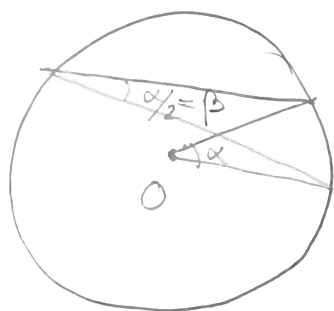


Mandag 24 august

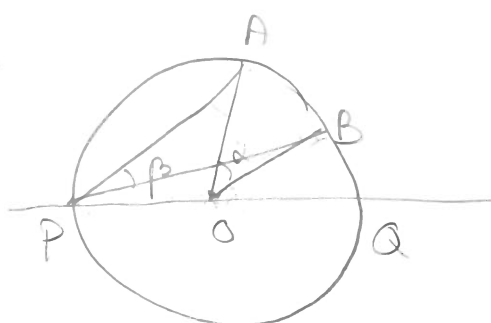
Central-periferi-setninger



α sentralvinkel; sentrum = toppunkt
 β periferivinkel; toppunkt p.c.
 periferi

$$\underline{\underline{\alpha = 2\beta}}$$

Beris



$$\angle AOQ = \pi - \angle POA$$

$$\angle BOQ = \pi - \angle POB$$

$$\begin{aligned} \Rightarrow \alpha = \angle AOB &= \angle AOQ - \angle BOQ \\ &= \pi - \angle POA - \pi + \angle POB \\ &= \angle POB - \angle POA \end{aligned}$$

$$PO = OA \Rightarrow \angle APO = \angle PAO \quad \text{og} \quad \angle BPO = \angle PBO$$

$$= OB \Rightarrow \angle APO + \angle POA + \angle PAO = \pi$$

$$\angle BPO + \angle POB + \angle PBO = \pi$$

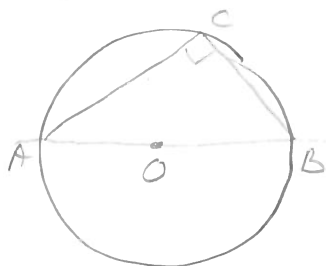
$$\Rightarrow \angle POA = \pi - 2\angle APO$$

$$\angle POB = \pi - 2\angle BPO$$

$$\text{dis } \alpha = \pi - 2\angle BPO - \pi + 2\angle APO = 2(\angle APO - \angle BPO) = 2\beta$$

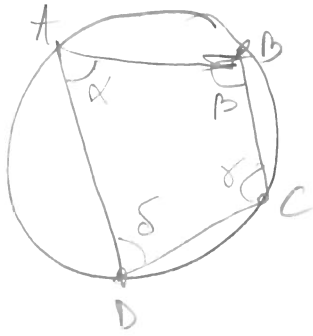
QED

Spesialtilfelle: Thales setning.



Buen AOB er π

$$\Rightarrow \angle ACB = \frac{\pi}{2}$$



$$\alpha = \frac{1}{2} \text{Bue}(\text{BCD})$$

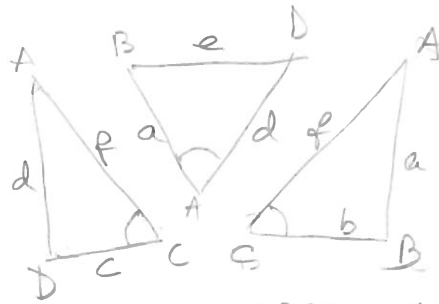
$$\gamma = \frac{1}{2} \text{Bue}(\text{BAD})$$

$$\Rightarrow \alpha + \gamma = \frac{1}{2} \text{Bue}(\text{BCDAB}) = \frac{1}{2} \cdot 2\pi = \pi$$

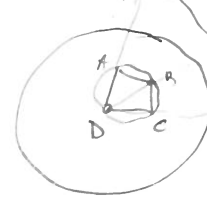
$$\Rightarrow \beta + \delta = \pi$$

$$\begin{aligned} A'B' &= AB \cdot \frac{DB'}{DA} \cdot \frac{DA \cdot DC}{DB} \\ B'C' &= BC \cdot \frac{DB'}{DC} \cdot \frac{DC}{DB} \\ A'C' &= AC \cdot \frac{DC}{DA} \cdot \frac{DA \cdot DB}{DB' \cdot DC} \end{aligned}$$

Ptolemais theorem



Berth. A' ved inversiön



$$\begin{aligned} A'B' &= AB \cdot \frac{DB'}{DA} \cdot \frac{DA \cdot DC}{DB} \\ B'C' &= BC \cdot \frac{DB'}{DC} \cdot \frac{DC}{DB} \\ A'C' &= AC \cdot \frac{DC}{DA} \cdot \frac{DA \cdot DB}{DB' \cdot DC} \end{aligned}$$

$$\angle DCA + \angle BAD + \angle BCA = \pi$$

$$\angle DAE + \angle ABD + \angle BDA + \angle CAB =$$

$$= \frac{1}{2} (\angle DAC + \angle DBC) + (\angle ABD + \angle ACD) + (\angle BDA + \angle BCA) + (\angle CAB + \angle CDB)$$

$$= \frac{1}{2} \cdot 2\pi = \pi$$

Parallelogram: $BD = e$ $DC + CB = c \cdot \frac{a}{f} + b \cdot \frac{d}{f}$

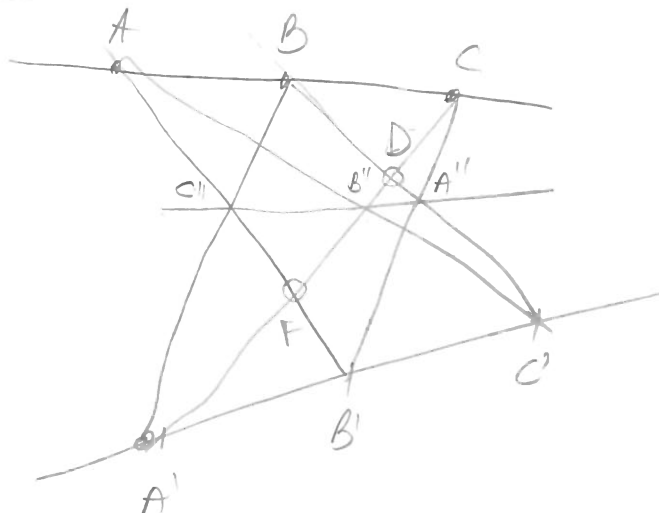
E

$$\Rightarrow ef = ac + bd$$

$$\text{dvs } BD \cdot AC = AB \cdot CD + BC \cdot AD$$

QED

Pappos sätning



Beweis Brucher Menelaos $p \in \Delta DEF$

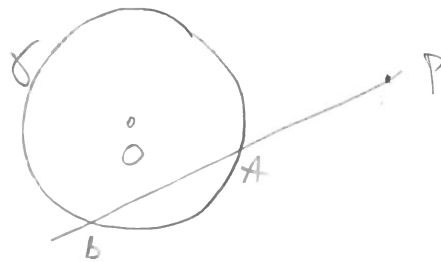
$$(A, B, C) \quad \frac{DC}{CF} \cdot \frac{FA}{AE} \cdot \frac{EB}{BD} = 1 \quad \frac{DC}{CE} \cdot \frac{EB}{BF} \cdot \frac{FA}{AD} = 1 \quad (A', B', C')$$

$$(A'', B'', C'') \quad \frac{DA''}{A''E} \cdot \frac{EB''}{B''F} \cdot \frac{FC}{CD} = 1 \quad \frac{DC}{CE} \cdot \frac{EA}{AF} \cdot \frac{FB''}{B''D} = 1 \quad (A, B'', C'')$$

$$(A'BC'') \quad \frac{DB}{BE} \cdot \frac{EC''}{C''F} \cdot \frac{FA'}{A'D} = 1$$

$$\begin{aligned} g. \quad & \frac{DC}{CF} \cdot \frac{FA}{AE} \cdot \frac{EB}{BD} \cdot \frac{DC}{CE} \cdot \frac{EB}{BF} \cdot \frac{FA}{AD} \cdot \frac{DA''}{A''E} \cdot \frac{EB''}{B''F} \cdot \frac{FC}{CD} \cdot \frac{DC}{CE} \cdot \frac{EA}{AF} \cdot \frac{FB''}{B''D} \cdot \frac{DB}{BE} \cdot \frac{EC''}{C''F} \cdot \frac{FA'}{A'D} \\ & = \frac{FB''}{B''D} \cdot \frac{DA''}{A''E} \cdot \frac{EC''}{C''F} = 1 \Leftrightarrow A'', B'', C'' \text{ kolinear} \end{aligned}$$

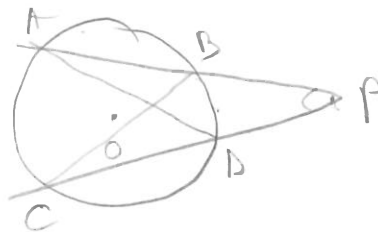
Punkts potens



P's potens mht. Sirkel γ

$$PA \cdot PB$$

Setzung



$$PA \cdot PB = PC \cdot PD$$

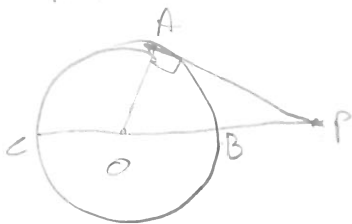
Beweis $\triangle CBP \sim \triangle ADP \Rightarrow$

$$\frac{CP}{AP} = \frac{BP}{DP}$$

$$\Rightarrow AP \cdot BP = CP \cdot DP$$

QED

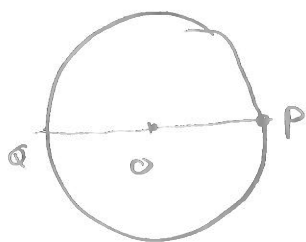
Spezialhilfe



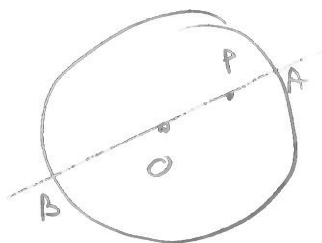
$$AP^2 = PC \cdot PB$$

$$= (PO + OA)(PO - OA) = PO^2 - OA^2$$

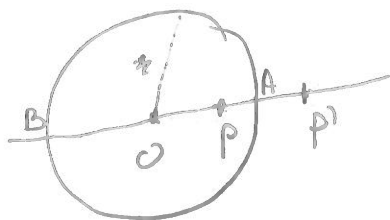
$$\text{d.h.} \quad \underline{AP^2 + OA^2 = OP^2}$$



Potens: $\overline{PP} \cdot \overline{PQ} = 0$
(P på periferin)



Potens $\overline{PA} \cdot \overline{PB} < 0$
(P inuti cirkeln)

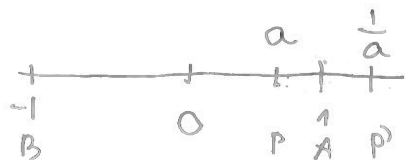


$$\overline{PO} \cdot \overline{P'O} = OA^2$$

P 's potens $\overline{PA} \cdot \overline{PB}$

$$\begin{aligned} &= (\overline{PO} + \overline{OA})(\overline{PO} + \overline{OB}) \\ &= \left(\frac{OA^2}{\overline{PO}} + OA\right) \left(\frac{OB^2}{\overline{PO}} + OB\right) \\ &= \frac{OA}{\overline{PO}} \cdot \frac{OB}{\overline{PO}} (\overline{OA} + \overline{PO})(\overline{OB} + \overline{PO}) \\ &= \frac{OA \cdot OB}{\overline{PO} \cdot \overline{PO}} \overline{PA} \cdot \overline{PB} \end{aligned}$$

Exempel



$$\frac{(1-a)(-1-a)}{-a} = \frac{(1-\frac{1}{a})(-1-\frac{1}{a})}{-\frac{1}{a}}$$

$$\frac{1-a^2}{a}$$

$$-\frac{1-(\frac{1}{a})^2}{\frac{1}{a}} = -a + \frac{1}{a} = \frac{1-a^2}{a}$$

Potens $(1-a)(-1-a) = (a-1)(a+1) = a^2 - 1$

dvs $a = 1 : \text{potens} = 0$

$a = 0 : \text{potens} = -1$