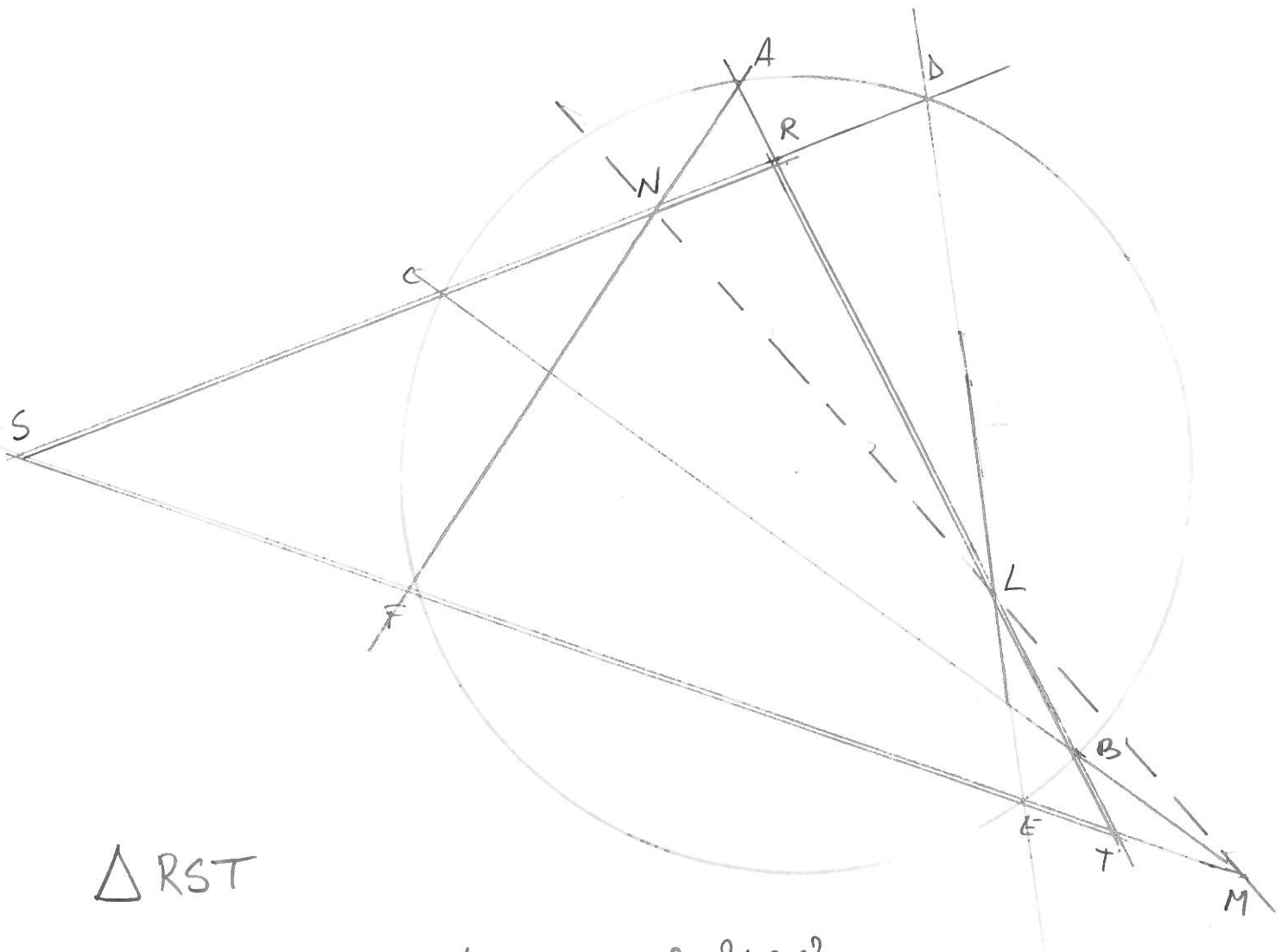




$\triangle RST$

Menelaos-tripler $\{E, L, D\}$ $\{F, A, N\}$ $\{M, B, C\}$

$$\text{gi} \quad \frac{\overline{SM}}{\overline{MT}} \cdot \frac{\overline{TL}}{\overline{LR}} \cdot \frac{\overline{RN}}{\overline{NS}} = - \frac{\overline{DS}}{\overline{RD}} \cdot \frac{\overline{ET}}{\overline{SE}} \cdot \frac{\overline{AR}}{\overline{TA}} \cdot \frac{\overline{FT}}{\overline{SF}} \cdot \frac{\overline{BR}}{\overline{TB}} \cdot \frac{\overline{CS}}{\overline{RC}}$$

Punkts potens:

$$\begin{aligned} \overline{RA} \cdot \overline{RB} &= \overline{RC} \cdot \overline{RD} \\ \overline{SC} \cdot \overline{SD} &= \overline{SE} \cdot \overline{SF} \\ \overline{TA} \cdot \overline{TB} &= \overline{TE} \cdot \overline{TF} \end{aligned}$$

-2	-3	-1	-3	-1	-2
1	2	3	2	3	1

$$= -1$$

$\Rightarrow$ Kolimpere M, L, N

År snakkes vi om?

Passer: Konstruere alle punkter med en gitt avstand til et fast punkt

Linjal: Konstruere linje gjennom to punkter

To linjer bestemmer punkt, linje og sirkel bestemmer 2 punkter

Gitt lengde 1: Kan konstruere alle rasjonale tall $\mathbb{Q}$ og alle tall av grad 2^m : $\mathbb{K}$

Konstruerbar linje: Gitt av to punkter: $\mathbb{K}^2$

Konstruerbar sirkel: Gitt av punkt i $\mathbb{K}^2$ og radius i $\mathbb{K}$.

Snitt av to konstruerbare linjer: punkt i $\mathbb{K}^2$

Snitt av konstruerbar sirkel og konstruerbar linje: 2 pkt i $\mathbb{K}^2$

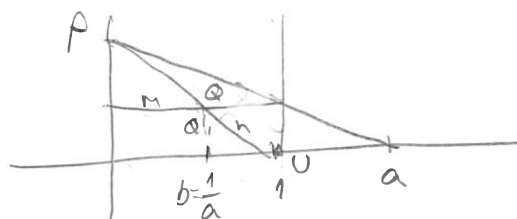
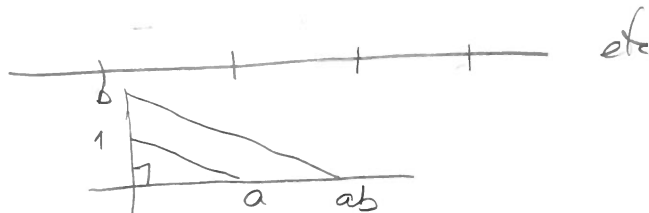
Snitt av to konstruerbare sirkler: 0-4 punkter i $\mathbb{K}^2$

Beskrivelse av $\mathbb{K}$: Noe starter med lengde 1

• Addisjon:

• Multiplikasjon:

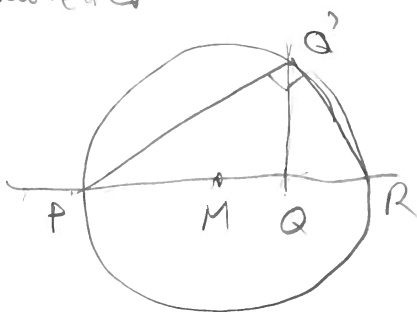
• Divisjon



$$\frac{b}{Pa} = \frac{1}{Pu} \quad \frac{PQ}{PU} = \frac{1}{a}$$
$$\Rightarrow b = \frac{PQ}{PU} = \frac{1}{a}$$

⇒ Kann Lage und Krümmung

• Kvadratröller



$$PQ = p$$

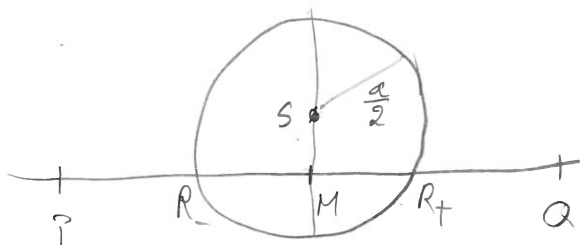
$$QR = q$$

$$\triangle PQQ' \sim \triangle Q'QR$$

$$\Rightarrow \frac{p}{QQ'} = \frac{QQ'}{q} \Rightarrow pq = (QQ')^2$$

$$\text{dus } QQ' = \sqrt{pq}$$

Kann lösen 2. gradsgleichung: $x^2 - ax + b^2 = 0$



$$PQ = a$$

$$PM = MQ$$

$$MS = b$$

$$\Rightarrow x_+ = R_+Q, x_- = R_-Q$$

Lösungen von $x^2 - ax + b^2 = 0$

$$R_+Q = MQ - MR_+$$

$$= \frac{a}{2} - \sqrt{SR_+^2 - SM^2} = \frac{a}{2} - \sqrt{\left(\frac{a}{2}\right)^2 - b^2}$$

$$= \frac{a - \sqrt{a^2 - 4b^2}}{2} \quad \text{rot: } x^2 - ax + b^2 = 0.$$

So Linie gegeben (x_1, y_1) und (x_2, y_2) : $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$$\text{dus } ax + by = c \quad a, b, c \in \mathbb{K}$$

Setztel $(x_0, y_0) \in \mathbb{K}$ Linie $ax + by = c$: (ante $b=1$: $y = c - ax$)

$$\text{mitt } (x - x_0)^2 + (y - y_0)^2 = (x - x_0)^2 + (c - ax - y_0)^2 = r^2$$

$$\Rightarrow x, y \in \mathbb{K}$$

Teorem

En regulær N -gon er konstruerbar hvis

$$N = 2^r p_1 \cdots p_s$$

hvor p_i er forskellige Fermat primtal, dvs $p_i = 2^{m_i} + 1$

Teorem

Et kompleks tal $z \in \mathbb{C}$ er konstruerbart (med passer og linjal) hvis rødkroppen $\mathbb{Q}(z)$ over $\mathbb{Q}$ har grad 2^m , dvs

$$\mathbb{Q} = K_0 \subseteq K_1 \subseteq \cdots \subseteq K^e = \mathbb{C}$$

hvor $K_i \subseteq K_{i+1}$ er kvadratisk $\mathbb{Q}(z)$

Basis: Konstruerbart $N = p_1^{m_1} \cdots p_s^{m_s}$ -gon $\Leftrightarrow$ konstruerbart $p_i^{m_i}$ -gon

Sett $\xi = e^{\frac{2\pi i}{p^m}}$. Da har vi

$$\deg(\mathbb{Q}(\xi)/\mathbb{Q}) = \varphi(p^m) = p^{m-1}(p-1)$$

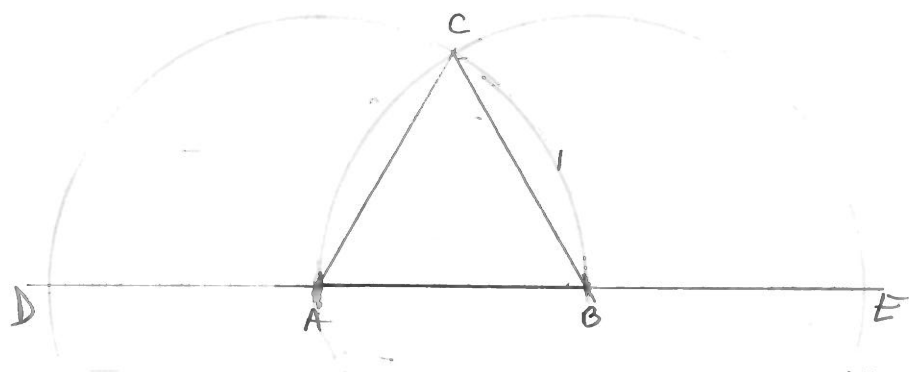
$$= 2$$

$$= 1 \quad p = 2^k + 1.$$

Infomat
NR-prisen

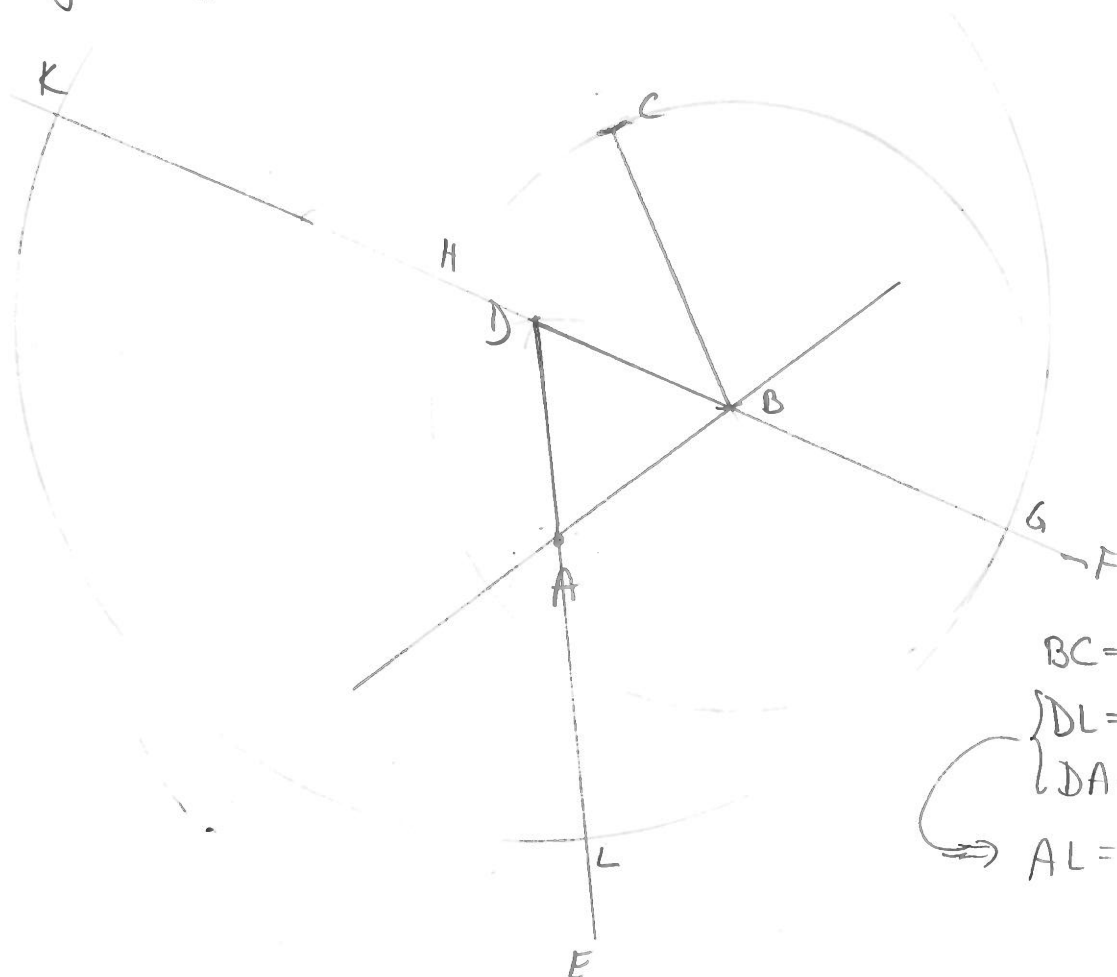
Flytte linjestykke med passer uten henholdsvis

1. Konstruere en likesidet trekant



$$\Rightarrow AC = AB, AB = BC \Rightarrow AC = AB$$

2. Flytte linjestykke til et gitt punkt



$$BC = BG$$

$$DL = DG$$

$$DA = DB$$

$$\Rightarrow AL = BG = BC$$

QED

Halvere ^{konstruierbar} Winkel
 $\cos \theta \in \mathbb{K}$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2\cos^2 \theta - 1$$

$$\Rightarrow \cos \theta = \sqrt{\frac{1 + \cos 2\theta}{2}}$$

Ikke tredele side $\cos 3\theta = \cos(2\theta + \theta)$

$$= \cos 2\theta \cdot \cos \theta - \sin 2\theta \cdot \sin \theta$$

$$= 2\cos^2 \theta - \cos \theta - 2(1 - \cos^2 \theta) \cos \theta$$

$$= 4\cos^3 \theta - 3\cos \theta = a \in \mathbb{K}$$

Generelt $4x^3 - 3x - a = 0$ irreduktibel
 3. grad 3

Kar g $\circ$: $3\theta = \frac{\pi}{2} \Rightarrow \cos 3\theta = 0$ som bet₃

$$4x^3 - 3x = x(4x^2 - 3) = 0 \quad \text{oh}$$

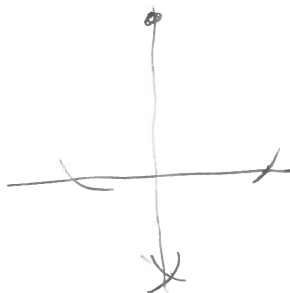
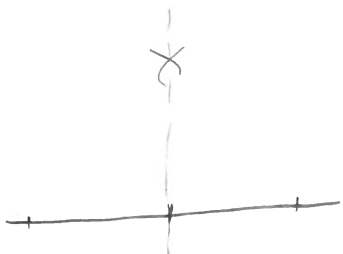
$3\theta = \pi \Rightarrow \cos 3\theta = -1$ som gn

$$4x^3 - 3x + 1 = 0$$

$$(2x-1)(2x^2+x-1)$$



Oppreise normal / nedfelle normal / konstruere parallell



Parallell: To gange normal

The punkter bestemmer stillet (a_1, b_1) (a_2, b_2) (a_3, b_3)

$$(x-a_1)^2 + (y-b_1)^2 = (x-a_2)^2 + (y-b_2)^2 = (x-a_3)^2 + (y-b_3)^2$$

↑ ↑
to ligninger

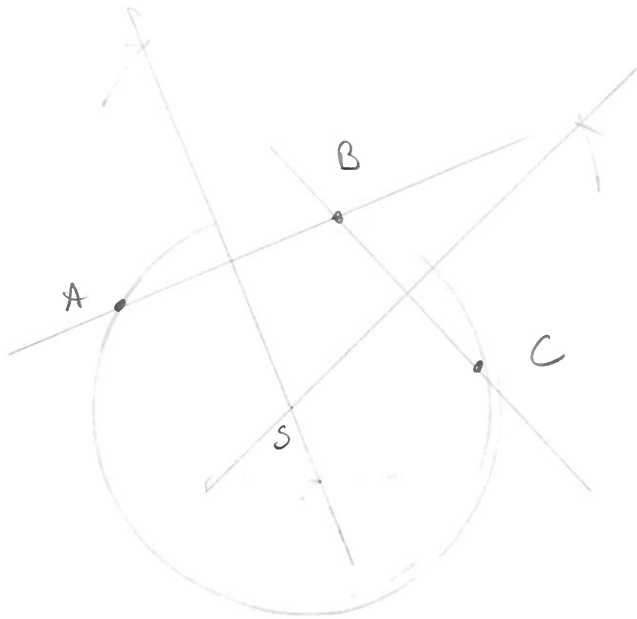
Eks. $(a_3, b_3) = (0, 0)$

$$(x-a_1)^2 + (y-b_1)^2 = x^2 - 2a_1x + a_1^2 + y^2 - 2b_1y + b_1^2 = x^2 + y^2$$

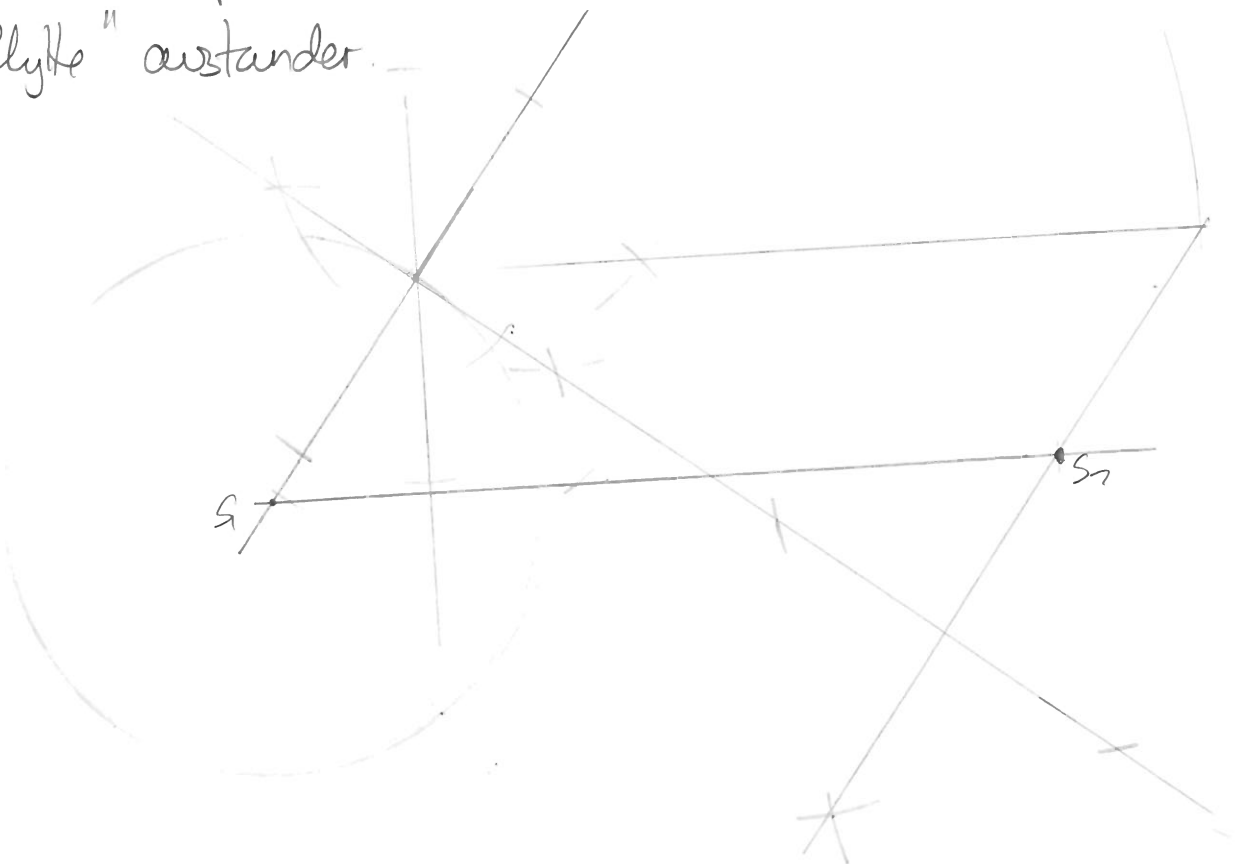
$$\text{dvs } 2a_1x + 2b_1y = a_1^2 + b_1^2$$

$$2a_2x + 2b_2y = a_2^2 + b_2^2$$

gør (x, y)

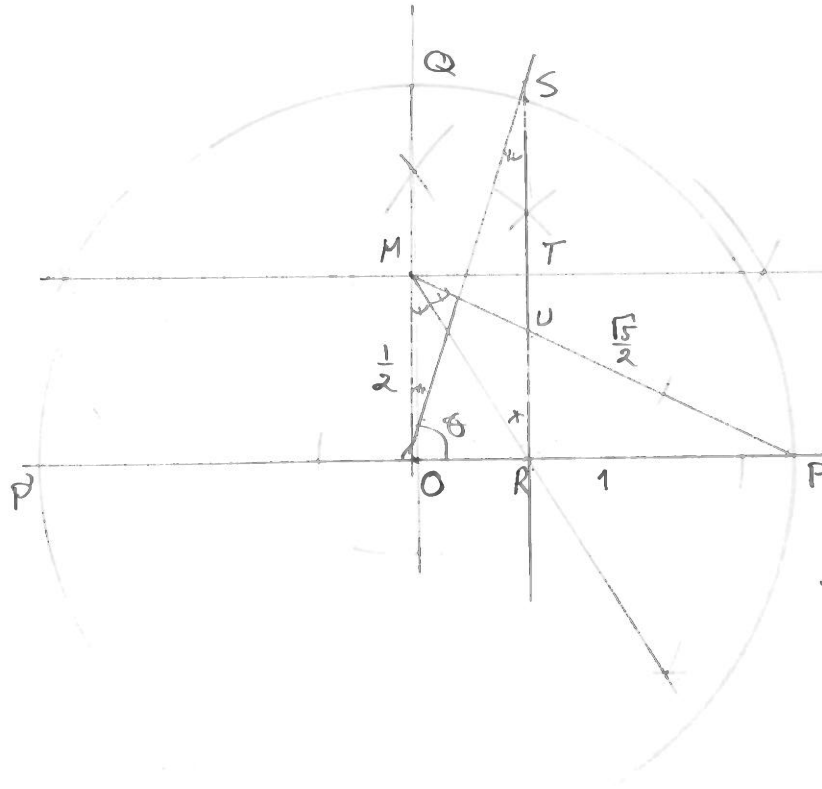


Hva hvis passerer ikke har hukommelse? Kan ikke
"flytte" afstand.



Konstruere regulär 5-kant

$$\cos \frac{2\pi}{5} = \frac{-1 + \sqrt{5}}{4}$$



$$\frac{OR}{RB} = \frac{\frac{1}{2}}{\frac{\sqrt{5}}{2}} = \frac{1}{\sqrt{5}}$$

$$\theta = \angle POS = \frac{2\pi}{5}$$

for 1) $\cos \theta = \frac{OR}{OS} = \frac{\frac{1}{1+\sqrt{5}}}{1}$
 $= \frac{\sqrt{5}-1}{5-1} = \frac{\sqrt{5}-1}{4}$