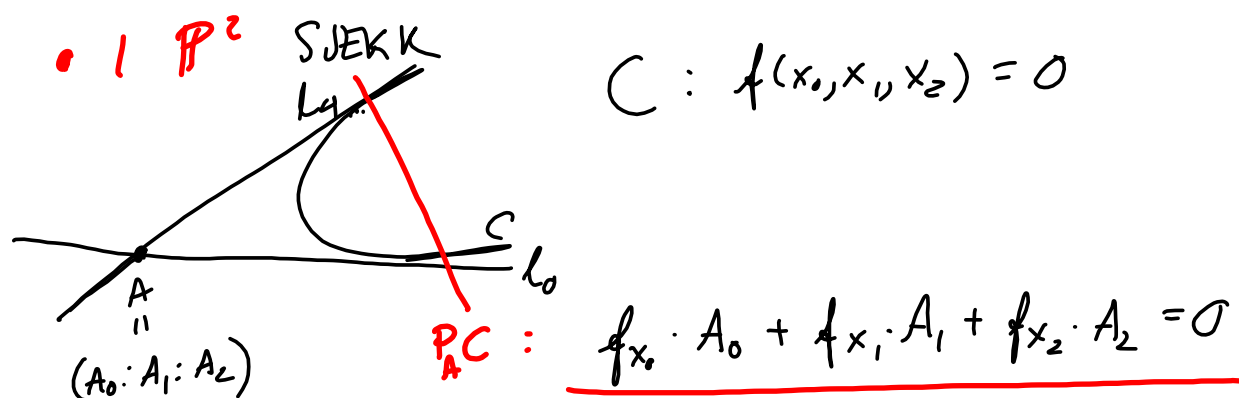
P^2

$$A = (1:0:1)$$

$$P_a = (0 : a : 1)$$

$$C: x_2^2 - x_0 x_1 = 0$$





$$A = x_2^2 - x_0 x_1 \quad \begin{aligned} f_{x_0} &= -x_1 \\ f_{x_1} &= -x_0 \\ f_{x_2} &= 2x_2 \end{aligned} \quad A = (1:0:1)$$

$$P_C : -x_1 + 2x_2 = 0 \quad \text{i } \mathbb{R}^2 : \begin{aligned} -y + z &= 0 \\ \underline{y} &= z \end{aligned}$$

$$P_C \cap C :$$

$$x_2^2 - 2x_2 \cdot x_0 = 0$$

$$x_2(x_2 - 2x_0) = 0$$

$$x_2 = 0 \text{ eller } x_2 = 2x_0$$

$$(1:0:0) \quad (1:4:2)$$

$$(\frac{1}{2}:2:1)$$

Så $(1:0:0)$ og $(\frac{1}{2}:2:1)$ er punkter på C
som har tangentlinjer gjennom A !

$$T_P C : f_{x_0}(P)x_0 + f_{x_1}(P)x_1 + f_{x_2}(P)x_2 = 0$$

for $P \in C$ ($f(P) = 0$)

$$T_{P_1} C : 0 \cdot x_0 + (-1) \cdot x_1 + 0 \cdot x_2 = 0$$

$$x_1 = 0 \leadsto l_0$$

$$T_{P_2} C : -2 \cdot x_0 - \frac{1}{2} x_1 + 2x_2 = 0$$

$$4 \cdot x_0 + x_1 - 4x_2 = 0 \leadsto l_4$$

6.6.10 B 3×3 matrise $\Rightarrow$ Polaritet
 $b(\vec{x}, \vec{y}) = \vec{x}^t B \vec{y}$
 For $P \in \mathbb{P}^2$ kalles linja $\{Q : b(Q, P) = 0\}$
 Polaren til P
 $\uparrow$
 Polen til linje

• Kmlert $B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ $P_1 = (1:0:0)$

- Skal finne P_2 og P_3 på polaren til P_1 .
 $P_1, P_2, P_3 \notin l$
 $P_1 P_2$ er polaren til P_3
 $P_1 P_3$ er polaren til P_2

① $Q = (x_0 : x_1 : x_2)$

$b(Q, P_1) = 0$

$$(x_0 \ x_1 \ x_2) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 0$$

$$\underline{x_0 = 0}$$

② $P_2 = (0 : y_1 : y_2)$

$P_3 = (0 : z_1 : z_2)$

③ Polaren til P_2

$$(x_0 \ x_1 \ x_2) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ y_1 \\ y_2 \end{pmatrix} = 0$$

$$(x_0 \ x_1 \ x_2) \begin{pmatrix} 0 \\ y_1 \\ -y_2 \end{pmatrix} = x_1 y_1 - x_2 y_2 = 0$$

$P_1 P_3$ = Polaren til P_2

$$\begin{vmatrix} x_0 & x_1 & x_2 \\ 1 & 0 & 0 \\ 0 & z_1 & z_2 \end{vmatrix} = 0 \quad \Rightarrow \quad x_1 z_2 - x_2 z_1 = 0$$

$$\underline{y_1 = -z_2 \quad y_2 = -z_1}$$

④ Polaren til P_3

$$(x_0 \ x_1 \ x_2) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ z_1 \\ -z_2 \end{pmatrix} = 0$$

$$(x_0 \ x_1 \ x_2) \begin{pmatrix} 0 \\ z_1 \\ -z_2 \end{pmatrix} = 0$$

$$x_1 z_1 - x_2 z_2 = 0$$

$P_1 P_2$ = polaren til P_3

$$\begin{vmatrix} x_0 & x_1 & x_2 \\ 1 & 0 & 0 \\ 0 & y_1 & y_2 \end{vmatrix} = 0 \quad \Rightarrow \quad -x_1 y_2 + x_2 y_1 = 0$$

$$\underline{z_1 = -y_2 \quad z_2 = -y_1}$$

• SÅ LANGT:

$P_1 = (1:0:0)$

$P_2 = (0:y_1:y_2)$

$P_3 = (0:-y_2:-y_1) = (0:y_2:y_1)$

$y_1 = 1$ og $y_2 = 2$ $P_2 = (0:1:2)$
 $P_3 = (0:2:1)$

SJEKK

- P_1, P_2, P_3 er ikke på linje:

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -3 \neq 0.$$

- P_2 og P_3 er på $x_0 = 0$

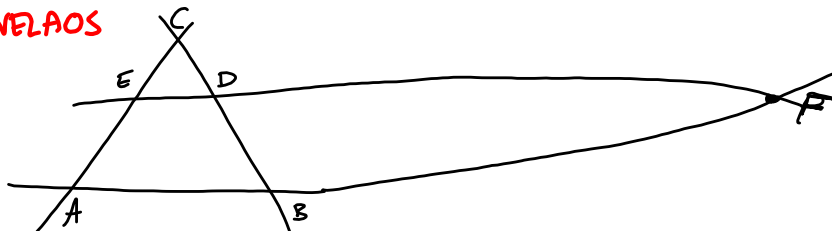
- Polaren til P_3 : $x_1 \cdot 2 - x_2 \cdot 1 = 0$, P_1 og P_2 oppfyller denne

- Polaren til P_2 : $x_1 - 2x_2 = 0$, P_1 og P_3 oppfyller denne

6.6.12 Menelaos & Ceva i P2

NYTT $\left\{ \begin{array}{l} \text{Et punkt } P \in l_{AB}: \\ \frac{\overline{AP}}{\overline{PB}} = -1 \Leftrightarrow P \text{ er i det uendelig fjerne} \end{array} \right.$

MENELAOS



$\Rightarrow$ Anta D, E, F kollinear og $F \in l_{AB}$ i det uendelig fjerne.

$$\begin{aligned} \frac{\overline{AP}}{\overline{PB}} \cdot \frac{\overline{BD}}{\overline{DC}} \cdot \frac{\overline{CE}}{\overline{EA}} &= \\ \frac{\overline{AP}}{\overline{PB}} &= -1 \\ \frac{\overline{BD}}{\overline{DC}} \cdot \frac{\overline{CE}}{\overline{EA}} &= \\ \Delta CED \text{ er liklik } \Delta CAB \text{ siden } l_{DE} \text{ og } l_{AB} &\text{ s\u00e5 er } l_{DE} \text{ og } l_{AB} \\ &\text{ parallelle} \\ \frac{\overline{CD}}{\overline{CE}} &= \frac{\overline{CB}}{\overline{CA}} \\ \frac{\overline{CE} + \overline{EA}}{\overline{CE}} &= \frac{\overline{CD} + \overline{DB}}{\overline{CD}} \\ 1 + \frac{\overline{EA}}{\overline{CE}} &= 1 + \frac{\overline{DB}}{\overline{CD}} \\ \frac{\overline{EA}}{\overline{CE}} &= \frac{\overline{BD}}{\overline{DC}} \\ &= (-1) \cdot \frac{\overline{EA}}{\overline{CE}} \cdot \frac{\overline{CE}}{\overline{EA}} = -1 \end{aligned}$$

$\Leftarrow$ Anta $\frac{\overline{AF}}{\overline{FB}} \cdot \frac{\overline{BD}}{\overline{DC}} \cdot \frac{\overline{CE}}{\overline{EA}} = -1$ og F i det uendelig fjerne

$$\begin{aligned} \frac{\overline{BD}}{\overline{DC}} \cdot \frac{\overline{CE}}{\overline{EA}} &= 1 \\ 1 + \frac{\overline{BD}}{\overline{DC}} &= \frac{\overline{EA}}{\overline{CE}} = \frac{\overline{AE}}{\overline{EC}} + 1 \\ \frac{\overline{BC}}{\overline{DC}} &= \frac{\overline{BD} + \overline{DC}}{\overline{DC}} = \frac{\overline{AE} + \overline{EC}}{\overline{EC}} = \frac{\overline{AC}}{\overline{EC}} \end{aligned}$$

s\u00e5 ΔCED er liklik ΔCAB
s\u00e5 l_{DE} er parallel med l_{AB}
og $F \in l_{DE}$ og D, E, F er kollinear.