

2012, oppg. 3

$$A = (1:0:2)$$

$$B_a = (0:2:a)$$

$$\ell_a: \begin{vmatrix} x_0 & x_1 & x_2 \\ 1 & 0 & 2 \\ 0 & 2 & a \end{vmatrix} = -4x_0 - ax_1 + 2x_2 = 0$$

skjærings $x_2 = 2x_0 + \frac{a}{2}x_1$ $x_0x_1 - x_2^2 = 0$

$$x_0x_1 - (2x_0 + \frac{a}{2}x_1)^2 = x_0x_1 - 4x_0^2 - 2ax_0x_1 - \frac{a^2}{4}x_1^2 = 0$$

$$-4x_0^2 + (1-2a)x_0x_1 - \frac{a^2}{4}x_1^2 = 0$$

abc-formel:
$$x_0 = \frac{-(1-2a) \pm \sqrt{(1-2a)^2 - 4a^2}}{-8} x_1$$

$$= \frac{-(1-2a) \pm \sqrt{1-4a}}{-8} x_1$$

$a = -1$

$$x_0 = \frac{1 \pm \sqrt{5}}{-8} x_1 \quad x_2 = 2 \frac{1 \pm \sqrt{5}}{-8} x_1 + \frac{1}{2} x_1$$

$$= \left(-\frac{3}{4} \mp \frac{\sqrt{5}}{4}\right) x_1$$

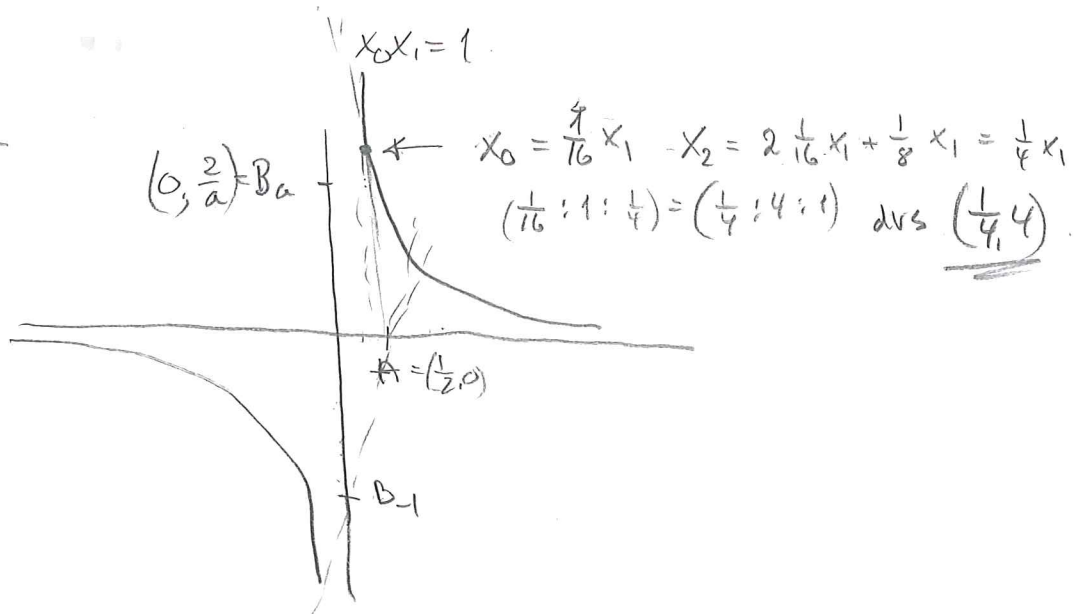
$$(1 \pm \sqrt{5} : -8 : 6 \pm 2\sqrt{5})$$

12

Tangens

$$D=0 \quad \text{dvs} \quad a = \frac{1}{4}$$

$x_2 = 1$



2005, oppg. 3

$$l: x+y-z=0$$

$$m: x+y-2z=0$$

$$n: y=0$$

$$k: x=0$$

$$A_1 = l \cap n, B_1 = m \cap n$$

$$A_2 = l \cap k, B_2 = m \cap k$$

$$n \cap k = C$$

$$D \in n_1(A_1 B_1, C D) = -1$$

$$A_1 = \begin{pmatrix} l & j & k \\ 1 & 1 & -1 \\ 0 & 1 & 0 \end{pmatrix} = (1:0:1)$$

$$B_1 = \begin{pmatrix} l & j & k \\ 1 & 1 & -2 \\ 0 & 1 & 0 \end{pmatrix} = (2:0:1)$$

$$C = \begin{pmatrix} l & j & k \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = (0:0:1)$$

$$D = (a:0:b)$$

$$C = 2A_1 - B_1 \quad D = (2b-a)A_1 + (a-b)B_1$$

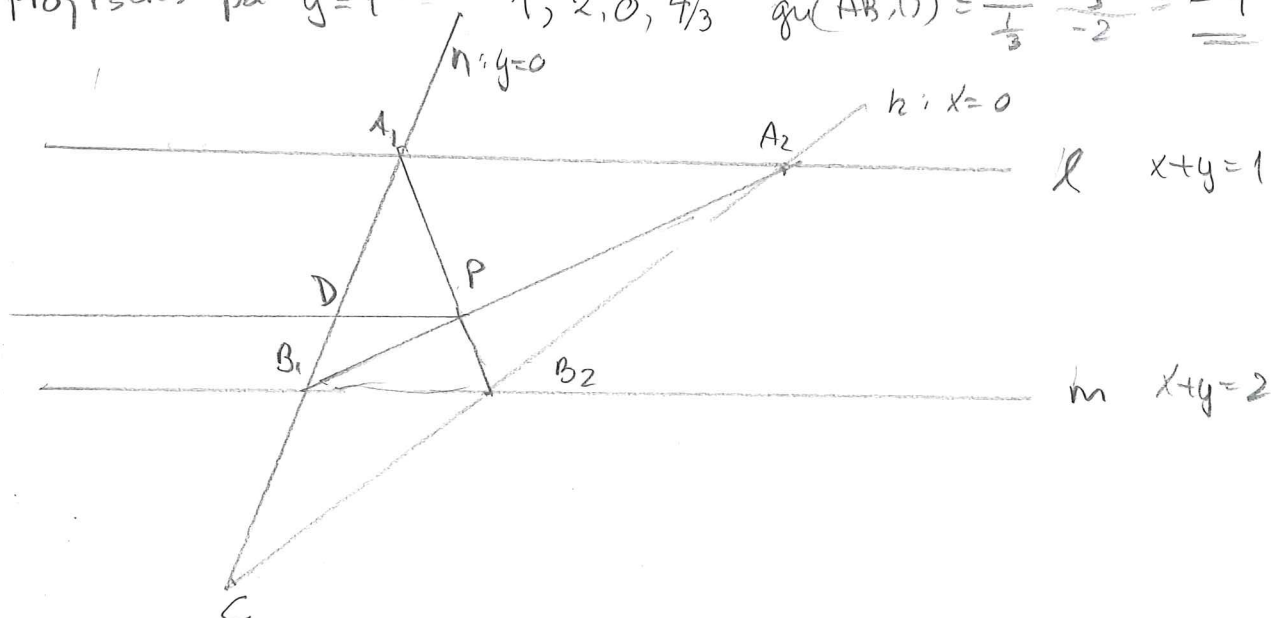
$$(AB, CD) = \frac{-1(2b-a)}{2(a-b)} - 1 \Leftrightarrow -2b+a = -2a+2b$$

$$3a = 4b$$

$$\Rightarrow D = (4:0:3)$$

Pa linja $y=0$: $A_1 = (1,1)$ $B_1 = (2,1)$ $C = (0,1)$ $D = (\frac{4}{3}, 1)$

Projiseres på $y=1$: $1, 2, 0, \frac{4}{3}$ $qu(AB, CD) = \frac{-1}{\frac{1}{3}} \frac{-\frac{2}{3}}{-2} = \underline{\underline{-1}}$



$$A_2 = \begin{vmatrix} l & j & k \\ 1 & 1 & -1 \\ 1 & 0 & 0 \end{vmatrix} = (0 : -1 : -1)$$

$$B_2 = \begin{vmatrix} l & j & k \\ 1 & 1 & -2 \\ 1 & 0 & 0 \end{vmatrix} = (0 : -2 : -1)$$

$$A_1 B_2 : \begin{vmatrix} x & y & z \\ 1 & 0 & 1 \\ 0 & -2 & -1 \end{vmatrix} = 2x + y - 2z = 0$$

$$A_2 B_1 : \begin{vmatrix} x & y & z \\ 0 & -1 & -1 \\ 2 & 0 & 1 \end{vmatrix} = -x - 2y + 2z = 0$$

$$P = \begin{vmatrix} l & j & k \\ 2 & 1 & -2 \\ -1 & -2 & 2 \end{vmatrix} = (-2 : -2 : -3) = (2 : 2 : 3)$$

$$DP = \begin{vmatrix} x & y & z \\ 4 & 0 & 3 \\ 12 & 12 & 3 \end{vmatrix} = -6x - 6y + 8z = 0 \quad \text{eller} \quad x + y - \frac{3}{2}z = 0$$

Parallel med lgm

Alternativt:

Ceva på $\triangle A_1 P B_1$, Cevalinjer $PD, A_1 A_2, B_1 B_2$

$$\gamma = \frac{\overline{A_1 D}}{\overline{D B_1}} \cdot \frac{\overline{B_1 A_2}}{\overline{A_2 P}} \cdot \frac{\overline{P B_2}}{\overline{B_2 A_1}} =$$

$$\text{Vi har } \frac{\overline{A_1 C}}{\overline{A_1 D}} \cdot \frac{\overline{B_1 D}}{\overline{B_1 C}} = -1 \Rightarrow \frac{\overline{A_1 D}}{\overline{D B_1}} = \frac{\overline{A_1 C}}{\overline{C B_1}}$$

$$\gamma = - \frac{\overline{A_1 C}}{\overline{C B_1}} \cdot \frac{\overline{B_1 A_2}}{\overline{A_2 P}} \cdot \frac{\overline{P B_2}}{\overline{B_2 A_1}}$$

Menelaos på $\triangle B_1 P A_1$ med $C \in A_1 B_1, B_2 \in P A_1, A_2 \in B_1 P$.
(disse er på linje), derfor

$$\frac{\overline{A_1 C}}{\overline{C B_1}} \cdot \frac{\overline{B_1 A_2}}{\overline{A_2 P}} \cdot \frac{\overline{P B_2}}{\overline{B_2 A_1}} = -1$$

$$\Rightarrow \gamma = -(-1) = 1 \quad \text{Kolincare eller Ceva}$$

2010, oppg. 2

$$P: V=6, f=5$$

$$a) V - e + f = 2 \Rightarrow e = V + f - 2 = 6 + 5 - 2 = 9$$

$$b) \text{ Anta } P \text{ har bare 3-kanter} \\ \Rightarrow e = \frac{3 \cdot f}{2} = \frac{15}{2} = 7,5 \text{ går ikke}$$

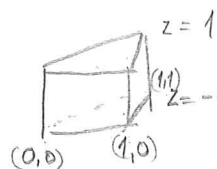
f_3 : antall 3-kanter

f_4 : antall 4-kanter

$$3f_3 + 4f_4 = 2e = 18$$

$$f_3 + f_4 = f = 5$$

$$\Rightarrow f_4 = 3, f_3 = 2$$



Hjørner: $(0,0,1), (1,0,1), (1,1,1),$
 $(0,0,-1), (1,0,-1), (1,1,-1)$

2011, oppg. 2

$$f=12 \text{ 4-kanter}$$

$$a) 4F = 2E \Rightarrow E = 24$$

$$\# \{(k,s) \mid k \in E, s \in F, k \subset s\} = 48 = 12 \cdot 4 = 4F \\ = 2E$$

$$b) V - E + F = 2 \Rightarrow V = 2 + E - F = 2 + 24 - 12 = 14$$

$$c) f \text{ 4-kanter har } 4f = 2e \text{ kanter til sammen} \\ \Rightarrow e = 2f$$

$$V - e + f = 2 \Rightarrow V = 2 + e - f = 2 + 2f - f = \underline{\underline{f+2}}$$

2014, oppg. 1

P: $f = 8$
 $f_6 = 4$
 $f_3 = 4$

a) Antall kanter: $6f_6 + 3f_3 = 2e$

$$\Rightarrow e = \frac{6f_6 + 3f_3}{2} = \frac{6 \cdot 4 + 3 \cdot 4}{2} = 18$$

Hjørner $v - e + f = 2 \Rightarrow v = 2 + e - f = 2 + 18 - 8 = \underline{12}$
Eulers formel

b)

V_i : hjørner av grad i
 $i = 3, 4, 5, 6$

$$V_3 + V_4 + V_5 + V_6 = v = 12 \quad | \cdot 3$$

$$3V_3 + 4V_4 + 5V_5 + 6V_6 = 2e = 36$$

$$3V_3 + 3V_4 + 3V_5 + 3V_6 = 36$$

$$V_4 + 2V_5 + 3V_6 = 0$$

Siden $V_i \geq 0 \Rightarrow V_4 = V_5 = V_6 = 0$

$$\Rightarrow \underline{V_3 = 12}$$

c)

Platonisk legeme: (v, e, f)

Kutte av hjørner: antall flater $f' = v + f = 8$

antall kanter $e' = 3e = 18$

$$\Rightarrow e = 6 \quad v + f = 8$$

$$\Downarrow$$
$$v = f = 4$$

dvs Tetraeder

