

MAT 2500

15.11.2023

H2020

① $E^2 (x, y)$

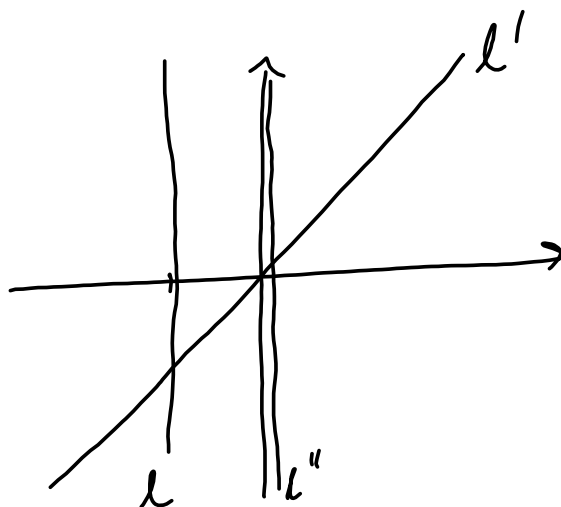
linjer

$$l: x = -1$$

$$l': y = x$$

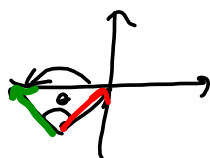
$$l'': x = 0$$

med speilinger



② (I) $S_l S_{l'} (0,0) = S_l (0,0)$ siden $(0,0) \in l'$
 $= \underline{\underline{(-2,0)}}$

- ②
- Hver speiling er or. rev. isometri, så sammensetn. er or. bev. isometri.
 - Punktet $l \cap l' = (-1, -1)$ er fikspunkt for begge speilingene og derfor for sammensetningen.
 - Da er $g = S_l S_{l'}$ or. bev. isometri med fikspunkt, som derfor er en rotasjon om $(-1, -1)$.
 - $g(0,0) = (-2,0)$ roteres om $(-1, -1)$



Vektorer: $\underline{(-2,0) - (-1,-1) = (-1,1)}$
 $\underline{(0,0) - (-1,-1) = (1,1)}$

Vinkel: $\cos \theta = \frac{(1,1) \cdot (-1,1)}{\sqrt{2} \cdot \sqrt{2}} = 0$

så $\theta = \underline{\underline{\frac{\pi}{2}}}$ eller $\underline{\underline{\frac{3\pi}{2}}}$

Rotasjon med omlopsvinkel mot klokke er $\theta = \frac{\pi}{2}$.

$\rho_{\frac{\pi}{2}, (-1, -1)} = S_l S_{l'}$

(b) (I) skal forklare at $S_{\ell''} S_{\ell} = t_{\vec{a}}$ for en $\vec{a}$.

- To or. rev. isometrier gir en or. bev. isometri
- $\ell'' \cap \ell = \emptyset$, så $S_{\ell''} S_{\ell}$ har ingen fikspunkter.
- Derfor er $S_{\ell''} S_{\ell}$ en translasjon

(II) Se hvor $(0,0)$ går

$$\left. \begin{aligned} S_{\ell''} S_{\ell} (0,0) &= S_{\ell''} (-2,0) \\ &= (2,0) \end{aligned} \right\} \text{ se dette på tegning}$$

$$\begin{aligned} t_{\vec{a}} (0,0) &= (2,0) = (0,0) + (a_1, a_2) \\ &\Rightarrow \underline{\underline{\vec{a} = (2,0)}} \end{aligned}$$

(c) (I) $\underline{t_{\vec{a}}} \underline{S} = \underline{S_{\ell''} S_{\ell} S_{\ell} S_{\ell'}} = S_{\ell''} S_{\ell'}$

id

(II) Fikspkt: $\ell'' \cap \ell' = (0,0)$

Rotasjonsvinkel:

$$\begin{aligned} S_{\ell''} S_{\ell'} (1,0) &= S_{\ell''} (0,1) \\ &= (0,1) \end{aligned}$$

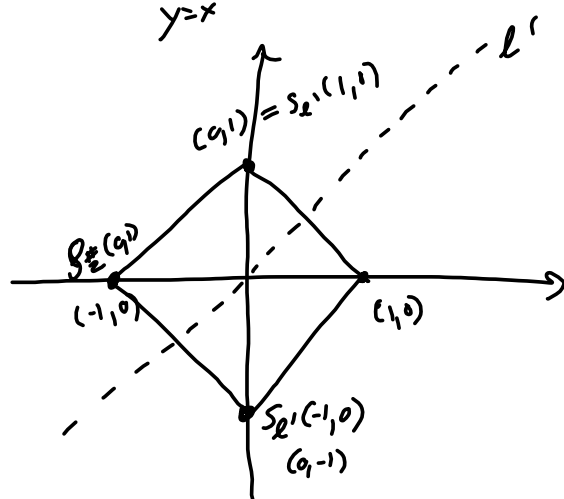


$$\cos \theta = \frac{(1,0) \cdot (0,1)}{1 \cdot 1} = 0 \quad \underline{\underline{\theta = \frac{\pi}{2}}}$$

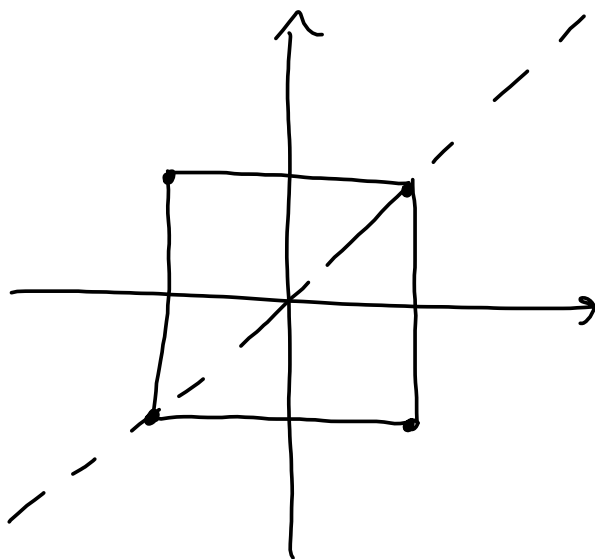
$$\underline{\underline{t_{\vec{a}} S = \rho_{\frac{\pi}{2}}}}$$

①

$s_{l'}$ og $\beta_{\frac{\pi}{2}}$ skal være en figur



$\left\{ \beta_{\frac{\pi}{2}}, \beta_{\pi}, \beta_{\frac{3\pi}{2}}, id \right\}$
 $\downarrow$
 $s_{e'}$
 $\downarrow$
 generators for D_4



③ a) $\mathbb{P}^2/\mathbb{R} \quad (x_0:x_1:x_2)$

$$A = (0:0:1) \quad L: \begin{vmatrix} x_0 & x_1 & x_2 \\ 0 & 0 & 1 \\ 0 & 3 & 2 \end{vmatrix} = 0$$

$$B = (0:3:2)$$

$$C = (1:0:1) \quad L': \begin{vmatrix} x_0 & x_1 & x_2 \\ 1 & 0 & 1 \\ 2 & 3 & 2 \end{vmatrix} = 0$$

$$D = (2:3:2)$$

$$\underline{x_0 = 0}$$

$$-3x_0 + 3x_2 = 0$$

$$\underline{x_0 - x_2 = 0}$$

$$P = L \cap L'$$

$$\vec{v}_P = \overset{L}{(1, 0, 0)} \times \overset{L'}{(1, 0, -1)}$$

$$= (0, 1, 0)$$

$$\underline{\underline{P = (0:1:0)}}$$

ALT:

$$L: x_0 = 0$$

$$L \cap L' - x_2 = 0$$

$$\underline{\underline{P = (0:1:0)}}$$

$$\textcircled{b} \quad C_t : x_0^2 - x_0 x_2 - t(2x_1^2 - 3x_1 x_2) = 0$$

$$\textcircled{I} \quad t \in \mathbb{R}$$

$$(0:0:1) = A \in C_t : 0^2 - 0 \cdot 1 - t(2 \cdot 0^2 - 3 \cdot 0 \cdot 1) = 0 \quad \underline{\text{ok}}$$

$$(0:3:2) = B \in C_t : 0^2 - 0 \cdot 2 - t(2 \cdot 3^2 - 3 \cdot 3 \cdot 2) = 0 \quad \underline{\text{ok}}$$

$$(1:0:1) = C \in C_t : 1^2 - 1 \cdot 1 - t(2 \cdot 0^2 - 3 \cdot 0 \cdot 1) = 0 \quad \underline{\text{ok}}$$

$$(2:3:2) = D \in C_t : 2^2 - 2 \cdot 2 - t(2 \cdot 3^2 - 3 \cdot 3 \cdot 2) = 0 \quad \underline{\text{ok}}$$

Alle pnt er pnt i C_t for alle $t \in \mathbb{R}$.

$$\textcircled{II} \quad P = (0:1:0) \in C_t$$

$$0^2 - 0 \cdot 0 - \hat{t}(2 \cdot 1^2 - 3 \cdot 1 \cdot 0) = 0$$

$$\underline{\underline{\hat{t} = 0}}$$

$$P \in C_0.$$

$$x_0^2 - x_0 x_2 - \underbrace{0(2x_1^2 - 3x_1 x_2)} = 0$$

$$x_0^2 - x_0 x_2 = 0$$

$$x_0(x_0 - x_2) = 0$$

$$\begin{array}{ccc} \nearrow & & \nwarrow \\ x_0 = 0 & & x_0 - x_2 = 0 \end{array}$$

$C_0 = L U L^T$: et degenerert kvadrant.

(C) $\mathbb{R}^2 \hookrightarrow \mathbb{P}^2$
 $(x, y) \mapsto (x : y : 1)$ $l_\infty : x_2 = 0$
 $\frac{x_0}{x_2} = x$
 $\frac{x_1}{x_2} = y$

(I) Se på $C_t \cap l_\infty$

$$x_0^2 - 2tx_1^2 = 0$$

$C_t \cap \mathbb{R}^2$ er ellipse når $C_t \cap l_\infty = \emptyset$.

$$\Leftrightarrow D < 0$$

$$b^2 - 4ac = 0^2 - 4 \cdot 1 \cdot (-2t) < 0$$

$$8t < 0$$

$$\underline{\underline{t < 0}}$$

Før $t < 0$ er $C_t \cap \mathbb{R}^2$ en ellipse.

(II) Se på $C_{t'}$ s.a. $(1 : 4 : 2) \in C_{t'}$

$$1^2 - 1 \cdot 2 - t'(2 \cdot 4^2 - 3 \cdot 4 \cdot 2) = 0$$

$$-1 - 8t' = 0$$

$$\underline{\underline{t' = -\frac{1}{8}}}$$

Før $t = -\frac{1}{8}$ er $(1 : 4 : 2) \in C_t$.

$t = -\frac{1}{8} < 0$, så $C_{-\frac{1}{8}}$ er en ellipse.

Skisse:

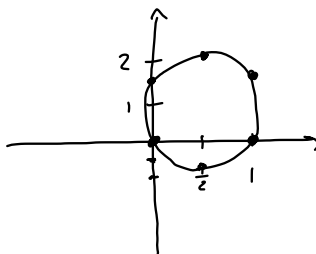
$$A = (0, 0)$$

$$B = (0, \frac{3}{2})$$

$$C = (1, 0)$$

$$D = (1, \frac{3}{2})$$

$$Q = (\frac{1}{2}, 2)$$



$$x^2 - x + \frac{1}{8}(2y^2 - 3y) = 0$$

$$(x - \frac{1}{2})^2 + \frac{1}{4}y^2 - \frac{3}{8}y = \frac{1}{4}$$

$$(x - \frac{1}{2})^2 + (\frac{1}{2}y)^2 - 2 \cdot \frac{1}{2}y \cdot \frac{3}{8} + (\frac{3}{8})^2 = \frac{1}{4} + (\frac{3}{8})^2$$

$$(x - \frac{1}{2})^2 + (\frac{1}{2}y - \frac{3}{8})^2 = \frac{25}{64}$$

$$\frac{(x - \frac{1}{2})^2}{(\frac{5}{8})^2} + \frac{(\frac{1}{2})^2(y - \frac{3}{4})^2}{(\frac{5}{8})^2} = 1$$

S

$$\underline{\underline{\frac{(x - \frac{1}{2})^2}{(\frac{5}{8})^2} + \frac{(y - \frac{3}{4})^2}{(\frac{5}{4})^2} = 1}}$$

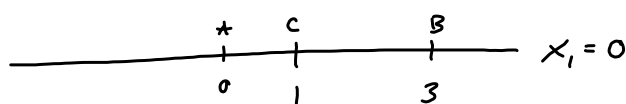
(4)

$$\begin{array}{ccc}
 \mathbb{R}^2 & & \mathbb{P}^2 \\
 x\text{-aksen } \cup \infty & \longleftrightarrow & x_1 = 0 \\
 (x, 0) & \longmapsto & (x : 0 : 1) \\
 \infty & \longmapsto & (1 : 0 : 0)
 \end{array}$$

$$l_\infty : x_2 = 0$$

(a)

$$\begin{array}{llll}
 A = (0, 0) & a = 0 & C = (1, 0) & c = 1 \\
 B = (3, 0) & b = 3 & D = (d, 0) & d = d
 \end{array}$$



(A, B) og (C, D) er harm. konj. linjer

$$\frac{c-a}{d-a} \cdot \frac{d-b}{c-b} = -1$$

$$\frac{1}{d} \cdot \frac{d-3}{-2} = -1$$

$$\frac{d-3}{d} = 2$$

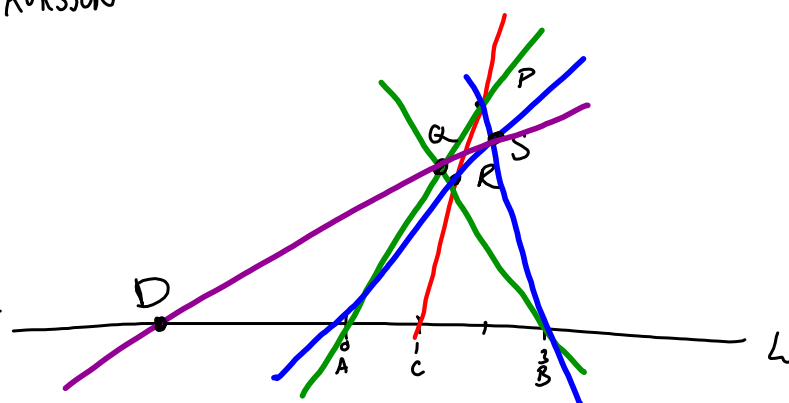
$$1 - \frac{3}{d} = 2$$

$$-\frac{3}{d} = 1$$

$$\underline{d = -3}$$

$$\underline{D = (-3, 0)}$$

KONSTRUKSJON



- ① Velg $P \notin L$ vilkårlig
- ② Velg $R \in L_{PC}$ vilkårlig
- ③ $Q = l_{AP} \cap l_{BR}$
- ④ $S = l_{PB} \cap l_{AR}$
- ⑤ $D = l_{AS} \cap L$

$$\begin{aligned} A = (0, 0) &= (0:0:1) & C = (1, 0) &= (1:0:1) \\ B = \infty &= (1:0:0) & D = (d, 0) &= (d:0:1) \end{aligned}$$

Projektwa :

$$C = pA + qB$$

$$(1:0:1) = p(0:0:1) + q(1:0:0) \quad \underline{p=q=1}$$

$$D = rA + sB \quad \text{or} \quad \frac{q_r}{p_s} = -1 \quad \text{gr } (AB; GD) \text{ harm. konj.}$$

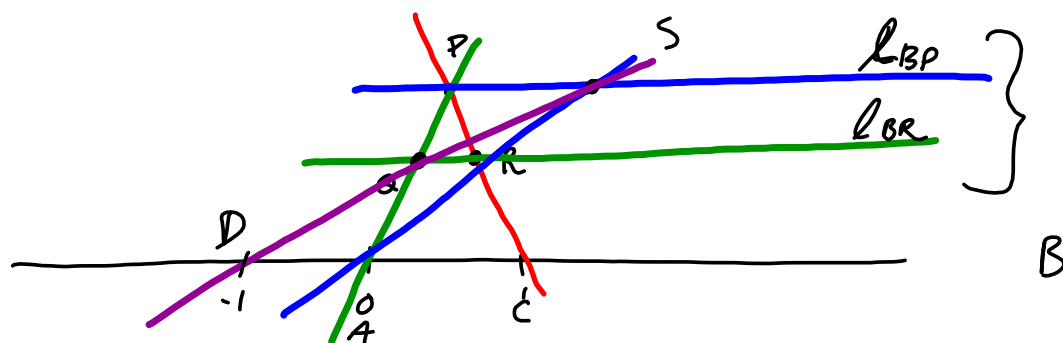
$$r = -5$$

↪ $D = -sA + sB$

$$\begin{aligned}(\underline{d}:0:\underline{1}) &= -s(0:0:1) + s(1:0:0) \\ &= s(1:0:-1) \\ &= (\underline{-1}:0:\underline{1})\end{aligned}$$

$$\underline{d = -1}$$

$$D = (-1, 0)$$



(1 a)

$$S_L \quad l: x = -1$$

$$= t_{(-1,0)} S_{L''} \quad \text{der } L'' : x = 0$$

$$S_{L''}(1,0) = (-1,0)$$

$$S_{L''}(0,1) = (0,1)$$

$$M_{L''} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$S_L \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} -x-1 \\ y \end{bmatrix}$$
