

# 2005, oppg. 4

$$(1,0) \in L_1$$

↑  
st. tall  $k$

$$y = k(x-1)$$

skjærer hverandre hvis de ikke er parallelle

$$(-1,0) \in L_2$$

↑  
st. tall  $\frac{1}{k}$

$$y = \frac{1}{k}(x+1)$$

$$k \neq \frac{1}{k}$$

$$\text{dvs } k \neq \pm 1$$

skjæringspunktet  $k(x-1) = \frac{1}{k}(x+1)$

$$\Leftrightarrow (k - \frac{1}{k})x = k + \frac{1}{k} \quad \text{dvs } x = \frac{k + \frac{1}{k}}{k - \frac{1}{k}} = \frac{k^2 + 1}{k^2 - 1}$$

$$y = k \left( \frac{k^2 + 1}{k^2 - 1} - 1 \right) = \frac{k}{k^2 - 1} (k^2 + 1 - k^2 + 1) = \frac{2k}{k^2 - 1}$$

Sei

$$\Rightarrow \left( \frac{k^2 + 1}{k^2 - 1}, \frac{2k}{k^2 - 1} \right)$$

$$\text{Vi har } \frac{(k^2 + 1)^2 - 4k^2}{(k^2 - 1)^2} = 1 \quad \text{dvs } x^2 - y^2 = 1$$

Alternativt:

$$\left. \begin{array}{l} y = k(x-1) \\ y = \frac{1}{k}(x+1) \end{array} \right\} y^2 = x^2 - 1 \Rightarrow x^2 - y^2 = 1$$

# 2012, oppg. 2

$$y^2 = 4x$$

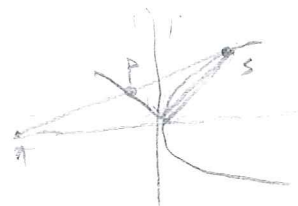
3 punkt på parabolen  $S = \left( \frac{t^2}{4}, t \right)$

$$A = (-12, 0)$$

$\ell$  normalen i O til linja AS.

$$O = (0, 0)$$

$$P = \ell \cap AS$$



$$\ell: S \left( -t, \frac{t^2}{4} \right)$$

$$AS: y = \frac{t}{\frac{t^2}{4} + 12} (x + 12) = \frac{4t}{t^2 + 48} (x + 12)$$

$$\ell \cap AS: \frac{st^2}{4} = \frac{4t}{t^2 + 48} (-st + 12) \Rightarrow st^2(t^2 + 48) = 16t(-st + 12)$$

$$st^4 + 48st^2 = -16st^2 + 192t$$

$$P: \frac{192}{t(t^2 + 64)} \left( -t, \frac{t^2}{4} \right)$$

$$st' = s = \frac{192t}{t^4 + 64t^2} = \frac{192}{t(t^2 + 64)}$$

$$= \left( \frac{-192}{-t^2 + 64}, \frac{48t}{t^2 + 64} \right) = \frac{48}{t^2 + 64} (-4, t)$$

" " " "

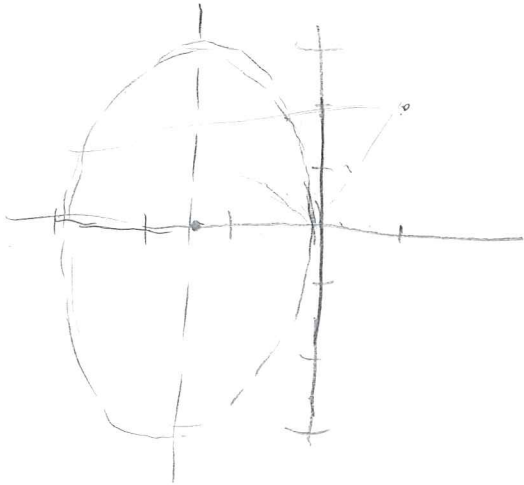
$$y^2 + (2x)^2 = \frac{48^2}{(t^2 + 64)^2} (t^2 + 64) = \frac{48^2}{t^2 + 64} = -12x$$

$$4x^2 + 12x + y^2 = 0$$

$$\text{dvs } 4\left(x + \frac{3}{2}\right)^2 + y^2 = 9$$

$$\frac{\left(x + \frac{3}{2}\right)^2}{\left(\frac{3}{2}\right)^2} + \frac{y^2}{3^2} = 1$$

Ellipse med sentrum:  $\left(-\frac{3}{2}, 0\right)$  og  
halvaksler  $\frac{3}{2}$  og  $3$ .



b) Symmetrilinjene:  $x = -\frac{3}{2}$ ,  $y = 0$

Eksempel 1)  $S = (1, 2)$   $l: y = -\frac{1}{2}x$   $AS: y - 2 = \frac{2}{13}(x - 1)$

$$l \cap AS: -\frac{1}{2}x - 2 = \frac{2}{13}(x - 1) \Rightarrow -13x - 52 = 4x - 4$$

$$\text{eller } 17x = -48 \Rightarrow x = -\frac{48}{17}$$

$$\text{og } y = +\frac{24}{17}$$

$$4\left(-\frac{48}{17} + \frac{3}{2}\right)^2 + \left(+\frac{24}{17}\right)^2 =$$

$$\left(-\frac{96}{17} + \frac{51}{17}\right)^2 + \left(\frac{24}{17}\right)^2 = \left(-\frac{45}{17}\right)^2 + \left(\frac{24}{17}\right)^2$$

$$= \frac{2025 + 576}{17^2} = \frac{2601}{17^2} = \frac{153}{17} = 9.$$

2)  $S = \left(\frac{1}{4}, 1\right)$   $l: y = -\frac{1}{4}x$   $AS: y - 1 = \frac{1}{\frac{1}{4} + 12}(x - \frac{1}{4})$

$$-1 \frac{1}{4}$$

$$y - 1 = \frac{4}{49}\left(x - \frac{1}{4}\right)$$

$$l \cap AS: -\frac{1}{4}x - 1 = \frac{4}{49}\left(x - \frac{1}{4}\right) \Rightarrow -49x - 196 = 16x - 4$$

$$\Rightarrow x = -\frac{192}{65}, y = \frac{48}{65}$$

$$4\left(-\frac{192}{65} + \frac{3}{2}\right)^2 + \left(\frac{48}{65}\right)^2 = \left(-\frac{384 + 195}{65^2}\right)^2 + \left(\frac{48}{65}\right)^2$$

$$= \frac{189^2 + 48^2}{65^2} = 9 \frac{63^2 + 16^2}{65^2}$$

$$= 9 \cdot \frac{4225}{65^2} = 9 \cdot \frac{169}{169} = \underline{\underline{9}}$$

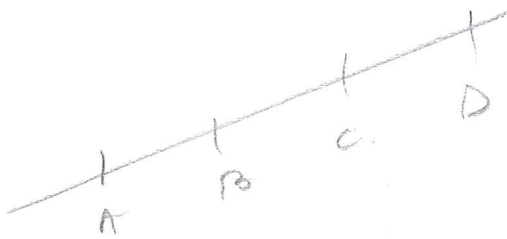
$$63 \cdot 63$$

$$\begin{array}{r} 189 \\ 378 \\ \hline 3969 \end{array}$$

$$\begin{array}{r} 3969 \\ 256 \\ \hline 4225 \end{array}$$

$$\begin{array}{r} 4225 \\ 4225 \\ \hline 0 \end{array}$$

2006, oppg. 1.



$$a) (AB, CD) = \frac{AC}{AD} \cdot \frac{BD}{BC} = 1$$

$$\Rightarrow AC \cdot BD = AD \cdot BC$$

$$(AB+BC)BD = (AB+BD)BC$$

$$AB \cdot BD + BC \cdot BD = AB \cdot BC + BD \cdot BC$$

$$\Rightarrow AB \cdot BD = AB \cdot BC \quad | \frac{1}{AB}$$

$$\Rightarrow BD = BC$$

$$\Rightarrow C = D$$

$$b) (AB, CD) = 0 \quad \frac{AC}{AD} \cdot \frac{BD}{BC} = 0 \Rightarrow A=C \text{ eller } B=D$$

$$c) (AB, CD) = \infty \quad \frac{AD}{AC} \cdot \frac{BC}{BD} = \frac{1}{\infty} = 0 \Rightarrow A=D \text{ eller } C=B$$

2008, oppg. 1

$$a) l = AB$$

$$A = (1, 1, 0)$$

$$B = (0, 1, 1)$$

Duale punkt:

$$l: \begin{vmatrix} x & y & z \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = x - y + z = 0$$

$$P_l = (1, -1, 1)$$

$$m: 2x - y + 3z = 0 \quad \{C\} = l \cap m$$

$$C = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 2 & -1 & 3 \end{vmatrix} = -2\vec{i} - \vec{j} + \vec{k} = (-2, -1, 1)$$

b)  $(AB, CD)$  er definert for punkter på linja  $l$ .

$$\text{Betn. } y \neq 0: A = (1, 0), B = (0, 1), C = (2, -1), D = (t, 1-t)$$

$$AD = t + 1$$

$$t = \dots$$

$$t = \dots$$

$$= 2011 - 1 + t = 2 \Rightarrow t = 3 \Rightarrow D = (3, -2)$$

Projiseres på x-aksen

$$a=1, b=0, c=2, d=t$$

$$(ab, cd) = \frac{c-a}{d-a} \frac{d-b}{c-b} = \frac{1}{t-1} \frac{t}{2} = -1 \quad \text{dvs } t = -2t + 2 \\ \text{eller } t = \frac{2}{3}$$

$$\text{som gir } \left(\frac{2}{3}, \frac{1}{3}\right) \text{ og } D = \left(\frac{2}{3} : 1 : \frac{1}{3}\right) = \underline{\underline{(2:3:1)}}$$

Vi har

$$(-2, -1, 1) = -2(1, 1, 0) + (0, 1, 1)$$

$$(2, 3, 1) = 2(1, 1, 0) + (0, 1, 1)$$

$$\Rightarrow (AB, CD) = \frac{1 \cdot 2}{-2 \cdot 1} = \underline{\underline{-1}}$$