

MAT 2580 - 08.11.2023

2014-4

(a) $A = (1:1:0)$
 $B = (0:1:1)$

③ Skal finne L_{AB} : punkter på $L_{AB}: (x_0:x_1:x_2)$

$$\begin{vmatrix} x_0 & x_1 & x_2 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 0$$

$$x_0 \cdot \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} - x_1 \cdot \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + x_2 \cdot \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 0$$

$$-x_0 - x_1 + x_2 = 0$$

Alternativt dualt:

$$A \times B: (1, 1, 0) \times (0, 1, 1) = (1, -1, 1)$$

④ Skal finne punkt(er) i $L_{AB} \cap C$
 der $C: x_0^2 + x_0 x_1 - x_1 x_2 - 7x_2^2 = 0$

$$L_{AB}: x_1 = x_0 + x_2$$

$$C \cap L_{AB}: x_0^2 + x_0(x_0 + x_2) - (x_0 + x_2)x_2 - 7x_2^2 = 0$$

$$\underbrace{(x_0 + x_2)(x_0 - x_2)}_{x_0^2 - x_2^2}$$

$$2x_0^2 - 8x_2^2 = 0$$

$$x_0^2 - 4x_2^2 = 0$$

$$x_0 = \pm 2x_2$$

Må alltid velge en koordinat $\neq 0 \dots = 1$!

$$x_2 = 1$$

$$x_0 = 2$$

$$x_1 = 3$$

$$x_2 = 1$$

$$x_0 = -2$$

$$x_1 = -1$$

$$P_1 = (2:3:1)$$

$$P_2 = (-2:-1:1)$$

$$U_1: x_1 = 1 \quad \frac{x_0}{x_1} = x \quad \frac{x_2}{x_1} = y$$

$$A = (1, 0)$$

$$B = (0, 1)$$

$$P_1 = (\frac{2}{3}, \frac{1}{3})$$

$$P_2 = (2, -1)$$

$$L_{AB}: y = 1 - x$$

$$C: x^2 + x - y - 7y^2 = 0$$

$$C \cap L_{AB}: x^2 + x - (1 - x) - 7(1 - x)^2 = 0$$

$$x^2 + 2x - 1 - 7 + 14x - 7x^2 = 0$$

$$-6x^2 + 16x - 8 = 0$$

$$3x^2 - 8x + 4 = 0$$

$$x = \frac{8 \pm \sqrt{64 - 4 \cdot 3 \cdot 4}}{2 \cdot 3}$$

$$x = \frac{2}{3} \quad \text{og} \quad x = 2$$

$$y = \frac{1}{3} \quad \text{og} \quad y = -1$$

$$P_1 = (\frac{2}{3}, \frac{1}{3}) \quad \text{og} \quad P_2 = (2, -1)$$

$$U_1: \frac{x_0}{x_1} = x \quad \frac{x_2}{x_1} = y$$

$$i \mathbb{P}^2: P_1 = (\frac{2}{3}:1:\frac{1}{3}) \quad P_2 = (2:1:-1)$$

$$P_1 = (2:3:1) \quad P_2 = (-2:-1:1)$$

⑥ Skal finne likningen til en tangent til C på formen $l_a: x_0 - ax_2 = 0$

$$\begin{array}{l} U_1 \\ x = ay \\ y = \frac{1}{a} \cdot x \end{array}$$

$$l_a: x_0 = ax_2$$

$$\begin{aligned} C \cap l_a: (ax_2)^2 + (ax_2)x_1 - x_1x_2 - 7x_2^2 &= 0 \\ (a^2 - 7)x_2^2 + (a-1)x_1x_2 &\stackrel{+0 \cdot x_2^2}{=} 0 \end{aligned}$$

l_a er tangent hvis " $b^2 - 4ac = 0$ "

$$(a-1)^2 - \underbrace{4 \cdot (a^2 - 7) \cdot 0}_0 = 0$$

$$(a-1)^2 = 0$$

$$\underline{a = 1}$$

For $a=1$ er $l_a: x_0 - x_2 = 0$ tangent til C .

● $\begin{array}{l} x_2 = 0 \\ x_0 = 0 \\ x_1 = 1 \end{array}$ Punkt der l_1 er tangent
er $(0:1:0)$
På $U_1: P = (0,0)$

● Vet at l_a går gjennom $P = (0:1:0)$
— " C — " —

$$T_P C: f_{x_0}(P)x_0 + f_{x_1}(P)x_1 + f_{x_2}(P)x_2 = 0$$

$$C: f=0$$

$$f_{x_0} = 2x_0 + x_1$$

$$f_{x_0}(P) = 1$$

$$f_{x_1} = x_0 - x_2$$

$$f_{x_1}(P) = 0$$

$$f_{x_2} = -x_1 - 14x_2$$

$$f_{x_2}(P) = -1$$

$$\underline{T_P C: x_0 - x_2 = 0}, \text{ si } \underline{a=1}$$

H2015-4 $(x_0 : x_1 : x_2) \in \mathbb{P}^2$

(a) $l_1 : \underline{x_0} - 2\underline{x_1} + \underline{x_2} = 0$

$l_2 : \underline{x_0} + 3\underline{x_1} - \underline{x_2} = 0$

① Skal finne $P \in l_1 \cap l_2$

$$\vec{v}_P = \begin{vmatrix} i & j & k \\ 1 & -2 & 1 \\ 1 & 3 & -1 \end{vmatrix} = i \begin{vmatrix} -2 & 1 \\ 3 & -1 \end{vmatrix} - j \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} + k \begin{vmatrix} 1 & -2 \\ 1 & 3 \end{vmatrix}$$

$$= i(-1) - j(-2) + k(5)$$

$$= (-1, 2, 5)$$

$$\underline{P = (-1 : 2 : 5) = (1 : -2 : -5)}$$

② $Q = (1 : 2 : 3)$

Skal finne l_{PQ}

$$l_{PQ} : \begin{vmatrix} x_0 & x_1 & x_2 \\ -1 & 2 & 5 \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$$x_0 \begin{vmatrix} 2 & 5 \\ 2 & 3 \end{vmatrix} - x_1 \begin{vmatrix} -1 & 5 \\ 1 & 3 \end{vmatrix} + x_2 \begin{vmatrix} -1 & 2 \\ 1 & 2 \end{vmatrix} = 0$$

$$-4x_0 + 8x_1 - 4x_2 = 0$$

$$\underline{x_0 - 2x_1 + x_2 = 0}$$

STEEK:

$P, Q \in l_{PQ}$

⑥ $C: x_0^2 + 2x_0x_1 + 3x_1x_2 - 3x_2^2 = 0$
 skal snitte med $l: x_1 - x_2 = 0$
 og bestemme antall punkter i $l \cap C$:

$$l: x_1 = x_2$$

$$C \cap l: x_0^2 + 2x_0x_2 + 3x_2^2 - 3x_2^2 = 0$$

$$x_0^2 + 2x_0x_2 + 0x_2^2 = 0$$

$$\text{Antall: } D = b^2 - 4ac$$

$$= 2^2 - 4 \cdot 1 \cdot 0$$

$$= 4 > 0 \text{ si 2 skjæringspunkt!}$$

$$\bullet x_0(x_0 + 2x_2) = 0$$

$$x_0 = 0$$

$$x_0 = -2x_2 = -2$$

$$x_2 = 1$$

$$x_2 = 1$$

$$x_1 = 1$$

$$x_1 = 1$$

$$P_1 = (0:1:1)$$

$$P_2 = (-2:1:1)$$

⑦ Affint plan $U: x_1 \neq x_2$

$$\mathbb{P}^2 \cong U_0 \cup U_1 \cup U_2$$

$$x_0 \neq 0, x_1 \neq 0, x_2 \neq 0$$

$$l_{\infty}: x_0 = 0, x_1 = 0, x_2 = 0$$

$$\mathbb{P}^2 \setminus l_{\infty}$$

$$l_{\infty}: x_1 - x_2 = 0$$

$$U: x_1 \neq x_2$$

Fra ⑥: $l_{\infty} \cap C$ har 2 snittpunkt!

Da er $C \cap U$ en hyperbel

MEN! Det kunne handle C var degenerert....

) < X

Kan ikke plutselig bli degenerert!

Sjekk på $U_0: x_0 \neq 0$

$$1 + 2x + 3xy - 3y^2 = 0$$

$$0x_2^2 + 3xy - 3y^2 + 2x + 1 = 0$$

$$M = \begin{bmatrix} 0 & \frac{3}{2} \\ \frac{3}{2} & -3 \end{bmatrix}$$

$$|M - \lambda I| = 0 \quad \left| \begin{bmatrix} -\lambda & \frac{3}{2} \\ \frac{3}{2} & -3-\lambda \end{bmatrix} \right| = 0$$

$$\lambda(3+\lambda) - \frac{9}{4} = 0$$

$$\lambda^2 + 3\lambda - \frac{9}{4} = 0$$

$$\lambda = \frac{-3 \pm \sqrt{9 - 4 \cdot 1 \cdot (-\frac{9}{4})}}{2}$$

$$\lambda = -\frac{3}{2}$$

En egenverdi som er reell....

Kan regne egenverdi og egenvektoren som før, men mye jobb...

$$A_a = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 0 & \frac{3}{2} \\ 0 & \frac{3}{2} & -3 \end{pmatrix}$$

$$\det A_a = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 0 & \frac{3}{2} \\ 0 & \frac{3}{2} & -3 \end{vmatrix} = 2 \cdot (-\frac{9}{4}) - 1(-3)$$

$$\neq 0, \text{ si } C \text{ er degenerert.}$$

Viljo heller $x_1 \neq 0$

$$C \cap U_1: x_0^2 + 2x_0 + 3x_2 - 3x_2^2 = 0$$

$$(x_0 + 1)^2 - 3(x_2 - \frac{1}{3})^2 = 1 - \frac{3}{9}$$

$$(x_0 + 1)^2 - 3(x_2 - \frac{1}{3})^2 = \frac{1}{3}$$

Standard form hyperbel.