

17. august 2026

1.

Hva er geometri?

(Jord-måling)

Nære stikkord:

Punkter, linjer, sirkler

Aksiomatisk plangeometri

Euklidisk geometri

Sfærisk / hyperbolisk geometri

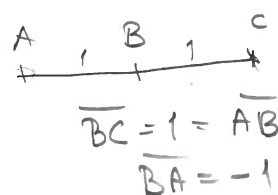
Projektiv geometri

Kap. 4.

Linjestykke AB

Lengde AB

Lengde med fortegn $\overline{AB}$



Kolineære punkter



$$\underline{AC = AB + BC}$$

Delingsforhold

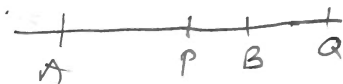


$$\frac{\overline{AP}}{\overline{PB}}$$

Lemma

P på AB er entydig bestemt av $\frac{\overline{AP}}{\overline{PB}}$

Bevis Anta P, Q oppfyller $\frac{\overline{AP}}{\overline{PB}} = \frac{\overline{AQ}}{\overline{QB}}$



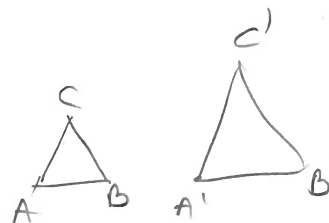
$$\Rightarrow \frac{\overline{AB}}{\overline{PB}} = \frac{\overline{AP}}{\overline{PB}} + \frac{\overline{PB}}{\overline{PB}} = \frac{\overline{AQ}}{\overline{QB}} + \frac{\overline{QB}}{\overline{QB}} = \frac{\overline{AB}}{\overline{QB}}$$

$$\Rightarrow \overline{PB} = \overline{QB} \Rightarrow \underline{P=Q}$$

Liketrukkede trekanter:

Parvis samme vinkler

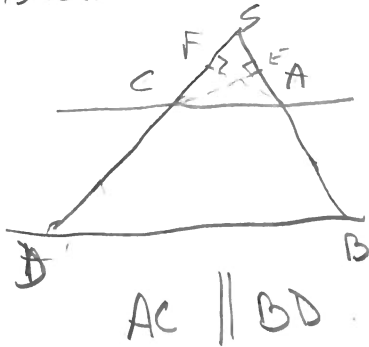
eller $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{AC}{A'C'}$



Kongruente Dreiecke:

- Liniennote
- Same Stempel
- Kann auch rotiert, gespiegelt, translatiert

Transversaltheorem:



$$\Rightarrow \triangle DBS \sim \triangle CAS$$

Beis:

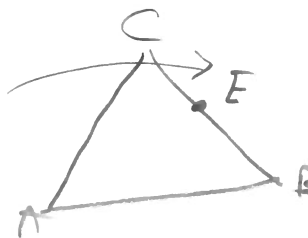
$$\frac{SCA}{CDA} = \frac{SCA}{CBA} \quad \frac{SCA}{SDA} = \frac{SCA}{SCB}$$

$$\frac{\frac{SC \cdot AF}{2}}{\frac{CD \cdot AF}{2}} = \frac{\frac{SA \cdot EC}{2}}{\frac{AB \cdot EC}{2}} \quad \frac{\frac{SC \cdot AF}{2}}{\frac{SD \cdot AF}{2}} = \frac{\frac{SA \cdot EC}{2}}{\frac{SB \cdot EC}{2}}$$

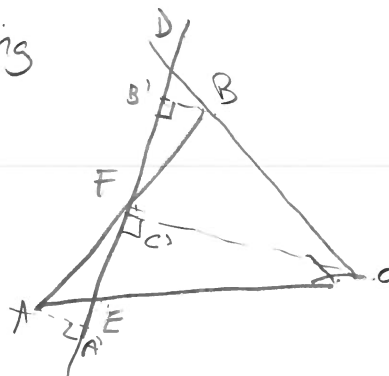
$$\frac{SC}{CD} = \frac{SA}{AB} \quad \frac{SC}{SD} = \frac{SA}{SB}$$

Menelaos:

Menelaos punkter



Menelaos setzung



E, F, D kollinear

$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = -1$$

$$\triangle DBB' \sim \triangle DCC'$$

$$\triangle AEA' \sim \triangle CEC'$$

$$\triangle AFA' \sim \triangle BFB'$$

$$\Rightarrow \frac{DB}{BB'} = \frac{DC}{CC'} \quad \frac{AE}{AA'} = \frac{CE}{CC'} \quad \frac{AF}{AA'} = \frac{BF}{BB'}$$

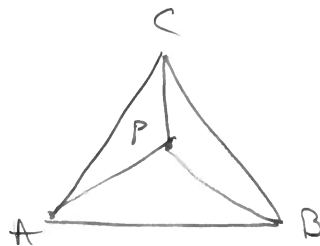
$$\Rightarrow 1 = \frac{DB}{BB'} \cdot \frac{CE}{DC} \cdot \frac{CE}{CC'} \cdot \frac{AA'}{AE} \cdot \frac{BB' \cdot AF}{AA' \cdot BF} = \frac{DB}{DC} \cdot \frac{CE}{AE} \cdot \frac{AF}{BF}$$

Med retninger blir det -1

Fordi ett av punktene (eller tre)
må ligge utenfor trekanten

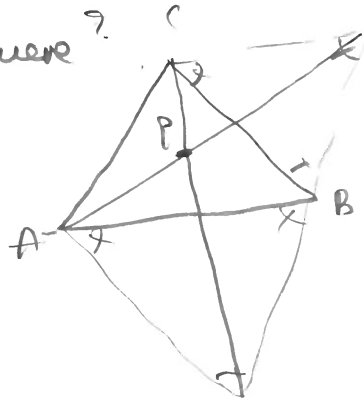
Toricelli-Fermat

Fermat punkt:



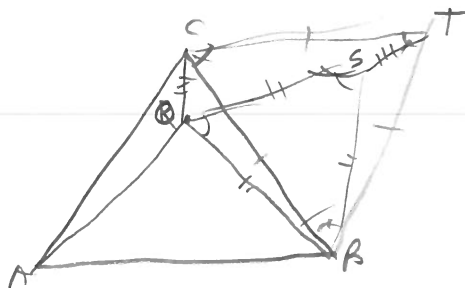
$AP + BP + CP$ minst mulig.

Hvordan konstruere?



Hvorfor minimum?

Q vilkårlig



$$AQ + BQ + CQ = AQ + QS + ST$$

Kortest vei er rett linje