

2022

1.

$$9x^2 - 18x + 4y^2 + 8y = 23$$

$$9(x^2 - 2x + 1) + 4(y^2 + 2y + 1) = 23 + 9 + 4 = 36$$

$$9(x-1)^2 + 4(y+1)^2 = 36$$

$$\frac{(x-1)^2}{4} + \frac{(y+1)^2}{9} = 1$$

2.



$$2\pi - 2 \times \frac{\pi}{3} - 2 \times \frac{3\pi}{5} = \frac{30\pi - 10\pi - 18\pi}{15} = \frac{2\pi}{15}$$

a)

$$V = \frac{4\pi}{\frac{2\pi}{15}} = \underline{\underline{30}}$$

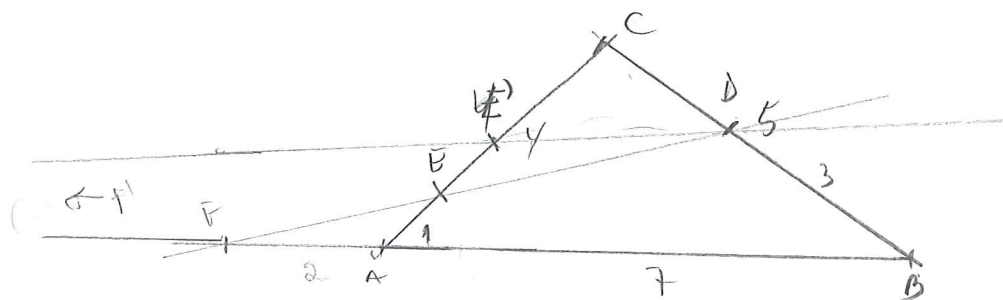
$$x - y = 1 - 2$$

b)

$$e = 30 \cdot 4 \cdot \frac{1}{2} = 60, \quad f = 2 - v + e = 2 - 30 + 60 = 32$$

$$\begin{cases} f_3 + f_5 = 32 \\ 3f_3 + 5f_5 = 2e = 120 \end{cases} \Rightarrow \begin{cases} 2f_5 = 24, f_3 = 12 \\ \Rightarrow f_3 = 20 \end{cases}$$

3.



a)

$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = \frac{-2}{9} \cdot \frac{3}{2} \cdot \frac{9}{1} = -1 \quad \text{Kollinearität von Menelaos}$$

b)

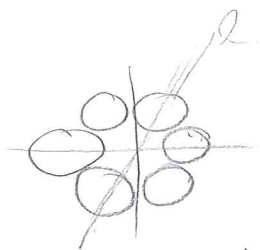
$$\frac{\overline{AF'}}{\overline{FB}} \cdot \frac{\overline{BD}}{\overline{DC}} \cdot \frac{\overline{CE'}}{\overline{EA}} = \frac{\overline{AF'}}{\overline{FB}} \cdot \frac{3}{2} \cdot \frac{2}{2} = \frac{3}{2} \frac{\overline{AF'}}{\overline{FB}} = -1 \Rightarrow \overline{AF'} = -\frac{2}{3} \overline{FB}$$

$$\overline{FB} = 7 - \overline{AF'} \Rightarrow \overline{AF'} = -\frac{2}{3}(7 - \overline{AF'})$$

$$\Rightarrow \frac{1}{3} \overline{AF'} = -\frac{14}{3} \Rightarrow \underline{\underline{\overline{AF'} = -14}}$$

4.

a)



symmetriegruppen: $G = \langle \rho_{\frac{\pi}{3}}, s \rangle$
 ρ spiegel an x-achse

b)

$$S_{\theta_1 + \theta_2} = \begin{pmatrix} \cos(\theta_1 + \theta_2) & -\sin(\theta_1 + \theta_2) \\ \sin(\theta_1 + \theta_2) & \cos(\theta_1 + \theta_2) \end{pmatrix}$$

$$= \begin{pmatrix} \cos \theta_1 & -\sin \theta_1 \\ \sin \theta_1 & \cos \theta_1 \end{pmatrix} \begin{pmatrix} \cos \theta_2 & -\sin \theta_2 \\ \sin \theta_2 & \cos \theta_2 \end{pmatrix}$$

c)

$$S_L = S_{\frac{\pi}{3}} S_{-\frac{\pi}{3}}$$

$$= \frac{1}{2} \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix} \frac{1}{2}$$

$$= \frac{1}{4} \begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} \begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} -2 & 2\sqrt{3} \\ 2\sqrt{3} & 2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

$$\det S_L = -\frac{1}{2} \cdot \frac{1}{2} - \frac{\sqrt{3}}{2} \left(+\frac{\sqrt{3}}{2} \right) = -\frac{1}{4} + \frac{3}{4} = \underline{\underline{-1}}$$

5.

$$L_{-1}: 2x_0 + x_1 - x_2 = 0$$

a)

$$L_1: 2x_0 + x_1 + x_2 = 0$$

$$P = (1; -2; 0)$$

$$L_1 \cap L_{-1} = \begin{vmatrix} i & j & h \\ 2 & 1 & -1 \\ 2 & 1 & 1 \end{vmatrix} = (2; -4; 0) = \underline{\underline{(1; -2; 0)}}$$

$$PQ = \begin{vmatrix} x_0 & x_1 & x_2 \\ 1 & -2 & 0 \\ 2 & 1 & 0 \end{vmatrix} = 5x_2 = 0 \quad \text{d.h.} \quad x_2 = 0.$$

b)

$$\text{Tangent: } 10x \frac{dx}{dt} + 10y \frac{dy}{dt} = 0$$

$$\left(\frac{2}{5}, \frac{1}{5} \right): 4 \frac{dx}{dt} + 2 \frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = -2 \quad y - \frac{1}{5} = -2 \left(x - \frac{2}{5} \right)$$

$$\Rightarrow y = -2x + 1.$$

$$\left(-\frac{2}{5}, -\frac{1}{5} \right) \quad y = -2x - 1$$

$$y - \frac{1}{5} = \frac{2}{5} \left(x - \frac{2}{5} \right) \quad y = \frac{1}{2}x \quad \text{st. fall } \frac{1}{2}$$

$$\frac{1}{2} \cdot -2 = -1$$

c)

$$5 \left(\frac{x_0}{x_2} \right)^2 + 5 \left(\frac{x_1}{x_2} \right)^2 = 1 \Rightarrow 5x_0^2 + 5x_1^2 - x_2^2 = 0$$

Schnitt mit $x_2 = 0$: Ein Punkt $x_0 = x_1 = 0$

d)

$\frac{1}{2}x_0 - x_1 = 0$ gar q' ongo. Polen er Wendepunkt Punkt normal p₀
 Polaron $\Rightarrow (2; 1; 0)$