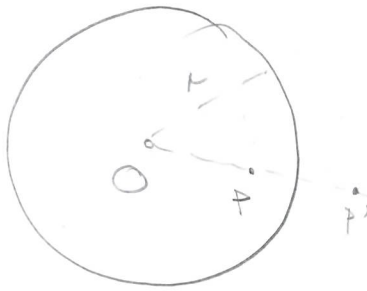


Torsdag 20. august



Inversjon over sirkel

$$OP \cdot OP' = r^2$$

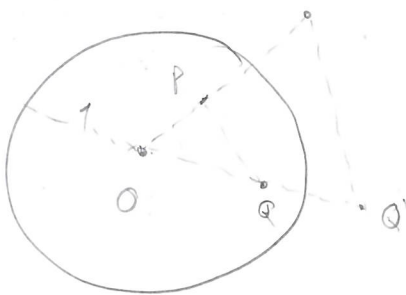
(kan anta $r=1$)

O, P, P' kolinear

$$P^{-1} = P', \quad P'^{-1} = P$$

Invers.

Invertere trekanter



$$\triangle OPQ \sim \triangle OQ'P'$$

- Felles vinkel

$$\frac{OP}{OQ} = \frac{\frac{1}{OP'}}{\frac{1}{OQ'}} = \frac{OQ'}{OP'}$$

$$\Rightarrow P'Q' = PQ \cdot \frac{OP'}{OQ}$$

Invertere sirkel (brukes komplekse tall)



Sirkel med radius r og senter s

Sirkel med radius $\frac{r}{s^2 - r^2}$ og senter $\frac{s}{s^2 - r^2}$

$$\|(x, y) - (s, 0)\|^2 = (x - s)^2 + y^2 = x^2 + y^2 - 2xs + s^2 = r^2$$

$$\left\| \frac{(x, y)}{x^2 + y^2} - \left(\frac{s}{s^2 - r^2}, 0 \right) \right\|^2 = \left(\frac{x}{x^2 + y^2} - \frac{s}{s^2 - r^2} \right)^2 + \left(\frac{y}{x^2 + y^2} \right)^2$$

$$Z = (x, y) \text{ la } t = s^2 - r^2$$

$$\|Z - s\|^2 = \|Z\|^2 - 2Zs + s^2 = r^2$$

$$\Rightarrow 2Zs = \|Z\|^2 + t$$

$$\left\| \frac{Z}{\|Z\|^2} - \frac{s}{t} \right\|^2 = \frac{1}{\|Z\|^2} - \frac{2Zs}{\|Z\|^2 t} + \frac{s^2}{t^2}$$

$$= \frac{1 \cdot t}{\|Z\|^2 t} - \frac{\|Z\|^2 + t}{\|Z\|^2 t} + \frac{s^2}{t^2}$$

$$= -\frac{\|Z\|^2}{\|Z\|^2 t} + \frac{s^2}{t^2} = -\frac{t}{t^2} + \frac{s^2}{t^2}$$

$$= \frac{r^2}{t^2}$$

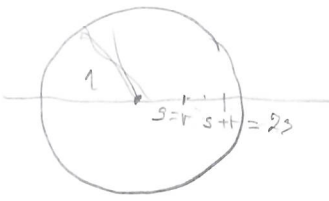
$$= \frac{x^2}{(x^2 + y^2)^2} - \frac{2xs}{(x^2 + y^2)(s^2 - r^2)} + \frac{s^2}{(s^2 - r^2)^2} + \frac{y^2}{(x^2 + y^2)^2}$$

$$= \frac{1}{x^2 + y^2} - \frac{x^2 + y^2 + s^2 - r^2}{(x^2 + y^2)(s^2 - r^2)} + \frac{s^2}{(s^2 - r^2)^2}$$

$$= \frac{s^2 - r^2 - x^2 - y^2 - s^2 + r^2}{(x^2 + y^2)(s^2 - r^2)} + \frac{s^2}{(s^2 - r^2)^2}$$

$$\Rightarrow \frac{s^2}{(s^2 - r^2)^2} - \frac{1}{s^2 - r^2} = \frac{s^2 - s^2 + r^2}{(s^2 - r^2)^2} = \frac{r^2}{(s^2 - r^2)^2}$$

Det følger:



$$r = s \Rightarrow$$

Invers av sirkel gjennom O
er sirkel med sentrum i $\frac{1}{O}$
og radius $\frac{1}{O}$ som går gjennom
punktet $(\frac{1}{2s}, 0)$ og normalt på x-aksen

$\Rightarrow$

Sirkel, ikke gjennom origo
Sirkel gjennom origo
linje

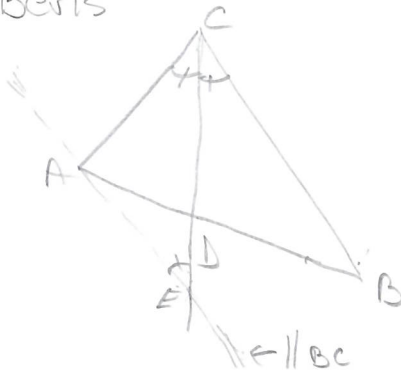
Sirkel, ikke gjennom origo
lett linje
Sirkel gjennom origo

Halveringslinje-sekningen



$$\frac{AD}{DB} = \frac{AC}{BC}$$

Beris



$$\triangle ADE \sim \triangle BDC$$

$$\Rightarrow \frac{AD}{BD} = \frac{AE}{BC}$$

$$\angle DCB = \angle AED = \angle DCA$$

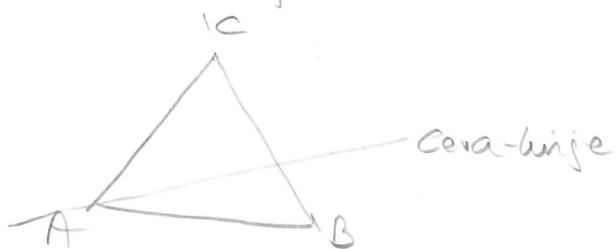
$$\Rightarrow AE = AC$$

$$\Rightarrow \frac{AD}{BD} = \frac{AC}{BC}$$

Tilsvarende motsatt ver

QED

Ceva-linje



Menelaos ca 70-140

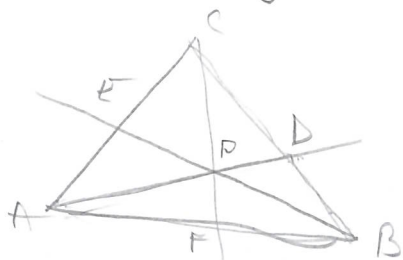
gresk, fra Alexandria
(døde i Roma)

Matematiker og astronom

Giovanni Ceva 1647 Milano
- 1734 Mantova

Matematiker og ingeniør

Cevas sætning



AD, BE, CF gennem fælles punkt



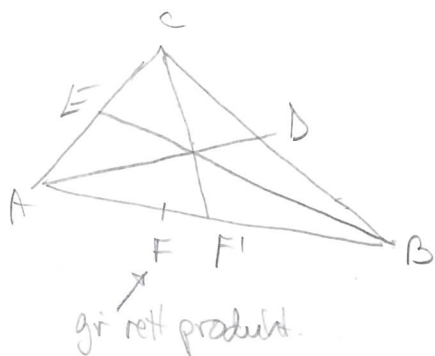
$$\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = 1$$

Beweis: Menelaos på $\triangle ABD$ og $\triangle ADC$

$$\frac{\overline{BC}}{\overline{CD}} \cdot \frac{\overline{DP}}{\overline{PA}} \cdot \frac{\overline{AF}}{\overline{FB}} = \frac{\overline{DB}}{\overline{BC}} \cdot \frac{\overline{CE}}{\overline{EA}} \cdot \frac{\overline{AP}}{\overline{PD}} = -1$$

$$\Rightarrow \frac{\overline{BC}}{\overline{CD}} \cdot \frac{\overline{DP}}{\overline{PA}} \cdot \frac{\overline{AF}}{\overline{FB}} = \frac{\overline{DB}}{\overline{BC}} \cdot \frac{\overline{CE}}{\overline{EA}} \cdot \frac{\overline{AP}}{\overline{PD}} = \frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = 1$$

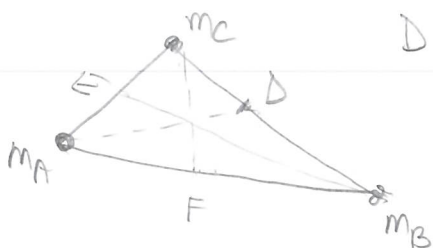
og modsat vei:



$$\Rightarrow \frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = \frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF'}{F'B} = 1$$

$$\Rightarrow \frac{AF}{FB} = \frac{AF'}{F'B} \Rightarrow F = F'$$

Cevas mekaniske bevis



D: tyngdepunkt for B og C

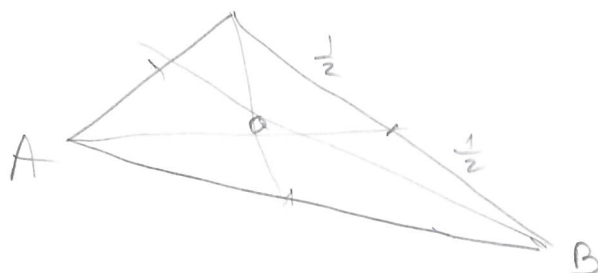
$$CD \cdot m_C = BD \cdot m_B$$

$$AF \cdot m_A = BF \cdot m_B$$

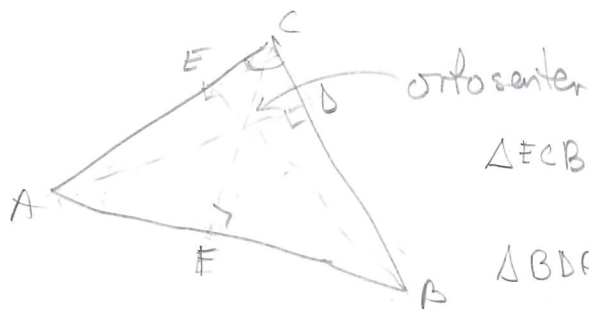
$$AE \cdot m_A = EC \cdot m_C$$

$$1 = \frac{m_C}{m_B} \cdot \frac{m_A}{m_C} \cdot \frac{m_B}{m_A} = \frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB}$$

Anvendelser av Ceva



Medianene går gjennom
ett felles punkt (tyngdepunkt)

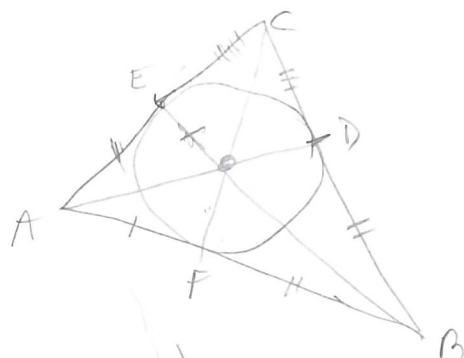
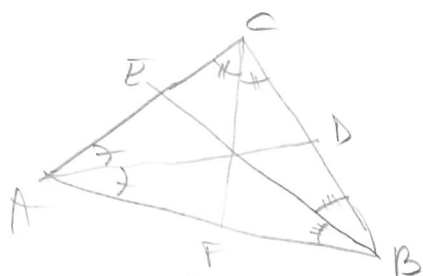


$$\triangle ECB \sim \triangle DCA \Rightarrow \frac{CE}{DC} = \frac{BE}{AD}$$

$$\triangle BDA \sim \triangle BEC \Rightarrow \frac{BD}{BE} = \frac{AD}{EC}$$

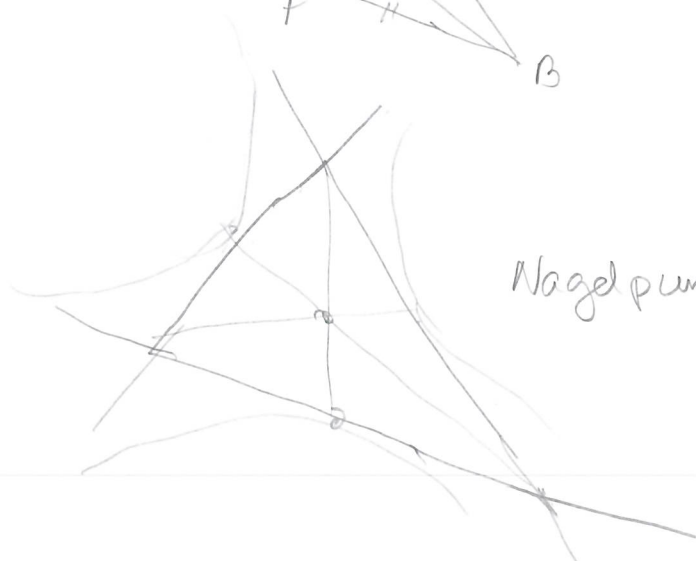
$$\triangle AFC \sim \triangle AEB \Rightarrow \frac{AF}{AE} = \frac{CF}{BE}$$

Halveringslinjer møtes i ett punkt



Gergonne punkt

Joseph Diez Gergonne
1771-1859



Nagel punkt

Christian Heinrich von Nagel
1803-1852