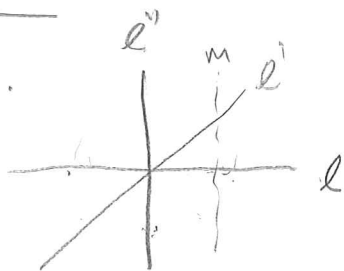


2018

1.



a) $S_{l''} \circ S_l$ rotation (sette sammen to refl. gir rotation)

Fix pt.: $l'' \cap l = (0,0)$

Vinkel $\angle(l, l'') = \frac{\pi}{2} \Rightarrow \text{Rot. vinkel} = \underline{\underline{\pi}}$

$S_{l'} \circ S_{l''} \circ S_l$ (tre speilinger gir speiling)

$= S_{l'} \circ S_{\pi}$

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ speiling om $y = -x$.

b)

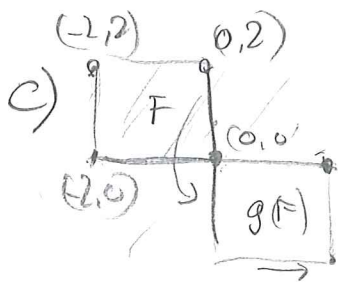
m: $x=1$

$g = S_m \circ S_{\pi}$

$= \frac{1}{2} S_{(1,0)} \frac{1}{2} S_{\pi}$

$$g \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \left(\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} \right) + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$



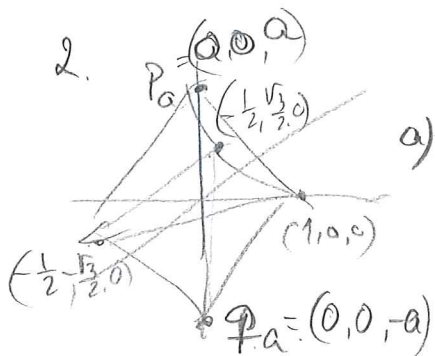
$$g \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad g \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$g \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} \quad g \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$$

Speiling om x-aksen
Speiling om y-aksen
glidespeiling

Symmetri: $S_{\pi}, S_{l'}, S_{l'} \circ S_{\pi}, \text{id}$

2.



$$a) \quad \|(1,0,0) - (-\frac{1}{2}, \frac{\sqrt{3}}{2}, 0)\| = \sqrt{(\frac{3}{2})^2 + (\frac{\sqrt{3}}{2})^2 + 0^2} = \sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{3}$$

$$\|(0,0,a) - (1,0,0)\| = \sqrt{1 + a^2} = \sqrt{3}$$

$$\text{dvs } a^2 + 1 = 3 \Rightarrow a^2 = 2$$

$$\Rightarrow \underline{\underline{a = \sqrt{2}}}$$

Ikke regulært fordi hjørner

ikke har samme konfigurasjon (3 eller 4).

b)

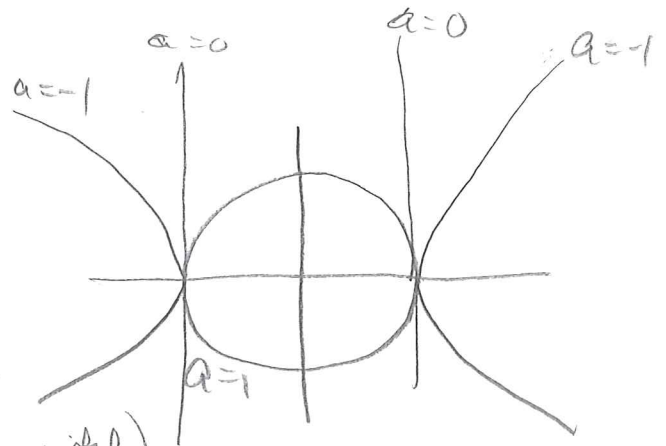
Symmetri generert av rotasjoner med vinkel $\frac{2\pi}{3}$ i xy-planen og rotasjoner med vinkel π i xz-planen

Samme som symmetri til trekant

$$(1,0,0) \quad (-\frac{1}{2}, \frac{\sqrt{3}}{2}, 0) \quad (-\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0)$$

3.

$$Q_a: X_0^2 + aX_1^2 - X_2^2 = 0 \quad a = -1$$



$$a) \quad Q_1 \cap (X_2=1) = (X_1^2 + Y^2 = 1)$$

$$Q_0 \cap (X_2=1) = (X^2 = 1)$$

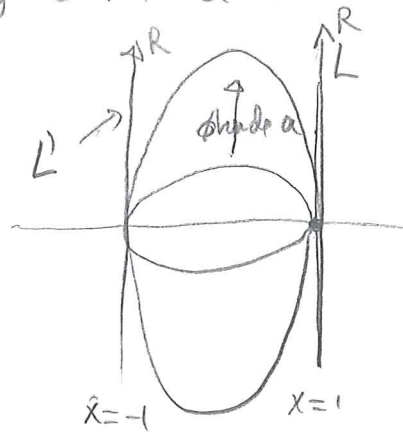
$$Q_{-1} \cap (X_2=1) = (X^2 - Y^2 = 1)$$

b) $a > 0$: Ellipse ($a=1$ degenerate)

$a=0$: Two lines

$a < 0$: Hyperbola

c) Degenerate: $a=0$: $X_0^2 - X_2^2 = (X_0 - X_2)(X_0 + X_2) = 0$
Two lines in $\mathbb{P}^2$



$$d) \quad P = (1:0:1) \quad Q = (1:1:1) \quad L = PQ = \begin{vmatrix} X_0 & X_1 & X_2 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -X_0 + X_2 = 0$$

$$L \cap Q_a : \begin{cases} X_0^2 + aX_1^2 - X_2^2 = 0 \\ -X_0 + X_2 = 0 \end{cases}$$

$$\Rightarrow aX_1^2 = 0 \quad \text{betr. } \underline{X_1=0} \text{ for } a \neq 0$$

$$e) \quad R = L \cap (X_2=0)$$

$$R \in L', \quad L' \cap Q_a = \text{eff. pt.}$$

$$G.H. \quad L' : pX_0 + qX_1 + rX_2 = 0$$

$$R \in L' : q=0 \Rightarrow L' : pX_0 + rX_2 = 0$$

$$L' \cap Q_a : p=0 \quad X_0^2 + aX_1^2 = 0 \quad \text{Two points for } a < 0$$

$$p \neq 0 \quad \left(\frac{r}{p} - 1\right)X_2^2 + aX_1^2 = 0 \quad \text{Eff. point for } r = p \quad (1:0:1)$$

$$R = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \bar{j} = (0:1:0)$$