

MAT 2500 22.11.2023

2017

③ $\mathbb{P}^2$

① $l_1: 2x_0 - x_1 + x_2 = 0$

$l_2: x_0 - 3x_1 + 4x_2 = 0$

$$\vec{v}_P = \begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 1 & -3 & 4 \end{vmatrix} = (2, -1, 1) \times (1, -3, 4)$$

$$= i(-1 \cdot 4 - (-3) \cdot 1) - j(2 \cdot 4 - 1 \cdot 1) + k(2 \cdot (-3) - 1 \cdot (-1))$$

$$= -i - 7j - 5k$$

$$= (-1, -7, -5)$$

SJEKK!

$$l_1(P): 2 \cdot 1 - 7 + 5 = 0 \checkmark$$

$$l_2(P): 1 - 3 \cdot 7 + 4 \cdot 5 = 0 \checkmark$$

$$P = (1:7:5)$$

② $Q = (0:0:1)$

$$l_{PQ}: \begin{vmatrix} x_0 & x_1 & x_2 \\ 1 & 7 & 5 \\ 0 & 0 & 1 \end{vmatrix} = 0$$

$$7x_0 - x_1 = 0$$

SJEKK!

$$P \in l_{PQ} \quad 7 \cdot 1 - 7 = 0$$

$$Q \in l_{PQ} \quad 7 \cdot 0 - 0 = 0$$

b)

$$C_a: x_0^2 + 2x_0x_1 - 4x_1^2 + ax_1x_2 = 0$$

$$l_b: 5bx_1 + x_2 = 0$$

$(a, b) \in \mathbb{R}^2$ parametre.

① Skal finne hvor $C_a \cap l_b \neq \emptyset$ ($\mathbb{R}$)
dvs. der $D \geq 0$

$$l_b: x_2 = -5bx_1$$

$C_a \cap l_b:$

$$x_0^2 + 2x_0x_1 - 4x_1^2 + ax_1(-5bx_1) = 0$$

$$x_0^2 + 2x_0x_1 - (4 + 5ab)x_1^2 = 0$$

2. gradslikning - homogen i 2. variable
- delvis med 2. gradslikn i 1. variabel

$$x_1 = 1 \quad x_0 = x$$

$$x^2 + 2x - (4 + 5ab) = 0$$

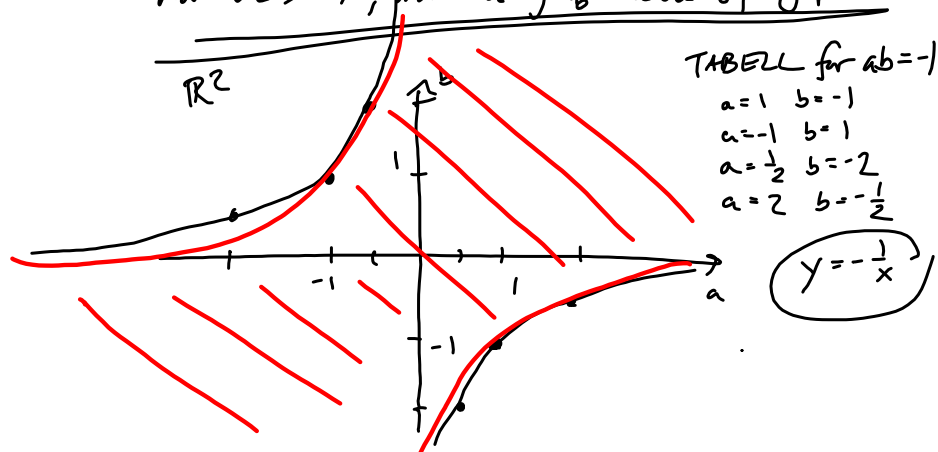
$$D = "b^2 - 4ac" = 2^2 - 4 \cdot 1 \cdot (-1)(4 + 5ab) \geq 0$$

$$4 + 20ab + 16 \geq 0$$

$$20ab \geq -20$$

$$ab \geq -1$$

Når $ab \geq -1$, har C_a og l_b reelle skjæringspunkt.



• er området vi er ute etter!

c)

Når er l_b tangent til C_a ?

• Når $D = 0$

$$ab = -1$$

④

a)

$$C: x^2 + y^2 = 1$$

 (x_s, y_s)

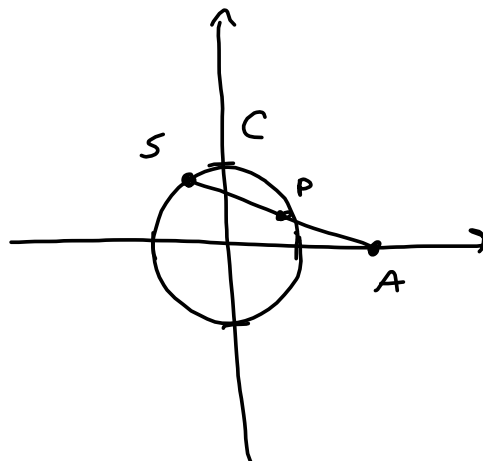
$$A = (2, 0)$$

 $s \in C$
 P midtpunkt SA .

"

 (x_p, y_p)

- $y_p = \frac{1}{2} y_s$
- $x_p = \frac{1}{2} (2 + x_s)$



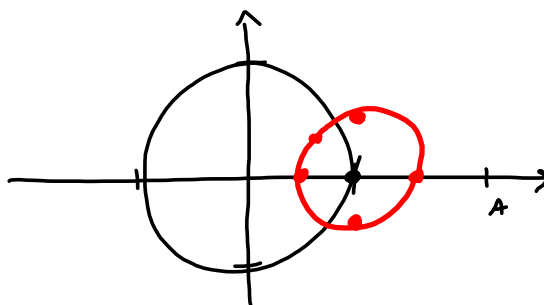
$$\left. \begin{array}{l} \bullet y_s = 2y_p \\ \bullet x_s = 2x_p - 2 \end{array} \right\} s \in C: x_s^2 + y_s^2 = 1$$

$$(2x_p - 2)^2 + (2y_p)^2 = 1$$

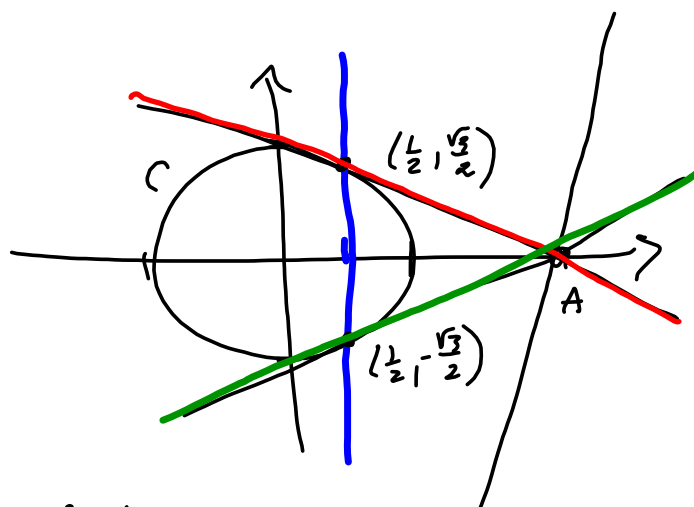
$$2^2(x_p - 1)^2 + 2^2 y_p^2 = 1 \quad | : 2^2$$

$$(x_p - 1)^2 + y_p^2 = \left(\frac{1}{2}\right)^2$$

Det geometriske stedet
til P er en sirkel med radius $\frac{1}{2}$
og sentrum $(1, 0)$



(b) Skal finne tangentene til C som går gjennom A .



$$C: x^2 + y^2 = 1$$

$$P_A C: a_x \cdot x + a_y \cdot y = 1$$

$$2x = 1$$

$$x = \frac{1}{2}$$

$$y = \pm \sqrt{1 - \left(\frac{1}{2}\right)^2} \quad (\text{sett inn i } C)$$

$$y = \pm \frac{\sqrt{3}}{2}$$

$$P_1 = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \quad P_2 = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

$$T_1 C: \frac{1}{2} \cdot x + \frac{\sqrt{3}}{2} \cdot y = 1$$

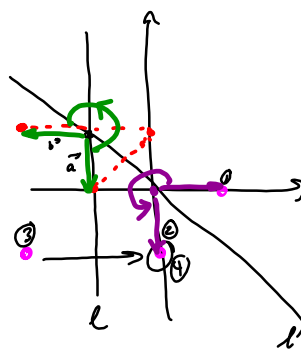
$$y = \frac{\sqrt{3}}{3}(2-x)$$

$$T_2 C: \frac{1}{2}x - \frac{\sqrt{3}}{2}y = 1$$

$$y = \frac{\sqrt{3}}{2}(x-2)$$

Kun utledes
for P_2 ...

① a) $E^2 \quad (x, y)$
 $l: x+1=0$
 $l': x+y=0$



$$g = S_l \circ S_{l'}$$

$$l \cap l' = P = (-1, 1) \quad \text{f.p.}$$

$$g(-1, 0) = (-2, 1)$$

$$\vec{a} = (-1, 0) - (-1, 1) = (0, -1)$$

$$\vec{b} = (-1, 0)$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|} = 0$$

$$\theta = \frac{\pi}{2} \text{ eller } \frac{3\pi}{2}$$

$$S_l \circ S_{l'} = \underline{\underline{g_{\frac{3\pi}{2}}(-1, 1)}}$$

② b) $t = t_{(2,0)}$

$$g' = t_{(2,0)} \circ S_l \circ S_{l'}$$

• (or. bevar. og 2 or. rer $\Rightarrow$ or. bevar. isometri)

Spiller om
origo er
firkantpunkt

$$\bullet g'(0, 0) = t_{(2,0)} \circ S_l \circ S_{l'}(0, 0)$$

$$= t_{(2,0)} \circ S_l(0, 0)$$

$$= t_{(2,0)}(-2, 0)$$

$$= (-2, 0) + (2, 0)$$

$$= \underline{(0, 0)}, \text{ s\u00e5 } (0, 0) \text{ er firkantpunkt.}$$

S\u00e5 g' er or. bevar. isometri med firkantpunkt origo, s\u00e5 er
rotasjon om origo.

Se hvor $(1, 0)$ g\u00e5r:

$$g'(1, 0) = t_{(2,0)} \circ S_l \circ S_{l'}(1, 0)$$

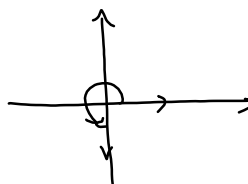
$$= t_{(2,0)} \circ S_l(0, -1)$$

$$= t_{(2,0)}(-2, -1)$$

$$= (0, -1)$$

$$\vec{a} = (1, 0)$$

$$\vec{b} = (0, -1)$$

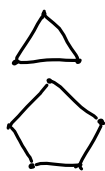


$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|} = 0$$

$$\theta = \frac{\pi}{2} \text{ eller } \frac{3\pi}{2}$$

$$g' = \underline{\underline{g_{\frac{3\pi}{2}}}}$$

H22 - (2)

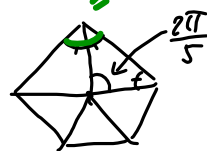
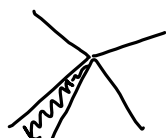


1 hvert hjørne notet
2 trekanter og 2 femkanter

Euler formel $2 = v - e + f$

Summere over $2e = \sum_v \deg v = 4v = \sum_f \deg f$

Descartes formel $D = 4\pi$



For et hjørne i P er $d_v = 2\pi - (2 \cdot \frac{\pi}{3} + 2 \cdot \frac{3\pi}{5})$
 $= 2\pi - (\frac{16\pi}{15} + \frac{18\pi}{15})$

$\frac{D}{\frac{2\pi}{15} \cdot v} = 4\pi$

$= \frac{2\pi}{15}$

$v = 30$

(b)

$2e = 4v = 120$

$e = 60$

$f - e + v = 2$

$f = 2 - 30 + 60$

$= 32$

$\begin{cases} f = T + F = 32 \end{cases}$

$\begin{cases} 3 \cdot T + 5 \cdot F = 2e = 120 \end{cases}$

$\begin{bmatrix} 1 & 1 & 32 \\ 3 & 5 & 120 \end{bmatrix} \xrightarrow{(-3)} \sim \begin{bmatrix} 1 & 1 & 32 \\ 0 & 2 & 24 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 1 & 32 \\ 0 & 1 & 12 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 20 \\ 0 & 1 & 12 \end{bmatrix}$

12 femkanter og 20 trekanter